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Multiscale Penalized Weighted Least-Squares Sinogram Restoration for Low-Dose X-Ray Computed Tomography

Jing Wang^{1,2}, Hongbing Lu³, Junhai Wen⁴, and Zhengrong Liang^{1,2,5}

¹ Department of Radiology, State University of New York, Stony Brook, NY 11794, USA

² Physics and Astronomy, State University of New York, Stony Brook, NY 11794, USA

³ Department of Biomedical Engineering/Computer Application Fourth Military Medical University, Xi'an, Shaanxi 710032, China

⁴ Department of Biomedical Engineering, Beijing Institute of Technology, Beijing 100081, China

⁵ Department of Computer Science, State University of New York, Stony Brook, NY 11794, USA

Abstract

In this paper, we propose a novel multiscale penalized weighted least-squares (PWLS) method for restoration of low-dose computed tomography (CT) sinogram. The method utilizes wavelet transform for the multiscale or multi-resolution analysis on the sinogram. Specifically, the Mallat-Zhong's wavelet transform is applied to decompose the sinogram to different resolution levels. At each decomposed resolution level, a PWLS criterion is applied to restore the noise-contaminated wavelet coefficients, where the penalty is adaptive to each resolution scale and the weight is updated by an exponential relationship between the data variance and mean at each scale and location. The proposed PWLS method is based on the observations that (1) noise in CT sinogram after logarithm transform and calibration can be modeled as signal-dependent variables and the sample variance depends on the sample mean by an exponential relationship; and (2) noise reduction can be more effective when it is adaptive to different resolution levels. The effectiveness of the proposed multiscale PWLS method is validated by both computer simulations and experimental studies. The gain by multiscale approach over single-scale means is quantified by noise-resolution tradeoff measures.

Keywords

Wavelet transform; multiscale analysis; PWLS; sinogram restoration; low-dose; X-ray CT.

I. Introduction

Low-dose computed tomography (CT) is clinically desired, especially for screening purpose and image-guided intervention. Minimizing X-ray exposure to patients has been one of the major efforts in the CT field [1]. A simple and cost-effective means among many other approaches to achieve low-dose CT is to lower the X-ray tube current (mAs) or deliver less X-rays. However the image quality with low mA acquisition protocol could be severely degraded due to the excessive X-ray quantum noise [2,3]. Many strategies have been proposed to reduce the noise, *e.g.*, by nonlinear noise filters [2-5] and statistics-based iterative image reconstructions (SIIRs) [6-8]. The nonlinear filters do not explicitly consider the statistical distribution of the noise and the SIIRs are notorious for their excessive computational demands

for large CT image size. Recently, statistics-based sinogram smoothing approaches [9-13] followed by filtered back-projection (FBP) image reconstruction have shown the potential for noise reduction and information preservation in low-dose CT imaging. These statistics-based sinogram-smoothing FBP-reconstruction approaches fully utilized the statistical information of the measured data while avoiding the time consuming re-projection and back-projection cycles in the SIIRs. So far, these sinogram-smoothing approaches have been implemented in a single-scale domain or the original sinogram space.

In the last three decades, wavelet theory on multiscale image analysis has been extensively studied and applied in image compression and restoration [14-21]. One main scheme of wavelet-based de-noising is based on thresholding or shrinkage of wavelet coefficients, where the wavelet coefficients are set to zero if they are less than some thresholds [14-17]. Another main scheme is based on multiscale singularity detection of the wavelet coefficients [18-20]. In this latter scheme, the wavelet coefficients are classified first into noise-related irregular and regular coefficients based on wavelet transform modulus (WTM) and then the noise-contaminated coefficients are restored based on certain criteria. Recently, an anisotropic diffusion filter on wavelet coefficients was proposed for speckle suppression and edge enhancement in ultrasound images [21]. Most of these previous wavelet-based noise-smoothing algorithms assume a Gaussian white noise for the image data and/or estimate the local statistics of the image data (*e.g.*, the variance of a pixel (x, y) from the neighboring pixels within a window centered at that pixel). However, the Gaussian white noise assumption is not valid for the low-dose CT sinogram, because the detected counts follow a Compound Poisson distribution [22] while the noise in sinograms after logarithm transform and other calibrations follows approximately a signal-dependent Gaussian distribution according to the analysis of repeated experimental measurements [3,9]. The estimation of variance or local statistics from neighboring pixels is not accurate because the variance of the repeatedly sampled data strongly depends on the mean value and their dependence can be described mathematically by a nonlinear relationship [3,9]. These invalid assumption and inaccurate estimation would be an obstacle for further development of the multi-resolution tomographic image reconstruction framework [23-26] toward low-dose CT application.

