INCOMPRESSIBILITY AT THE EARTH'S MANTLE-CORE BOUNDARY

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(1) *Introduction.*—In this paper, the values of variables at the base of the mantle and top of the core in the Earth models to be considered will be indicated by the subscripts 1 and 2, respectively; k and ρ will denote the incompressibility and density; let $\Delta k = k_2 - k_1$ and $\Delta \rho = \rho_2 - \rho_1$.

In an earlier Earth model,¹ Model A, $k_1 = 6.5 \times 10^{12}$ dyn/cm² and $k_2 = 6.2 \times 10^{12}$ dyn/cm², giving $\Delta k = -0.3 \times 10^{12}$ dyn/cm² and $\Delta k/k_1 \approx -5$ per cent. The values of k in Model A are derived from the density distribution and the seismically determined distributions of the P and S velocities α and β . These values of k do not depend on any theory or additional observational evidence relating to compressibility in the Earth. The sources of error contributing to the determination of Δk on the Model A procedures are listed in § (2).

The fact that $\Delta k/k_1$ is so small in Model A, along with certain other evidence,^{2, 3} led to the formulation of a compressibility-pressure hypothesis, one important part of which is that k varies continuously through the Earth, as well as smoothly with respect to the pressure p , below 1000 km depth. The hypothesis need be considered to hold only as a useful first approximation to actual conditions in the Earth, but even so has several important implications, including, for example, being instrumental in providing the initial evidence that the Earth's inner core has significant rigidity. In 1950 an Earth model B was constructed⁴ in which the hypothesis was taken as a central feature, and which exhibits various characteristics of an Earth with a solid inner core.

As a consequence of additions to older seismic data, including observations of the orbits of artificial satellites and of the periods of free Earth oscillations excited by the Chilean and Alaskan earthquakes of 1960 and 1964, as well as other evidence also essentially unconnected with compressibility theory, it is now possible to examine the assessment of Δk fairly closely, and so test one of the important items of evidence on which the k - p hypothesis rests. The purpose of the present paper is to examine the amendments needed to the previously assessed Δk and, in particular, to show how on current evidence k appears indeed⁵ to be practically continuous at the mantle-core boundary.

(2) *Corrections to Model A Estimate of Δk .*—The Model A estimate of Δk is derived from the values taken for α and β and the values inferred for ρ over the period 1940–42 in the vicinity of the mantle-core boundary, N say. In the present paper, except to a small extent in § (6), errors in α and β near N will be disregarded as being probably small in their effect on Δk compared with other errors in the Model A structure.

The values of ρ near N are tied up with the determination of the density distribution of the whole Earth. Thus corrections to the Model A estimate of Δk arise principally from corrections to the Earth's density distribution. The main sources of corrections needing to be considered are:

- (i) revision of the estimated value of y , where $I = yMR^2$, I , M and R being the moment of inertia, the mass, and the radius of the Earth;
- (ii) allowance for abnormalities of the region D'' ($2700 < z < 2900$ km (approx.), where z denotes the depth below the surface);
- (iii) revision of the P velocity and density distributions in the lower core ($z > 4500$ km, approx.);
- (iv) revision of the estimated radius R_c of the Earth's core; and
- (v) modifications to the structure assumed for Model A in the regions A , B and C ($0 < z < 1000$ km, approx.).

For convenience, the separate contributions to $\Delta k/k_1$ arising from (i) to (v) will be denoted as $(\Delta k/k_1)_y$, $(\Delta k/k_1)_{D''}$, $(\Delta k/k_1)_{LC}$, $(\Delta k/k_1)_{Rc}$, $(\Delta k/k_1)_{ABC}$, respectively.

These five contributions are examined in §§ (3)–(7). Section (8) contains a brief consideration of other possible contributions to changes in $\Delta k/k_1$.

The examination is facilitated by using certain tables in a previous paper,⁶ to be referred to as Paper I. Each of these tables relates to a particular Earth model, and it will be convenient in the present paper to refer to the models of Paper I by their table numbers; thus, e.g., "Model 17" is specified by Table 17 of Paper I.

Reference will later be made to the coefficient η in the density gradient relation

$$d\rho/dz = \eta g \rho / \phi, \quad (1)$$

where g denotes the gravitational intensity and $\phi = \alpha^2 - 4\beta^2/3$.

