A POSSIBLE RELATION BETWEEN COSMOLOGICAL QUANTITIES AND THE CHARACTERISTICS OF ELEMENTARY PARTICLES

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In 1937, Dirac¹ proposed that the close numerical agreement of the ratio of the electrical to gravitational forces between a proton and an electron with the age of the universe expressed in atomic time units might not be a coincidence. He suggested that the agreement implied some deep relation between the elementary charge and masses of the fundamental particles of matter on the one hand and the large-scale cosmological behavior of the universe on the other hand. The former ratio is

$$e^2/(Gm_P m_e) = 0.23 \times 10^{40}, \quad (1)$$

whereas the latter is

$$t/\tau = 0.67 \times 10^{40}, \quad (2)$$

if the age of the universe is taken as ~ 2 eons and the unit of time $\tau = 1$ tempon $= e^2/(m_e c^3) = 0.94 \times 10^{-23}$ sec. Dirac inferred from this near equality that the gravitational constant might be expected to vary inversely as the epoch unless one is willing to assign a peculiar significance to the present age value. Since Dirac's proposal, a number of studies² have indicated that there are serious difficulties with respect to geological and astronomical evidence unless the gravitational constant is in fact constant through the history of the universe.

It is the purpose of this paper to indicate that there is another relation between the characteristics of the universe and the ratio of the electrical to gravitational forces between elementary particles. Moreover, this new relation does not imply any temporal variation of fundamental constants. According to calculations developed in a recent paper by Alpher, Gamow, and Herman,³ if one has an adiabatically expanding homogeneous, isotropic universe whose present properties agree with observation, then the mass densities of matter and of radiation, which vary differently with epoch, would have been equal at a time

$$t_c \cong 10^6 \text{ years} \cong 3 \times 10^{36} \text{ tempons.} \quad (3)$$

The time t_c appears to be a significant constant in models of the universe as the epoch of initiation and growth of perturbations leading to structural differentiation.⁴ In the present context we note that the value of t_c is very close to the value

$$e^2/(Gm_P^2) \cong 1 \times 10^{36}, \quad (4)$$

representing the ratio of electrical to gravitational forces between protons. Since t_c is in fact a constant, one is dealing with the equality of constants in equations (3) and (4) rather than with variables as in Dirac's work, so that temporal variation of physical constants is not involved. Again, however, as with

Dirac's observation, either this is all a remarkable coincidence or else it has much deeper significance.

In the work described in reference 3, another dimensionless constant characteristic of the cosmological model of the universe was developed, namely, the ratio C_r/C_m of the heat capacities of radiation to that of matter. As noted there, one can also interpret this ratio in terms of the ratio of pressures of radiation and matter, with $p_r/p_m = (1/8)C_r/C_m$, or the dimensionless entropy per baryon, with $S/(nk) = (1/2)C_r/C_m$, or as the number of photons per baryon, with $n_{ph}/n \cong (1/7)C_r/C_m$. It was shown that, except for very early times in the expansion, this ratio remained constant throughout the expansion with a numerical value of $\sim 10^{10}$ for reasonable choices of parameters describing the model of the universe. If C_r/C_m is a characteristic cosmological constant, then one should be able to obtain its value of $\sim 10^{10}$ from the number 10^{36} .

This can in fact be accomplished in the following way (we use the notation of ref. 3). One writes the heat capacities of radiation and of matter as

$$C_r = 4aT^3 \text{ ergs}/(\text{cm}^3 \text{ deg K}) \quad (5)$$

and

$$C_m = (3/2)nk \text{ ergs}/(\text{cm}^3 \text{ deg K}), \quad (6)$$

respectively, where $a = 8\pi^5 k^4/(15c^3 h^3) \text{ ergs}/(\text{cm}^3 \text{ deg K}^4)$ is the radiation density constant, T the temperature of matter and radiation, and n the number density of particles (we consider only the baryons present). In the adiabatic expansion of the universe,