In this paper, we propose a novel multiscale penalized weighted least-squares (PWLS) method for the restoration of low-dose CT sinograms in the wavelet domain. The widely-used wavelet transform of Mallat-Zhong [27] is first applied to decompose the sinogram image into different resolution scales. In the wavelet domain, noise is dominant in those wavelet coefficients at the lower decomposition levels [27]. Based on this observation, a PWLS criterion is applied to restore those wavelet coefficients where the penalty is adaptive to each decomposition level and the weight, which reflects a strong dependence of the data variance on the mean, is updated at each scale and location. The multiscale PWLS method fully utilizes the statistical information of the sinogram data [3,9] and also considers the characteristics of the signals from multiscale decomposition of the data [27].

In the following section, we first introduce the dyadic wavelet transform of Mallat-Zhong [27] and the PWLS criterion in the wavelet domain. We then describe the proposed multiscale PWLS minimization strategy. In Section III, computer simulation and experimental results with comparison between multiscale PWLS approach and single-scale PWLS means are presented, followed by discussions and conclusions in Section IV.

II. Methods

A. Mallat-Zhong's Dyadic Wavelet Transform

In the wavelet transform, a function $f(x)$ can be decomposed to different resolution levels by diluting and shifting a mother wavelet function $\Psi(x)$ [28]:

$$W_{a,b} f(x) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{x-b}{a}\right) dx \quad (1)$$

where a is a scale parameter, b is a translation parameter, and the mother wavelet satisfies:

$$\int_{-\infty}^{\infty} \psi(x) dx = 0. \quad (2)$$

In [27], Mallat and Zhong presented an over-complete dyadic wavelet transform, where the mother wavelet is the first derivative of a smoothing function $\theta(x)$. For a two-dimensional (2D) wavelet transform, the two wavelet functions $\Psi^x(x,y)$ and $\Psi^y(x,y)$ are defined as:

$$\Psi^x(x, y) = \frac{\partial \theta(x, y)}{\partial x} \quad (3)$$

$$\Psi^y(x, y) = \frac{\partial \theta(x, y)}{\partial y}. \quad (4)$$

Let

$$\Psi_{2^j}^x(x, y) = \frac{1}{2^{2j}} \Psi^x\left(\frac{x}{2^j}, \frac{y}{2^j}\right) \quad (5)$$

$$\Psi_{2^j}^y(x, y) = \frac{1}{2^{2j}} \Psi^y\left(\frac{x}{2^j}, \frac{y}{2^j}\right) \quad (6)$$

denote the dilation of the 2D wavelets. The wavelet transform of a sinogram $f(x,y)$ at scale 2^j has two components which are defined as:

$$W_{2^j}^x f(x, y) = f * \Psi_{2^j}^x(x, y) \quad (7)$$

$$W_{2^j}^y f(x, y) = f * \Psi_{2^j}^y(x, y) \quad (8)$$

where $*$ denotes the convolution operation. With an associated scaling function $\phi_{2^j}(x, y)$, the coarse approximation of $f(x,y)$ at scale 2^j can be calculated as:

$$S_{2^j} f(x, y) = f * \phi_{2^j}(x, y). \quad (9)$$

A J -level 2D dyadic wavelet transform of sinogram $f(x,y)$ is then referred to a set of functions $\left\{ S_{2^j} f(x, y), \left(W_{2^j}^x f(x, y) \right)_{1 \leq j \leq J}, \left(W_{2^j}^y f(x, y) \right)_{1 \leq j \leq J} \right\}$, where $S_{2^j} f(x, y)$ represents the approximation of $f(x,y)$ at scale 2^j and $\left\{ \left(W_{2^j}^x f(x, y) \right)_{1 \leq j \leq J}, \left(W_{2^j}^y f(x, y) \right)_{1 \leq j \leq J} \right\}$ represents the detail sinogram data at scale 2^j .

