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On the making of sharp longitudinal dose profiles with helical tomotherapy

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Abstract

Since the beam width on the helical tomotherapy machine produced by TomoTherapy, Inc. is typically a few centimeters in the longitudinal direction (into the bore), the optimizer must choose to have a relatively high intensity local to the inside edge of a tumor or planning treatment volume (PTV) when avoiding an immediately adjacent organ at risk (OAR) either superior or inferior. By using a stand alone version of the TomoTherapy dose calculator, a realistic beam is applied to idealized deconvolution schemes including the MATLAB Optimizer Toolbox™ for a simple one dimensional PTV with adjacent OARs. The results are compared to a clinical example on the TomoTherapy planning station. It is learned that a Gibbs phenomenon type of oscillation in the dose within the tumor under these special circumstances is not unique to TomoTherapy, but is related to the attempt to form a sharp dose gradient – sharper than the beam profile with typical optimization constraints set to achieve a uniform dose as close as possible to the prescription. The clinical implication is that the Gibbs induced cold spots force the dose to increase in the PTV if a typical PTV dose-volume constraint is used. It is recommended that the dose prescription be smoothed prior to optimization or the dosimetric goals for an OAR adjacent to the PTV are such that a sharp dose falloff is not demanded, especially if the user either reduce the requirements that such an OAR be of both high importance and immediately adjacent to the PTV edge.

Keywords

Tomotherapy; Optimization; Dose Conformity; Dose Uniformity

1. Introduction

Tomotherapy is an intensity modulating technology for radiation cancer therapy. A multileaf collimator is rotated around the patient either helically or at discrete couch positions, junctioned to each other. The later technique is termed serial tomotherapy (Carol 1995), and because of beam divergence, the junctioning is difficult over a wide part of a transverse cross-section. The advantage of helical tomotherapy (Mackie *et al* 1993, Mackie *et al* 1999, Mackie *et al* 2006) over serial tomotherapy is that the junctioning of slices is spread throughout the volume because of the helical pattern of delivery. If there were no optimized modulation, then the sharpest dose gradient or edge that one could expect longitudinally, or into the bore of the device, would be the beam width itself because of the continuous motion of the couch. TomoTherapy, Inc., Madison, WI which makes the only helical tomotherapy machine, currently let's the user choose 1, 2.5, or 5 cm beam widths. The 2.5cm setting is the most commonly used for treatments. However, with optimization and modulation, a much sharper edge can be produced. In principle, the sharpest edge possible would be the penumbra of a single beam; however, a practical restriction on modulation reduces the maximum dose gradient. This paper discusses

how this sharp edge is produced with typical optimizer constraints set to achieve a uniform dose as close as possible to the prescription and the consequences for the tumor dose uniformity.

In order to study this problem, three levels of investigation are pursued. First, the actual TomoTherapy™ planning station is used, and it is the object of the study. A typical representative case is chosen for the calculation, but there is no reason why the results presented would not apply to any sufficiently large tumor. Next, the investigation is moved to a simplified 1-dimensional (1-D) treatment. This validity of simplifying a helical delivery into 1-D for this type of study is based on a previous work (Kissick *et al* 2007). The 1-D treatments include both the use of MATLAB's Optimization Toolbox™ to explore the effects of using a very different optimizer and a second idealized approach (Snyder *et al* 1992) is reproduced here, and is shown to be qualitatively consistent.

An observed trade-off that we see between conformity and uniformity in certain situations is shown to be fundamental to the situation with typical optimizer constraints. There is a theoretical basis for expecting a Gibbs phenomenon like oscillation or ringing to occur when performing an ill-posed deconvolution: that is exactly what we are doing when we ask an optimizer to produce an edge sharper than the edge of our beam. Finally, some suggestions are presented to help in the reduction of the unwanted ringing inhomogeneities, even though they may be small enough to be ignored in many situations anyway. This study is really restricted to large or long tumors relative to the longitudinal beam width size longitudinally.