$$n = (\rho_m/m_P)L^{-3} \text{ cm}^{-3}, \quad (7)$$

where ρ_m is the present density of matter, L is proper distance normalized to unity at the present epoch, and

$$T = T_0 L^{-1} = (c^2 \rho_r/a)^{1/4} L^{-1} \text{ }^\circ\text{K}, \quad (8)$$

where T_0 and ρ_r are the present temperature and radiation mass density, respectively. Substituting (7) and (8) into (5) and (6) yields

$$C_r/C_m = [8a^{1/4}c^{3/2}m_P/(3k)]\rho_r^{3/4}\rho_m^{-1} = 4.945 \times 10^4 \rho_r^{3/4}\rho_m^{-1}. \quad (9)$$

But since $\rho_r \propto L^{-4}$ and $\rho_m \propto L^{-3}$, then

$$\rho_r^{3/4}\rho_m^{-1} = \text{const.} = \rho_r^{3/4}\rho_m^{-1} \quad (10)$$

during the expansion, and C_r/C_m is truly a constant characterizing the cosmology through present matter and radiation densities and universal constants.

In reference 3, a solution of the equations describing an open Friedmann-Lemaitre model was presented. This solution relates the epoch of the expansion, t , to proper distance, L , with present mass densities of matter and radiation, ρ_m and ρ_r , as well as Hubble's constant, H_0 , as parameters. This rather cumbersome relation reduces for early times in the expansion to

$$t_{\text{early}} \cong (32\pi G \rho_r/3)^{-1/2} L^2 \text{ sec}, \quad (11)$$

while for late times in the expansion

$$t_{\text{late}} \cong [H_0^2 - (8\pi G/3)(\rho_{m^*} + \rho_{r^*})]^{-1/2} L \quad \text{sec.} \quad (12)$$

With the present epoch defined by $L = 1$, and with reasonable values for ρ_{m^*} and ρ_{r^*} , equation (12) yields the present age of the universe as

$$t_0 \cong H_0^{-1}. \quad (13)$$

This does not provide any obvious entree for replacing quantities in C_r/C_m (in (9)), with t_0 . On the other hand, we suggest that (11) gives a valid approximation to the time of crossover, as defined by $L = \rho_{r^*}/\rho_{m^*}$, so that

$$t_c \cong (32\pi G/3)^{-1/2} \rho_{r^*}^{3/2} \rho_{m^*}^{-2}. \quad (14)$$

This substituted in (9) yields

$$C_r/C_m = [8a^{1/2} c^{3/2} m_P / (3k)] (32\pi G/3)^{1/4} t_c^{1/2} = 1.912 \times 10^3 t_c^{1/2}, \quad t_c \text{ in sec,} \quad (15)$$

or, with t_c in tempons and replacing the radiation density constant,

$$\begin{aligned} C_r/C_m &= [2^5 \pi^{3/2} G^{1/4} e m_P / (3^{3/2} 5^{1/4} c^{3/4} h^{3/4} m_e^{1/2})] t_c^{1/2} \\ &= 5.863 \times 10^{-9} t_c^{1/2}, \quad t_c \text{ in tempons.} \end{aligned} \quad (16)$$

To check that (15) is in fact valid despite the approximation in (14), we have calculated the present age, t_0 , and the crossover time, t_c , without approximation from equation (9) of reference 3 for a range of parameters $50 \leq H_0 (\text{km/sec/Mpc}) \leq 150$, $10^{-31} \leq \rho_{m^*} (\text{gm/cm}^3) \leq (\rho_{m^*})_{cl}$, and $2 \leq T_{\text{now}} (^\circ\text{K}) \leq 4$, where $(\rho_{m^*})_{cl}$ is the upper limit of ρ_{m^*} for an open model of the universe.⁵ A plot of $\log C_r/C_m$ calculated from equation (9) of this paper versus $\log (t_0/t_c)$, calculated as just indicated, yields a family of straight lines which can be represented by