To compute the discrete dyadic wavelet transform, Mallat and Zhong [27] have developed a fast algorithm called *à trous algorithm* [29], which involves the convolution of the sinogram with a set of low-pass filter H and high-pass filters G and D :

$$W_{2^{j+1}}^x f(x, y) = S_{2^j} f(x, y) * (G_j, D_j) \quad (10)$$

$$W_{2^{j+1}}^y f(x, y) = S_{2^j} f(x, y) * (D_j, G_j) \quad (11)$$

$$S_{2^{j+1}} f(x, y) = S_{2^j} f(x, y) * (H_j, H_j) \quad (12)$$

where $S_{2^0} f(x, y)$ is the original sinogram and D is chosen as the Dirac filter. Notations G_j and H_j represent their respective discrete filters obtained by putting $2^j - 1$ zeros between consecutive coefficients of their defined filter forms G or H [27]. Operation $A * (D, G_j)$ donates the separable convolution of the rows and columns of the sinogram A with the 1D filters D and G . At each 2^j decomposition level, we obtain $\left\{ S_{2^j} f(x, y), \left(W_{2^j}^x f(x, y) \right)_{1 \leq j \leq J}, \left(W_{2^j}^y f(x, y) \right)_{1 \leq j \leq J} \right\}$ by the use of equations (10)-(12). Noise reduction by a PWLS criterion is then performed on the wavelet coefficients at each decomposition level (see the next section). The estimation of the ideal sinogram from a noisy one is finally performed by the inverse wavelet transform on all the wavelet coefficients at all decomposition levels. The inverse wavelet transform on $\left\{ S_{2^j} f(x, y), \left(W_{2^j}^x f(x, y) \right)_{1 \leq j \leq J}, \left(W_{2^j}^y f(x, y) \right)_{1 \leq j \leq J} \right\}$ can be obtained by the following recursive formula:

$$S_{2^{j-1}} f(x, y) = W_{2^j}^x f(x, y) * (K_{j-1}, L_{j-1}) + W_{2^j}^y f(x, y) * (L_{j-1}, K_{j-1}) + S_{2^j} f(x, y) * (\tilde{H}_{j-1}, \tilde{H}_{j-1}) \quad (13)$$

where K and L are the high-pass filters defined in [27] and $\tilde{H}$ is the conjugate filter of H . The coefficients of filters H, G, K and L can be found in [27].

B. Penalized Weighted Least-Squares Restoration of Low-Dose CT Sinogram

The noise in low-dose CT projection (after logarithm transform and system calibration) can be modeled approximately as signal-dependent Gaussian noise, and the variance of the noise can be determined by an exponential formula [9,13,30]:

$$\sigma_i^2 = \alpha_i \times \exp(p_i / \eta) \quad (14)$$

where p_i is the mean and σ_i^2 is the variance of the projection data at detector channel i , and both η and α_i are object-independent parameters, which are completely determined by the system or manufacture configuration. Notation α_i is a parameter adaptive to different detector channels or bins. Based on the noise properties, a PWLS cost function [30-32] can be constructed in the sinogram space as:

$$\Phi(p) = \frac{1}{2} (z - p)' \Sigma^{-1} (z - p) + \beta R(p). \quad (15)$$

The first term in equation (15) is a weighted least-squares (WLS) measure, where $'$ denotes the transpose operator, p is the vector of ideal projection $\{p_i\}$ to be estimated, and vector z is the system-calibrated measured projection data $\{z_i\}$ (after logarithm transform and system calibration). Matrix Σ is a diagonal matrix with the i th element of σ_i^2 , i.e., an independent Gaussian distribution for each measurement z_i . The second term in equation (15) is a smoothness penalty or *a priori* constraint, where β is the smoothing parameter which controls the degree of agreement between the estimated and the measured data. In this paper, the quadratic smoothness penalty is used [31]:

$$R(p) = \frac{1}{2} p' R p = \frac{1}{2} \sum_i \sum_{k \in N_i} w_{ik} (p_i - p_k)^2 \quad (16)$$

where N_i indicates the set of nearest neighbors of the i th pixel in the 2D sinogram and w_{ik} reflects the relative contributions of the nearest neighbors.

The weight in the WLS term of equation (15) plays the role to control the contribution of different measured data, where less reliable data contribute less in the PWLS cost function. In

this study, the variance of each measured datum is chosen as its weight as defined above. By this choice, the PWLS cost function is equivalent to the penalized maximum likelihood (pML) or maximum *a posteriori* probability (MAP) criterion for independent Gaussian distributed noise. This is consistent with the experimental results [3, 9]. The presented PWLS cost function of (15) is similar to the likelihood function derived in [7, 33] for Bayesian reconstruction of CT images, but with different implementations on the penalty term and on the WLS weight. The PWLS cost function of [7, 33] was implemented in the image domain where the penalty was imposed on the nearby image voxels and the WLS weight was pre-computed independent from the data mean or signal. Other similar implementations in the image domain are described in [31, 32]. Our PWLS cost function of (15) is described in the sinogram space and its WLS weight is determined by the experimental result of equation (14). The implementation is presented below.