2. Methods and Materials

There are three levels of investigation each with comparable constraints set to achieve a uniform dose as close as possible to the prescription. The first level is clinically motivated: an anonymized lung tumor is bounded superior and inferior with large, but contrived organs at risk (OARs) and is planned within the TomoTherapy Hi-ART™ treatment planning system. The next two levels investigation use a simplified model of the measured on-axis beam longitudinal tomotherapy beam profile described more below. This profile is used in the MATLAB optimization toolbox with a 1-D simplified version of the first level of investigation case. The third and final level of investigation uses another iterative approach, but also using the same simplified beam profile model with width adjusted.

In these investigations, we assume that the longitudinal dynamics are roughly separable from the transverse dynamics. This was shown in reference Kissick *et al* 2007, and it is true for a typical low pitch, p , defined as the couch translation relative to the beam width at isocenter for a full gantry rotation ($p < 0.5$). In this typical regime, transverse modulations are on a time scale an order of magnitude finer than longitudinal modulations. In other words, the TomoTherapy device generally operates in a 'tight helix' regime. By concentrating on the longitudinal or slice-to-slice modulations, the faster, transverse, within a slice, modulations are effectively averaged over, and the 1-D approximation has validity. The fact that all three levels of investigation are qualitatively consistent also self-consistently demonstrates the validity of this approximation.

2.1. TomoTherapy Planning Station

On a research dedicated TomoTherapy Hi-ART™ treatment planning station at the University of Wisconsin Hospital, an anonymous lung cancer patient was used to explore what would happen if one were to add long ($>$ few longitudinal beam widths) adjacent organs at risk (OARs) superior and inferior to the tumor. The Fig. 1 shows the general set-up of the plan for a central sagittal slice. The tumor length is several times longer than the longitudinal beam width, set at 2.5cm. The pitch was set at a constant 0.287 (determined to be an optimal value for our clinic (Kissick *et al* 2005)). The prescribed tumor dose was set such that 95% of the PTV will receive

60 Gy. This is a “hard constraint” whereby the optimizer will apply any global scaling of the beam fluence necessary to ensure adherence to this constraint. The “Avoidance end caps” are the OARs for which the maximum dose goal is set to zero. The optimizer is, in effect, asked to make a sharper edge than can be done physically. The typical additional constraints, used for most treatments, were set for all of the optimizations: the maximum and minimum doses were both set equal to the prescription dose of 60 Gy with equal penalties set to unity. The OARs likewise had the maximum and minimum doses set to zero with equal unity penalties.

The importance ratio between the tumor and the OARs (superior and inferior had the same importance) was varied. In the current version of the optimizer, the beamlets whose weight is set to be optimized extend several centimeters beyond the target extents, thus allowing beamlets with a “beam’s eye view” not intersecting the target to contribute dose to the target edges via scatter and lateral electron electron transport.

Several hundred iterations of optimization did not improve or alter the results from about 50 iterations, so we choose 63 iterations for every run to be consistent, once it was learned that this was more than enough for this particular study that is focused on larger, slice to slice, modulations.

The modulation factor is defined as the ratio of the maximum to the average leaf opening time for all non-zero leaf opening values per projection. This factor was varied from its highest possible value of 5 to its lowest possible value of 1. At a modulation factor of 1, the only modulation that the leaves can make is open at the same amount of time (or arc length) as all the other open leaves or the leaves are closed. At the highest level of 5, this factor is set to allow for the most flexibility in the optimizer possible, but the optimizer must work harder to find the best solution. Nothing substantially changed in the results above 50 iterations for this problem even at a modulation factor setting of 5.

The OARs importance ratio to the tumor importance ratio was varied from 1:10 to 10:10 at steps of $n:10$, $n=1,2,3,\dots$. Several well-spaced modulation factor settings were also tried for several importance ratios, but not all possible combinations were calculated.