$$C_r/C_m = \text{const.} \times H_0^{-1/2} (t_c/t_0)^{1/2}, \quad (17)$$

while a plot of $\log (C_r/C_m)$ from equation (9) versus $\log (t_c)$, again from reference 3, yields essentially a single straight line, as shown in Figure 1, which can be represented by

$$C_r/C_m = 6 \times 10^9 t_c^{1/2}, \quad t_c \text{ in tempons.} \quad (18)$$

Since $t_0 \cong H_0^{-1}$, both H_0 and t_0 drop out of equation (17), and equations (18) and (17) are equivalent. Clearly the approximation for t_c is excellent so long as $t_c \gtrsim 10^{15}$, which is well above the value likely to be correct for the universe.

Thus, we have the remarkable result that on the one hand, the cosmological quantity t_c , the matter-radiation density crossover time in atomic units, is numerically equal to $e^2/(Gm_P^2)$, a quantity characteristic of elementary particles. On the other hand, the independent cosmological quantity C_r/C_m , which is clearly a critical constant in singling out a particular model from all possible models of the expanding universe, is also uniquely described in terms of t_c and thence $e^2/(Gm_P^2)$.

The significance of this close relation between universal macro- and micro-quantities may lie in the fact that it states that the universe emerged from its

FIG. 1.—Ratio of heat capacities of radiation and matter in an open expanding universe versus the epoch of crossover of matter and radiation mass densities, computed per equation (9) of this paper and of reference 3, respectively, with $2 \leq T_{\text{now}}(^{\circ}\text{K}) \leq 4$, $10^{-31} \leq \rho_m(\text{gm/cm}^3) \leq (\rho_m)_{cl}$ where $(\rho_m)_{cl}$ is the density above which the model is closed (for $H_0 = 100 \text{ km/sec/Mpc}$, $T = 3^{\circ}\text{K}$, $(\rho_m)_{cl} \cong 1.8 \times 10^{-29} \text{ gm/cm}^3$).

early stages with $\sim 10^9$ photons per baryon and that it infers that the physics of the interactions between baryon pairs and high-energy photons should yield such a number from first principles.

Summary.—The time of equality of mass densities of matter and radiation in the expanding universe, when expressed in atomic time units, is found to be numerically equal to the ratio of electrical to gravitational forces between protons. Moreover, it is shown to be possible to obtain the value of the ratio of heat capacities of radiation to that of matter from this time of equality of densities and thence to relate to the ratio of forces between protons. Since the heat capacity ratio singles out a particular model of the universe, one then has a unique relation between quantities characteristic of the universe and of elementary particles.

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* *Editorial note:* We regret the necessity of recording that Professor Gamow died on August 20, 1968, shortly after this paper had been communicated to these PROCEEDINGS.

¹ Dirac, P. A. M., *Nature*, **139**, 323 (1937); *Proc. Roy. Soc. (London)*, **A165**, 198 (1938).

² Teller, E., *Phys. Rev.*, **73**, 801 (1948); Pochoda, P., and M. Schwarzschild, *Ap. J.*, **139**, 587 (1964); Gamow, G., these PROCEEDINGS, **57**, 187 (1967).

³ Alpher, R. A., G. Gamow, and R. Herman, these PROCEEDINGS, **58**, 2179 (1967).

⁴ Field, G. B., and L. C. Shepley, *Astrophys. Sp. Sci.*, **1**, 309 (1968); Alpher, R. A., and R. Herman, General Electric Research and Development Center Rept. 68-C-307, 1968.

⁵ While attention is restricted here to open models as suggested by current values $T_{\text{now}} = 3^{\circ}\text{K}$, $\delta_m = 7 \times 10^{-31} \text{ gm/cm}^3$ and $H_0 = 50$ to 75 km/sec/Mpc , the conclusions hold for Newtonian models and for closed models with δ_m only slightly greater than $(\delta_m)_{cl}$. For the Newtonian case one obtains (15) and (16) with an added numerical factor of $(6\pi G)^{1/4}/(2^{3/2} - 18^{1/2} + 2)^{1/2}$.