Minimization of the presented PWLS cost function can be performed efficiently by the iterative Gauss-Seidel (GS) updating strategy [7,31], although other numerical calculation algorithms could be applied. The iterative formula for the solution of minimizing equation (15) is given by:

$$p_i^{(n+1)} = \frac{z_i + \beta \sigma_i^2 \left(\sum_{k \in N_i^-} w_{ik} p_k^{(n+1)} + \sum_{k \in N_i^+} w_{ik} p_k^{(n)} \right)}{1 + \beta \sigma_i^2 \sum_{k \in N_i} w_{ik}} \quad (17)$$

where index n represents the iterative number, N_i^- denotes the left and upper nearest neighbors of pixel i (in 2D presentation) whose index number is less than i , N_i^+ denotes the right and lower neighbors of pixel i whose index number is larger than i , and N_i was defined before and now specifically denotes these four nearest neighbors of pixel i . In a multiscale PWLS approach, both β and σ_i^2 can be adaptive to different resolution levels. Implementation of the multiscale PWLS algorithm is detailed below.

C. Multiscale PWLS Method

After applying the wavelet transform, noise is contaminated in the wavelet coefficients, especially in those coefficients at the lowest scale ($j=1$). At the same time, the signal is dominant in the coarsest approximation $S_{2^J} f(x, y)$ of the original sinogram image

$S_{2^0} f(x, y)$. Based on these observations, we propose to apply the PWLS criterion for

restoration or de-noising of the wavelet coefficients with different penalty strengths and WLS weights at different scales, while preserving the signal-dominant component $S_{2^J} f(x, y)$ as much as possible. This multiscale PWLS approach could potentially improve performance of the direct single-scale PWLS method as described in Section II.B above.

As reported in [3,9,33], the noise in low-dose CT sinogram image can be modeled as a Gaussian distribution and the variance of the noise is signal-dependent. It has been shown [34] that the sum of two Gaussian random variables is still a Gaussian variable with its variance being the sum of the variance of the two Gaussian variables. Based on these properties of Gaussian variables and the wavelet transform equations (10) and (11), the noise in wavelet coefficients still follows the Gaussian distribution and the variance of the noise can be calculated as:

$$V W_{2^{j+1}}^x f(x, y) = V S_{2^j} f(x, y) * (G_j^2, D) \quad (18)$$

$$V W_{2^{j+1}}^Y f(x, y) = V S_{2^j} f(x, y) * (D, G_j^2) \quad (19)$$

$$V S_{2^{j+1}} f(x, y) = V S_{2^j} f(x, y) * (H_j^2, H_j^2) \quad (20)$$

where $V W_{2^j}^X f(x, y)$, $V W_{2^j}^Y f(x, y)$ and $V S_{2^j} f(x, y)$ denote the variance of $W_{2^j}^X f(x, y)$, $W_{2^j}^Y f(x, y)$ and $S_{2^j} f(x, y)$ respectively. The filter coefficients of G_j^2 and H_j^2 equal to the square of the filter coefficients in G_j and H_j correspondingly. $V S_{2^0} f(x, y)$ represents the variance of the initial projection data, and its value can be calculated from equation (14). For each projection data, the local mean value (using a 3×3 window) was used as p_i in equation (14) for estimation its variance. Since the noise in wavelet coefficients still follows the Gaussian distribution, we are able to construct the corresponding PWLS cost function (15) for wavelet coefficients at each resolution scale. In order to consider different signal-to-noise ratios (SNRs) in the wavelet coefficients at different scales, the penalty parameter becomes $\beta / 2^j$ at scale j , *i.e.*, being adaptive to each scale. The optimal wavelet coefficients are then estimated by minimizing the PWLS cost function using the iterative GS updating strategy according to equation (17) with z_i being the sinogram data at the corresponding resolution level.