2.2. Idealized Optimizations

By using a stand-alone version of the TomoTherapy Hi-ART™ (Lu *et al* 2005) dose calculator, the longitudinal beam profile was calculated for a local machine at 2.5cm (actually 2.47cm at isocenter, at a depth of 15cm in water) beam width. We discovered that one can model this profile very effectively with a simple fit to two Gaussian convolved ‘rect’ functions below:

$$Dose_{2.47\text{cm}}(z) = rect(z / 2.775\text{cm}) \otimes \{G_p(z, \sigma_p) + G_s(z, \sigma_s)\} + 0.0035$$

$$G(z, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-z^2 / 2\sigma^2\right\} \quad (1)$$

where $\sigma_p=0.35\text{cm}$, and $\sigma_s=6\sigma_p$. The ‘p’ stands for primary, and the ‘s’ stands for scatter. In Fig. 2.b, this model from Eq. 1 is compared to the stand-alone dose calculator calculation. In Fig. 2.a, the simplified 1-D set-up (dose prescription) used in the MATLAB 6.5 (Math Works, inc., Natick, MA, USA) Optimization Toolbox™ is shown. The optimization is a least squares minimization of the calculated minus the prescription dose. The calculation used a 100 point grid. The tumor here is much larger than a longitudinal beam width, and the avoidance end caps (OARs) are adjacent to the tumor or planning treatment volume (PTV).

The importance ratio of the OARs to the tumor is varied from 0 to 1 in this second level of investigation. We had more control over the calculation with MATLAB and were able to vary the prescription edge’s sharpness. This is a regularization technique similar to that used by

(Snyder *et al* 1992, Snyder *et al* 1997). In the later stages of the investigation of this level, the edge was smoothed to the extent of the primary component of the beam profile, with a Gaussian kernel having the same variance as the primary beam. There is also a modulation factor which was varied to see consistency with the TomoTherapy planning station, but was set at 3 otherwise.

The model here is 1-D, so there is no pitch setting. As mentioned above, a 1-D approximation to the tight helix delivery is reasonable for the low pitches used clinically.

We also implemented another iterative deblurring method described by (Snyder *et al* 1992, Snyder *et al* 1997). The reader is referred to those references for details. In effect, any optimization is solving the following problem, a Fredholm integral equation of the first kind for the ‘intensity,’

$$dose(x) = \int beam(x | x') \cdot intensity(x') dx', \quad (2)$$

such that the ‘dose’ is as close as possible to the ‘prescription’:

$$prescription(x) \approx \int beam(x | x') \cdot intensity_{optimal}(x') dx'. \quad (3)$$

The ‘beam’ here needs to be realistic – it represents a ‘point spread function’ in analogy to imaging. The ‘prescription’ is typically sharper than the beam, and hence the problem is usually ill-posed: there is no ‘optimal intensity’ that can possibly match the ‘prescription’ exactly.

The ‘beam’ and ‘prescription’ functions used in this third level of investigation are the same ones depicted in Fig. 1. In this Snyder *et al* method, however, there is no modulation factor, and the importance ratio is always unity, and so it represents a special case of the former method. The algorithm described in (Snyder *et al* 1992) for a non-negatively constrained deblurring of images. It uses an iterative, monotonic convergence to minimize a quantity similar to a cross-entropy measure that accommodates functions whose integrals are not equal. The basic Snyder *et al* method (Snyder *et al* 1992, Eq. 10) iteration is as follows for iteration step = k:

$$intensity_k(x) = intensity_{k-1}(x) \frac{1}{H(x)} \sum_x \left[\frac{beam(x | x')}{dose_{k-1}(x')} prescription(x') \right], \quad (4)$$

with $H(x) = \sum_x beam(x | x')$ and the dose is calculated according to

$$dose_k(x) = \sum_x beam(x | x') intensity_k(x').$$

In either the forward or the inverse direction, the beam profile is the main spatial constraint of the system and is unchanged. It defines the resolution in the same way as the point spread function does for imaging.

The results from this method are used to demonstrate the robustness of the observed phenomenon – the phenomenon is not particular to any given optimizer. It represents a deeper result.

3. Results

All three levels of investigation show that there is a consequence to asking for the impossible from the system: optimizer or any method of deconvolution. If the system is asked to define an edge sharper than the beam profile will allow, then the consequence will be some

inhomogeneity within the tumor similar to Gibbs phenomenon. A tradeoff is observed between conformity and uniformity. There are suggestions to reduce this problem, presented in the discussion.