In § (9), attention is drawn to direct evidence on Δk from the Earth model HB_1 (refs. 8 and 9). Conclusions are summarized and discussed in § (10).

(3) *Effect of the Revised Moment of Inertia.*—The Model A value 0.3335 of y has to be corrected¹⁰ to 0.3309 as a consequence of observations of artificial-satellite orbits. Reducing y is expected to result in some transfer of mass from the mantle to the core. Hence the correction will be expected to increase the Model A value of $\Delta\rho$, and hence also of Δk , α and β being kept unchanged. This in fact is found to be the case.

The magnitude of $(\Delta k/k_1)_y$ can be estimated using Models 17 and 18 of Paper I. (Models 10 and 11 would do equally well.) These two models have certain common principles in their construction, including the stipulation that Δk is zero. The only stipulated difference is that $y = 0.3309$ in 17, and 0.3335 in 18. In conjunction with the stipulation that $\Delta k = 0$ in both models, this results in markedly different density variations in the upper mantle. For the purposes of the present comparison, we retain the stated different values of y in the two models, but deviate from the stipulation $\Delta k = 0$ in Model 18 to the extent necessary to make its upper mantle structure agree essentially with that of Model 17. The changed value of Δk gives the correction required. The Models 17 and 18 were selected because they belong to a class which has a particularly simple form of mantle structure. This consists principally of a region B in which ρ varies fairly smoothly, separated by a simple finite discontinuity in ρ from a combined C - D region throughout which ρ again varies fairly smoothly. This structure is

sufficiently complex for the immediate purpose, and sufficiently simple to enable Δk to be readily computed.

The first step is to replace the Model 18 distribution of ρ for $0 < z < 515$ km by that in Model 17, thereby lowering the depth of the Model 18 upper-mantle discontinuity in ρ from 298 km to 515 km. The remaining step is to determine average density increments in the mantle below 515 km and in the core, which will restore the correct values of M and I . The two steps result in an increase of 0.12 gm/cm^3 , or 2.2 per cent, in the original Model 18 value of ρ_1 , and a decrease of 0.185 gm/cm^3 , or 1.9 per cent, in ρ_2 . Since α and β are being kept unchanged, the same proportional changes are required in k_1 and k_2 . Since also $k_1 \approx k_2$, we obtain $(\Delta k/k_1)_y \approx +4.1$ per cent.

It is desirable to check this result with other types of models. Direct comparison of the values of k_1 and k_2 in the Earth models^{1, 8} A' and A'' , which also differ only in having $y = 0.3335, 0.3309$, respectively, is found to give $(\Delta k/k_1)_y \approx +4.3$ per cent. (Values of k (and other variables) for A'' are to be published in a later paper.) The uncertainty in taking $(\Delta k/k_1)_y = +4.2$ per cent is probably not greater than about 0.2 per cent.

(4) *Effect of the Region D'' .*—In Model A , the density increment inside D'' is 0.10 gm/cm^3 and η is taken equal to unity. The observational near-zero gradients of α and β inside D'' , however, require⁷ that $\eta_{D''} \approx 3.0$ and hence that the Model A value of ρ_1 be increased by 0.20 gm/cm^3 , or 3.5 per cent. The abnormal density changes inside D'' have no significant effects outside D'' , so that ρ_2 is not sensibly affected. Hence $(\Delta k/k_1)_{D''} \approx -3.5$ per cent. The only uncertainty here needing possible future consideration is in the observational values of da/dz and $d\beta/dz$ in the lowest few hundred kilometers of the mantle.

