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DO BLOOD CAPILLARIES EXHIBIT OPTIMAL BUMPINESS?

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Abstract

A recent analytical solution of the three-dimensional Stokes flow through a bumpy tube predicts that for a given bump area, there exists an optimal circumferential wavenumber which minimizes flow resistance. Comparison of this theory with representative histological measurements from human tissue sections reveals that the geometry of normal human capillaries and arterioles indeed lies quite close to this predicted optimal configuration. This raises the interesting question of whether less well-developed circulatory systems, such as that found in newly formed solid tumors or during embryonic development, may deviate from the optimal bumpiness predicted to minimize flow resistance.

Keywords

capillaries; arterioles; Stokes flow; flow resistance

Blood flow in mammalian organisms proceeds through the vascular tree from the heart to the aorta, large arteries, main branches, terminal arteries, arterioles, to the capillaries. On the venular side the vessel diameters continually increase from the capillaries to the venules, terminal veins, main branches, large veins, back to the heart. The capillary bed, with its vastly increased total surface area relative to the larger vessels, is crucial for efficient oxygen and nutrient transport, and temperature regulation. The human body contains millions of arterioles and billions of individual capillaries. Comparison of typical blood flow velocities, vessel diameters, and simple application of Poiseuille's law reveals that the arterioles and capillaries contribute the majority of the resistance to flow within the vascular bed (Cooney, 1976). A recent closed form solution of the three-dimensional Stokes flow through a bumpy tube (Wang, 2006) allows an analysis of whether the geometry of human capillaries and arterioles exist in an optimized state to maximize blood flow for a given pressure drop, or whether some additional evolutionary factor may have led to their current shape. In this Letter to the Editor I apply the Stokes flow solution of Wang (2006) to representative histological measurements of blood microvessels to explore how well the theory of bumpy tube flow predicts experimentally observed vessel shapes.

1. Three-dimensional Stokes flow in a bumpy tube

Recently, Wang (2006) considered the three-dimensional Stokes flow in a periodic bumpy-walled tube. Previous analytical investigations considered tubes with wall corrugations in one direction only, either in the longitudinal (Shah and London, 1978; Phan-Thien, 1981) or

transverse (Deiber and Schowalter, 1979; Phan-Thien, 1980) direction. The model of Wang consists of a tube wall with periodic bumps described by the nondimensionalized equation

$$r = 1 + \varepsilon \sin(n\theta) \sin(\alpha z) \quad (1)$$

where the radial (r) and longitudinal (z) spatial variables have been scaled by the mean tube radius a , the bump amplitude is $\varepsilon = b/a \ll 1$, n is the circumferential wavenumber, $\alpha = 2\pi a/l \neq 0$, and l is the longitudinal wavelength. By performing a regular perturbation in the small parameter ε , Wang was able to obtain a closed form solution for the velocity distribution in terms of modified Bessel functions up to the second order correction to the mean flow rate. Interestingly, this solution predicts that for a given dimensionless bump area $A = \pi^2/n\alpha$, there exists an optimal circumferential wavenumber n for which the flow resistance is minimized (or, equivalently, the flow rate maximized for a given total pressure drop). In the next section, I compare these predicted optimal bump size and spacings to representative experimental measurements of real human microvessels obtained from the histology literature, to determine how closely the Stokes flow solution correctly anticipates the vessel geometry.