3.1. TomoTherapy Planning Station

Shown in Fig. 3 are screen shots of the dose volume histograms DVHs, dose contour plots, and a graph of the delivery sinograms – all for various choices of the importance ratios OARs/tumor. One can observe that as the importance of the OARs is increased and the optimizer is working to eliminate dose there, a hot spot develops just inside the superior and inferior edges of the tumor. In fact, a closer look will show that there is a Gibbs like ringing or oscillation happening near these sharp edges.

Of course, if the modulation factor is 1, then the optimizer can only open or close a leaf, and there are not enough degrees of freedom to allow the optimizer to form a sharp edge. In Figs. 3 and 4, there are plots of the delivery sinogram integrated over all leaves, plotted versus projection. there are 51 projections per rotation, and there are $1/p$ rotations per delivered ‘slice.’ If the modulation factor is large enough, then a higher importance ratio of OAR to tumor will produce more ringing or inhomogeneity in the tumor. The DVHs show this trend as well. The conformity improves with more OAR importance (if enough modulation allowed), but at the expense of uniformity in the tumor.

In effect, the optimizer is increasing the intensity of the ‘beam’ function near the edge to make it relatively sharper, but then it must compensate just after this. The phenomenon is basically the same as Gibbs ringing as it will be described later. However, this phenomenon is the result of the user also imposing a strong requirement that the maximum and minimum dose be as close as possible to the prescription, as is usually the case.

3.2 Clinical Considerations

It must first be emphasized that the cases studied here were specifically designed to enhance the Gibbs ringing by putting in mock OARs immediately adjacent to the PTV, and are therefore not representative of a real clinical case. That said, the potential clinical implications of this Gibbs ringing for TomoTherapy planning and treatment stem from cold spots which appear within the target volume about one beam-width inside the PTV superior and inferior. This is illustrated in Fig. 5b where only the prescription dose of 60 Gy or greater has the red color in the PTV. In the slice shown, the two cold spots resulting from the Gibbs ringing are seen as gaps in the red colorwash. Because the TomoTherapy treatment planning system is constrained to deliver a specified dose to a given percent of the target volume, the presence of this ringing requires a scaling of the total dose so that the prescription to the target is met. While this scaling generally results in an integrated boost being delivered to a large percentage of the target volume, if care is not taken to constrain the minimum dose as well, the effects of the remaining cold spots can still have a deleterious impact on the probability of tumor control.

Specifically, for the cases shown in Fig. 5a, a dose of 60 Gy was prescribed to 95% of the PTV (volume equal to 498 cc) for both high and low OAR to PTV importance ratio cases. In this example the modulation factor was set to its maximum value of five. For the 10:10 importance ratio case (shown in Fig. 5b), the maximum, minimum, average, and standard deviation of the dose in the PTV are 73.9 Gy, 29.17 Gy, 64.11 Gy, and 3.54 Gy respectively. For the 1:10 case these values are 67.4 Gy, 42.7 Gy, 61.73 Gy, and 1.64 Gy respectively. One can see that in the 10:10 case the planning system scaled the total dose upwards to compensate for the cold spots produced by the ringing, however due to the nature of the 95% prescription (95% of the PTV is getting at least the prescription dose) and the lack of constraint on minimum dose, small cold spots were still present; even in the 1:10 importance ratio case.

In order to assess the relative quality of the treatment plans generated using the different optimization parameters, the equivalent uniform dose (EUD) as well as the tumor control probability (TCP) were calculated for both cases. The EUD (calculated using the formalism given by Niemierko, 1997) was found to be 58.8 Gy for the 1:10 importance ratio case and 47.1 Gy for the 10:10 importance ratio case. These calculations assumed a reference dose of 2 Gy and a 2 Gy surviving fraction of 0.48. In addition, fractionation effects were included by first normalizing the differential dose-volume histogram to a 2 Gy reference dose using the normalized total dose (NTD) concept. This assumes a full linear quadratic description of cell survival vs. dose, a standard 2 Gy fractionation schedule, and an α/β ratio of 10 for the PTV.