(5) *Effect of Amendments to α and ρ in the Lower Core.*—In Model A , the central density ρ_0 was (arbitrarily) taken as 17.3 gm/cm^3 , whereas the favored value now is of the order of 13 gm/cm^3 . Values of α in the lower core have also been substantially revised.¹¹⁻¹³ The immediate effect on Δk of reducing ρ_0 can be estimated using Models 10 and 17 of Paper I, for which $\rho_0 = 15.08$ and 13.00 , respectively. Proceeding after the fashion in § (3), we find that when the upper mantle structure is kept essentially the same, $\Delta k/k_1$ is increased by about 1.0 per cent when ρ_0 is reduced from 15.0 to 13.0 gm/cm^3 . On a simple proportional basis, this would give $(\Delta k/k_1)_{LC} \approx 2.2$ per cent, corresponding to a change in ρ_0 from 17.3 to 13.0 gm/cm^3 . This estimate of $(\Delta k/k_1)_{LC}$ needs, however, to be increased because of the substantial revision of the P velocity distribution in the lower core; the revision causes particularly large changes from the density variation in the region F' ($4980 < z < 5120$ km) of Model A . An independent comparison of Models A and A' (in Model A' , $\rho_0 = 12.3 \text{ gm/cm}^3$) leads to a preferred estimate of about +2.8 per cent for $(\Delta k/k_1)_{LC}$ when 13.0 gm/cm^3 is taken as the preferred value of ρ_0 . The principal uncertainty here probably arises from the uncertainty in ρ_0 . (Taking 14.0 instead of 13.0 gm/cm^3 for ρ_0 would reduce the estimate to about +2.2 per cent; taking the Model HB_1 value⁹ of 12.46 gm/cm^3 for ρ_0 would give about +3.1 per cent.)

(6) *Effect of Change in the Core Radius.*—Model 17 of Paper I has $R_c = 2878$ km, $\Delta\rho = 4.20 \text{ gm/cm}^3$, and $\Delta k = 0$. Consider a modified model in which R_c is

changed to 2898 km and the densities are increased from the Model 17 values by X , $X - 4.20$ gm/cm³, and Y in the ranges $515 < z < 2878$, $2878 < z < 2898$, and $z > 2898$ km, respectively, X and Y being determined so as to keep M and I unchanged. We find X , $Y = +0.006$, $+0.052$ gm/cm³. The increases in ρ_1 , ρ_2 for the modified model are $X + X'$, $Y + Y'$, where X' , Y' are allowances for the change in the depth at which ρ_1 , ρ_2 are estimated. Using the Model 17 values of $d\rho/dz$ in the region D and at the top of the core, we obtain X' , $Y' = +0.031$, $+0.030$ gm/cm³. The increases in ρ_1 , ρ_2 are therefore 0.037, 0.082 gm/cm³, or 0.65, 0.83 per cent, respectively. The corresponding contribution to $(\Delta k/k_1)_{Rc}$ is thus about -0.2 per cent per 20-km increase in R_c .

In addition to the direct density effect, changing R_c requires adjustments to be made to the over-all P velocity distribution in order to keep to a minimum consequential changes to the travel times of seismic phases such as PKP . The adjustments may include changes in α near the boundary N , more particularly on the core side. The adjustments made for this purpose in constructing Model HB_1 would give an additional contribution of order $+0.5$ per cent to $(\Delta k/k_1)_{Rc}$; the actual contribution could possibly be a little greater.

Hence $(\Delta k/k_1)_{Rc}$ may be taken as zero, with an uncertainty of order 0.5 per cent.

(7) *Effect of Changes in the Crust and Upper Mantle.*—Observations of free Earth oscillations indicate that for spherically symmetrical model representations of the Earth, it is more suitable⁸ to take the thickness of the crust (region A) as 15 km than as the continental value of 33 km used in Model A . Thus the Model A values of ρ need to be increased by about 0.5 gm/cm³ for $15 < z < 33$ km, with appropriate adjustments below $z = 33$ km. The resultant effect on $\Delta k/k_1$ is found to be a contribution of $+0.3$ per cent.

The free Earth oscillation data also⁸ require some density changes inside the regions B and C , including an appreciable reduction in the Model A value of unity taken for η inside most of the region B . It is not easy to compute the consequent correction to $\Delta k/k_1$ precisely, but scrutiny of Tables 3 and 12 of Paper I indicates that it is small, probably not more than $+0.2$ per cent.

Thus $(\Delta k/k_1)_{ABC} \approx +0.5$ per cent, with uncertainty about 0.2 per cent.