2. Comparison of the Stokes flow theory to representative histological measurements

A preliminary review of human histology micrographs available in the literature yields some geometric measurements for comparison with the Stokes flow theory. Most available images of intact capillaries and arterioles are in the form of cross-sectional slices at a fixed longitudinal position, although some cross-sectional images along the longitudinal axis are available to provide the necessary dimension l (Bevelander and Ramaley, 1974; Bloom and Fawcett, 1975; Kelly, Wood and Enders, 1984). Capillaries can be accurately represented as a quasi-periodic bumpy tube with circumferential wavenumber of $n = 1$, since in these smallest blood vessels a single endothelial cell wraps around the entire tube circumference and the endothelial cell nucleus protrudes into the vessel lumen to provide the occasional large bump. For capillaries of inner diameter ranging from $2a = 5.86\text{--}11.6\text{ }\mu\text{m}$, average longitudinal wavelengths of $l = 38.5\text{--}48.4\text{ }\mu\text{m}$ were measured. Arterioles, on the other hand, are the somewhat larger microvessels that feed into the capillary bed, and they appear with ruffled inner surfaces comprised of a well defined circumferential and longitudinal periodicity. Arterioles with inner diameter of $2a = 35.3\text{--}40.0\text{ }\mu\text{m}$ were characterized with circumferential wavenumbers of $n = 26\text{--}25$ (respectively) and circumferential-to-longitudinal bump aspect ratio of 2.38. The post-capillary venules, the other vessel type for which the non-inertial assumptions of Stokes flow would also apply, do not exhibit any well-defined circumferential or longitudinal periodicity and thus are not considered further in this report. Typical three-dimensional tube surfaces utilizing the capillary and arteriolar histological measurements are plotted in Figure 1, which appear as accurate mathematical representations of the true vessel shapes.

When the optimal circumferential wavenumber (i.e., that which minimizes flow resistance) is plotted as a function of the dimensionless bump area A , a remarkable level of agreement is observed between the theoretically predicted optimal geometry and the experimentally determined microvessel geometries. Thus, one may conclude from this analysis that the capillaries and arterioles in normal human tissues, that represent over two-thirds of the total flow resistance through the entire vascular network, exhibit a three-dimensional bumpiness which minimizes the flow resistance through these vessels.

3. Conclusions

In this Letter to the Editor, I have compared the predictions of the Stokes flow theory for three-dimensional bumpy tubes to histological measurements of human capillaries and arterioles to

find that these microvessels exhibit nearly optimal geometry for minimizing the overall flow resistance. Although these are preliminary findings and do not represent a comprehensive analysis, they do raise some intriguing questions which deserve further study. For instance, it would be interesting to more fully characterize and compare the microvessel geometry measured from different organs to determine whether any significant differences exist. Similarly, the capillaries of lower organisms such as amphibians and invertebrates could be compared to test whether they are also optimized for flow resistance or are less efficient. Solid tumors induce rapid ingrowth of microcapillaries through the cellular production and release of biochemical angiogenic factors. It has been well documented that the overall structure of tumor microvessels is less well organized than in normal tissue, with larger pores that make the capillaries in tumors more leaky to fluid and macromolecules. Future work could focus on the quantification of these cancerous microvessel geometries to examine whether such newly formed, “unnatural” capillaries fall outside of the optimal bump configuration predicted from Stokes flow theory. The transport characteristics of solid tumor microvessels are particularly important since, somewhat counterintuitively, one must provide adequate oxygen supply to such tumors to deliver effective radiotherapy. Finally, the theoretical result that there exists an optimal circumferential wavenumber which minimizes the flow resistance in capillaries and arterioles with bumpy walls has important implications in the field of tissue engineering. Specifically, there is great current interest in engineering vascularized tissue implants to replace damaged tissues such as ischemic heart muscle, spinal cord, or the diseased liver. Considering that nanostructured and microstructured artificial surfaces can be used to control cell morphology and multicellular organization (see, for instance, King, 2006), arranging optimally placed bumps may represent a powerful tool for the creation of artificial capillary beds which do not place unnecessarily large demands on the host cardiovascular system.

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Figure 1.

Left: representative capillary geometry with $n = 1$, $\varepsilon = 0.15$, and $\alpha = 0.5$. Right: representative arteriolar geometry with $n = 25$, $\varepsilon = 0.12$, and $\alpha = 10$.

Figure 2.
Comparison of the Stokes theory for three-dimensional bumpy tube flow with experimentally determined microvessel geometries.