It should be noted that in both cases the EUD is lower than the prescription dose of 60 Gy. As discussed by Tomé and Fowler, this would likely result in both plans being unsuitable for treatment (Tomé and Fowler 2002); however, because the nature of this study is comparative, it is enough to simply note that the EUD is reduced significantly in the 10:10 case as compared to the 1:10 case.

To further investigate the impact of this loss in EUD, the TCP was calculated for both cases using the model described by Webb and Nahum (Webb and Nahum 1993). This model is based on the surviving number of clonogens and requires a dose-volume histogram, an initial clonogen density, and a 2 Gy surviving fraction. For these calculations, we again assume a 2 Gy surviving fraction of 0.48, and utilize the same normalized total dose-volume histograms used for the EUD calculations in order to properly account for fractionation effects. To obtain the initial clonogen density, a TCD_{50} (total dose required to control 50% of tumors) value was assumed and the initial number of clonogens calculated. While normally, the value for TCD_{50} would come from observed clinical data, for our study we chose to set this value equal to the EUD for the 1:10 ratio case. Doing this essentially maximizes the sensitivity of the TCP to changes in the dose distribution by allowing access to the linear region of the TCP curve and therefore providing a conservative estimate of TCP change. With these assumptions in place, the PTV clonogen density was calculated to be 3.05×10^6 /cc.

With these numbers in hand, the TCP was calculated for both the 1:10 and 10:10 importance ratio cases and the values were found to be 52.75% and 0.00% respectively. While providing a “worst case” scenario, this calculation illustrates the dramatic effect these cold spots can have on the probability of tumor control. Furthermore, these results parallel those found by Tomé and Fowler in 2002. Unfortunately, the gain in longitudinal dose gradient was only minimal for this unacceptable cost. The longitudinal dose gradients increase several percent by using the higher OAR importance (exact amounts vary along these edges from 5% to 10%). In order to utilize the planning systems ability to create sharp longitudinal edges, one must constrain the minimum dose, which will be lowered by the Gibbs ringing effect, in order to obtain a clinically acceptable plan.

3.3. Idealized Optimizations

The MATLAB optimizer produced qualitatively the same result when the prescription edges were sharp like the TomoTherapy prescription edges. The beam width was varied from a few millimeters to 5 cm and the ringing was still there. Shown in Fig. 6a when the edges are sharp, the ringing is pronounced. In Fig. 6b, when the edge is pre-smoothed to a level consistent with the primary beam smoothing, the ringing diminishes considerably. A very smooth edge would make the ringing vanish, even with equal importance between the OAR and the tumor, any modulation factor, and no added space between the OAR and the tumor. In fact, there was a gain in uniformity with some loss in conformity with pre-smoothed edges: termed ‘regularization’ in imaging deblurring. For many situations, conformity is already compromised with an internal or planning treatment volume margin, and the uniformity loss

probably more troublesome. Therefore, regularizing the problem may produce a net gain for most situations where a margin is added to the clinical treatment volume.

The general trend for sharp prescription edges with the 1-D idealized MATLAB optimizer calculations is displayed in Fig. 7 in DVH form.

The ringing we see in the optimization procedures are also present in the Snyder *et al* approach, see Fig. 8. In fact, compare this figure and Fig. 6 with Fig. 1 in (Snyder *et al* 1992).

4. Discussion

The convolution theorem states that the deconvolution problem is exactly equivalent when specified correctly in the frequency domain. Because the Fourier transform of a Gaussian is itself another Gaussian and that represents the edge of our beam (see Eq. 1), the kernel in the convolution, it has a quickly dropping spectrum at higher frequencies. A sharp, discontinuous, edge, by contrast, will fall off much less quickly than that in frequency space ($\sim 1/k$). The problem will be ill-posed at these higher frequencies needed to represent the sharp edge of the prescription, because of the very small associated kernel at high frequencies. So, even a purely numerical optimization solution will act like to produce a Gibbs like phenomenon. In fact, J. Willard Gibbs explained this phenomenon which was originally observed by Albert Michelson in 1898 who was using a machine to compute the Fourier representation of a square wave with a purely iterative method.