(8) *Other Contributions to Δk .*—The detailed density distribution in the lower core depends to some extent on the degree of reliability of the k - p hypothesis^{2, 3} in this part of the Earth. Deviations from continuity of k between the outer and inner core are likely to be small for two reasons. First, such deviations are not expected to exceed any deviation at the mantle-core boundary, and the present paper indicates that Δk is in fact very small. Secondly, there is independent evidence¹⁴ that any increase in k between outer and inner core (above that due to smooth variation) is unlikely to exceed 5 per cent. Such an increase would have negligible effect on the estimate of $\Delta k/k_1$.

Some uncertainty also attaches to dk/dp in the lower core, where Birch's finite-strain theory¹⁵ gives $dk/dp \approx 3$, in contrast to evidence from seismology and the k - p hypothesis which indicate values of order 4 to 5. Comparison of models B_2 and 15 of Paper I, which have central values of 4.2 and 5.0 for dk/dp , shows, however, that the effect on $\Delta k/k_1$ is again negligible. (Models B_1 and 8

of Paper I could also be used, but it is to be noted that Tables 5, 6, and 8 need certain corrections, which will be provided in ref. 16.)

No further significant corrections to $\Delta k/k_1$ seem likely to be required unless the coefficient η should be found to differ significantly from the generally assumed value of unity inside the region D' ($1000 < z < 2700$ km) or E' ($2900 < z < 4500$ km). All present evidence indicates that η is in fact close to unity in these regions. (Landisman, Satô and Nafe¹⁷ recently concluded from free Earth oscillation data that η is substantially less than unity inside a large part of D' , but later work^{8, 9} has shown that the evidence can be otherwise interpreted.)

Thus it is expected that the results in §§ (3)–(7) cover all significant corrections likely to be required to the Model A value of $\Delta k/k_1$.

(9) Δk in the Earth Model HB_1 .—An approach to estimating $\Delta k/k_1$ can also be made fairly directly through Model HB_1 , which incorporates the principal corrections needed to the Model A value of $\Delta k/k_1$ apart from allowances for the region D'' . The HB_1 values⁹ of k_1 and k_2 are 6.352 , 6.521×10^{12} dyn/cm², respectively. Allowance for D'' is found to increase k_1 by 0.21×10^{12} dyn/cm², with negligible effect on k_2 . Thus Model HB_1 , corrected for the region D'' , has $\Delta k_1/k_1 \approx -0.6$ per cent.

(10) *Summary and Discussion*.—The total corrections obtained in §§(3)–(8) amount to +4 per cent, and so require the Model A value of -5 per cent for $\Delta k/k_1$ to be increased to -1 per cent. This revised estimate of $\Delta k/k_1$ agrees well with the estimate for Model HB_1 as modified in § (9). With the various sources of uncertainty taken into account, it seems rather unlikely that any further changes in $\Delta k/k_1$ will exceed a total of 1 to 2 per cent. The details in §§ (3)–(7) exhibit the magnitudes and relative importance of the contributions from the various sources listed in § (2).

Thus the total evidence on the density and P and S velocity distributions in the Earth favors the important conclusion that any discontinuous change in k that may occur at the Earth's mantle-core boundary is fairly small even compared with that in Model A. The evidence is fully compatible with the change's being zero. Hence, with respect to smooth variation of k below 1000 km depth, the Earth appears to fit the k - p hypothesis significantly better than does Model A, which was the historical source of the hypothesis. Details in the present paper appear to show that this conclusion can be shaken only if $d\rho/dz$ is unexpectedly anomalous inside the lower mantle (excluding the region D'') and/or the outer core.

By implication, the conclusion strengthens the evidence that the Earth's inner core is solid. Furthermore, the need for treating A -type and B -type Earth Models¹ as distinct classes is largely removed. The A models when revised to fit modern evidence now conform closely to the central hypothesis of the original Model B .

Application of Birch's finite-strain formulae¹⁵ to the Earth gives Δk significantly positive. Hence, insofar as the net change from the original Model A estimate of Δk , as derived in the present paper, is from a negative towards a positive value, an important discrepancy from Birch's theory is reduced in magnitude. Nevertheless, Birch's formulae require Δk to be not only positive,

but also appreciably larger¹⁴ than seems possible on the present calculations, so that full agreement is not achieved.

Calculations based on those in Paper I but allowing for nonzero Δk , are at present being carried out. It is hoped that through these calculations it will prove possible to set a closely determined upper bound to Δk , and thence to indicate needed corrections to the finite-strain formulae.

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