The Gibbs like ringing we observe is also observed in Snyder *et al* 1992 and Snyder *et al* 1997 in the context of image de-blurring with non-negativity-constrained deconvolution techniques. This effect is best stated by (Snyder *et al* 1997) as follows: “The kernel of the deconvolution problem has an eigenspectrum that very rapidly (exponentially quadratically) decreases towards zero. Frequency components of the intensity which lie beyond a certain frequency where the eigenspectrum falls below the spectrum of the noise, which is due to finite machine precision, cannot be recovered by deconvolution, no matter how cleverly the deconvolution problem is solved This leads to a Gibbs overshoot at the discontinuity of the intensity ...” The situation is depicted graphically in Fig. 9.

Therefore, the Gibbs like ringing, responsible for the inhomogeneity in the tumor or PTV, is not the result of any particular optimizer.

There is, however, a way to improve the situation through regularizing the problem to make it less ill-posed. By smoothing the prescription prior to optimization with a Gaussian kernel similar to the beam function, then the above situation may not occur; we observe the success of this approach for the MATLAB case, see Fig. 6. In addition, (Snyder *et al* 1992, see Fig. 1 of that reference) also observes the same beneficial effect. However, going too far with smoothing the prescription could lead to a significant reduction in edge resolution. In that sense, it may be better to live with some inhomogeneity. There may be an optimal middle ground where some prescription smoothing, say at the level of the lateral secondary electron scattering range, could not possibly reduce an already fundamental blurring and would only be helpful.

The TomoTherapy user can make some decisions to improve the situation. The critical structure and PTV borders are chosen by doctors, dosimetrists, and physicists, and the importance of various structures can be chosen by the person doing the treatment planning as well. When faced with any situation similar to the idealized cases written about in this paper like a large and important avoidance just touching the tumor either superior or inferior, consider a sacrifice of either the OAR importance or using OAR dose goals that do not demand an immediate falloff to zero dose. In clinical optimization, for an OAR abutting a PTV, the maximum dose goal is often set equal to the minimum dose goal for the PTV, along with one or more dose-

volume goals for dose values less than the maximum. This effectively prevents a discontinuity in the prescription dose at the PTV-OAR boundary. If the optimizer is producing something similar to the Gibbs ringing discussed here, then the user is simply asking for more than the system can produce. However, the dose inhomogeneity associated with the Gibbs ringing may be small enough to ignore.

Another way to increase the dose gradient outside the PTV would be the addition of dynamic jaw controls as was originally proposed for TomoTherapy, called “running stop/start” (see Fig. 14 of reference Yang *et al* 1997). However, this feature would be difficult to implement. There are probably few cases that push the needs for both a very sharp gradient longitudinally and simultaneously a high dose uniformity in the PTV.

For small tumors, such as in radiosurgery cases, a uniform PTV dose is typically not an important requirement. In fact, it is often desirable to incur a central dose much higher than the margin dose in order to increase overall cell kill. In such a situation, the penalty for the maximum PTV dose can be relaxed and/or the maximum PTV dose can be set to a value much higher than the minimum or prescription dose. The dose gradient at the edges of the PTV are typically higher in this case than when a uniform PTV dose is demanded, as the PTV edges are in a rapid falloff region of the dose distribution rather than in the shoulder region. Without the goal of uniform PTV dose or constant average PTV dose, Gibbs ringing is less likely to occur since the optimizer will not attempt to counteract the effect of elevated dose just inside the PTV boundary.

If the optimizer is being asked to keep the inferior and superior dose as low as possible, and one imposes the additional constraints that maximum and minimum doses are to be as close as possible to the prescription dose, then the ringing will occur under the conditions discussed in this paper. The hard ‘constraint’ of a, typically, 95% of the PTV to receive the prescription dose normalizes (shifts) the dose after the optimization and is responsible for much of the clinical impact of this phenomenon.

5. Conclusions

By confirming that a Gibbs like phenomenon shown on a representative TomoTherapy planned case is not particular to the dynamics of the TomoTherapy optimizer, we learn that there is a fundamental limit to the sharpness of a longitudinal (superior-inferior) dose edge. That limit corresponds to sharpness of the edge of the beam, not the entire width of the beam since optimization will act to spike the intensity just inside the edge of the PTV. The problem is that the prescription has very sharp edges and therefore requires a broader frequency spectrum than can be provided in *any* deconvolution algorithm. The fundamental system constraint is the longitudinal edge sharpness of the beam (the kernel). A Gibbs like phenomenon will occur because the deconvolution problem becomes ill-posed when the prescription is sharper than the kernel. The Gibbs phenomenon will manifest as a ringing in the dose at the PTV edge: inhomogeneities. With associated typical dose-volume constraints that act to control the minimum PTV dose, the average PTV dose could increase as a result of these inhomogeneities.

A tradeoff exists between conformity and uniformity with typical optimizer constraints set to achieve a uniform dose as close as possible to the prescription. If one does not require the prescription edge to be so sharp, say by lowering the OAR importance, reducing the modulation factor, or by reducing the demand for a rapid reduction in dose outside the target, then the inhomogeneities are less severe. A hypothetical next generation of helical tomotherapy which would provide for pre-optimization prescription smoothing (regularization) and also dynamic jaws, would allow for more success in making sharp longitudinal dose profiles while retaining excellent dose uniformity.

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Figure 1.

The TomoTherapy calculations use a real lung tumor without margins (PTV=GTV) and two mock OARs immediately inferior and superior to the tumor.

Figure 2.

A sharp longitudinal dose profile (a) and a realistic longitudinal beam profile (b) obtained by matching to the TomoTherapy dose calculator simulation of an actual machine, both are used for idealized modeling.

Figure 3.

The results of TomoTherapy treatment planning are summarized. The normalized cumulative dose volume histograms (DVHs) are screen captured from the planning station together with a dose contour plot for three values of importance ratios. The dose inhomogeneities within the tumor are a few percent, and are noticeable on the DVHs. The integration of the delivery sinograms over all leaves for each projection (51 projections per complete gantry rotation) are displayed lower right. A typical beam width of 2.5 cm is used with the maximum modulation factor or 5. Notice that the intensity spikes inside the tumor edge when a large OAR importance is used. The intensity spike is the cause of the dose inhomogeneity in the tumor which shows an oscillation similar to a Gibbs like ringing. At the bottom left, the ringing is expanded for a better view with the OARS shown for the 9:10 case. The top 3 figures show a progression of OAR importance leading to more Gibbs ringing and increased PTV DVHs since the system adjusts for 95% of the PTV to receive 60 Gy.

Figure 4.

Leaf integrated delivery sinograms from the TomoTherapy treatment planning station are presented for extremes in the importance ratios of OAR to tumor for the minimum modulation factor (a,b) and a typical modulation factor (c,d). Notice that the intensity spikes needed for a sharp dose profiles and the Gibbs like ringing require both a high OAR importance and a large enough modulation factor.

Figure 5.

Representative mid-section final dose profiles for the TomoTherapy case previously described (a) are shown through the line indicated by the big arrow through (b). The red color scale in (b) for an importance ratio of OAR:PTV of 10:10 is set to 60 Gy (prescription) or above.

Figure 6.

The MATLAB Optimization Toolbox™ calculated dose is presented relative to the sharp edge prescription (a) and the pre-smoothed prescription (b). The prescription smoothing is at a degree comparable to the beam edge in (b). Notice that the dose uniformity is greatly improved in (b), but the conformity is nearly the same.

Figure 7. Using the MATLAB Optimization Toolbox™ with a sharp edge prescription, the integral DVHs for both the OAR (normal tissue) and the tumor are presented for an importance ratios (OAR importance to tumor importance) of 0, 0.1, and 1. Notice that there is a tradeoff between uniformity and conformity as the importance ratio is varied.

Fig. 8.

An unmodulated (constant intensity) beam is compared to the method defined by (Snyder *et al* 1992) relative to the sharp edge prescription.

Figure 9.

A graphical explanation of why it is difficult to deconvolve a discontinuous prescription with a continuous beam profile (kernel). The discontinuity will always extend much further in frequency space than the continuous kernel.