The flowchart of the presented multiscale PWLS method for noise reduction or restoration of low-dose CT sinogram is illustrated in Figure 1 and the implementation of the multiscale PWLS algorithm is summarized as follows:

1. Calculate the variance image of the original CT sinogram $V S_{2^0} f(x, y)$ according to equation (14).
2. Decompose the CT sinogram to $\left\{ S_{2^J} f(x, y), \left(W_{2^j}^X f(x, y) \right)_{1 \leq j \leq J}, \left(W_{2^j}^Y f(x, y) \right)_{1 \leq j \leq J} \right\}$ according to equations (10), (11) and (12), where J is set to 3 in the following studies.
3. Calculate the variance image of wavelet coefficients $V W_{2^j}^X f(x, y)$ and $V W_{2^j}^Y f(x, y)$ according to equations (18), (19) and (20).
4. Restore the wavelet coefficients at each scale by the PWLS criterion via the equation (17).
5. Compute the de-noised CT sinogram by inverse wavelet transform on the PWLS-processed wavelet coefficients, *i.e.*, determining $S_{2^J} f(x, y)$ according to equation (13).
6. Reconstruct the low-dose CT image from the de-noised sinogram by FBP algorithm.

III. Results

In this section we evaluate the proposed multiscale PWLS algorithm for low-dose CT sinogram restoration by using both computer simulations and experimental studies.

A. Computer Simulations

Computer simulation studies were performed to test the performance of the proposed algorithm using a modified 2D Shepp-Logan head phantom, see Figure 2(a). The phantom array size is $512 \times 512 \text{ mm}^2$ (i.e., the pixel side size is 1 mm) and the parameters of the ellipsoids (or objects) in the phantom can be found in [12]. A total of 984 fan-beam projections for a 2π rotation and 888 bins for each projection view were used in the simulation to generate the CT sinogram. Each projection value along a ray through the phantom was computed based on the known densities and intersection lengths of the ray with the geometric shapes of the objects in the phantom. The detector arrays are on an arc concentric to the X-ray source with a distance of 949 mm and the detector cell spacing is 1.0239 mm. The distance of X-ray source to the center of rotation was 408 mm. This detector configuration simulates a commercial CT system. After the ideal or noise-free sinogram was generated, a signal-dependent Gaussian noise was added according to equation (14). Figure 2 shows the results of FBP reconstruction using different noise reduction methods. Figure 2(b) shows the standard FBP reconstruction (i.e., using the Ramp filter at 100% Nyquist frequency cutoff) from the noisy sinogram, where strong streaking artifacts are presented. Figure 2(c) is a conventional FBP reconstruction from the noisy sinogram using the traditional low-pass Hanning filter at 80% Nyquist frequency cutoff. It can be observed that the traditional Hanning filter could not efficiently suppress the noise artifacts. Figure 2(d) shows the de-noised image from Figure 2(b) using a soft-threshold method with db4 wavelets from the wavelet toolbox in Matlab7.0 (R14). Figure 2(e) shows the standard FBP reconstruction from the de-noised sinogram by the soft-threshold method with db4 wavelets of the wavelet toolbox. Both the threshold-based wavelet denoising algorithms can not effectively remove the noise and artifacts. Figure 2(f) shows the standard FBP result from the direct or single-scale PWLS smoothed sinogram and Figure 2(g) shows the standard FBP result from the proposed wavelet-based PWLS restored sinogram. For both the proposed multiscale PWLS method and the single-scale PWLS approach, the value of the smoothing parameter β was chosen ranging from 1×10^{-10} to 1×10^{-5} . Figure 2(f) and Figure 2(g) show the visually optimal results in the selected β value range. Both the direct PWLS approach and the multiscale PWLS method can suppress the noise-induced artifacts effectively. The multiscale method shows slightly better performance over the single-scale approach in terms of noise reduction and streak artifact suppression, as indicated by the arrows in Figure 2(f) and Figure 2(g). We further calculated the signal to noise ratio (SNR), which is defined as mean divided by standard deviation, of a rectangular region of interest (ROI) in both Figure 2(f) and Figure 2(g). The SNR is 89.7 and 94.5 in Figure 2(f) and Figure 2(g) respectively. More quantitative comparison studies will be performed in Section C via noise-resolution trade-off and full-width-at-half-maximum (FWHM) measurements.

B. Experimental Studies

We also show the effectiveness of the proposed multiscale PWLS method upon a 10 mAs low-dose CT noisy sinogram acquired from an anthropomorphic shoulder phantom. The sinogram of the phantom was acquired by a commercial multi-slice spiral CT scanner with fan-beam curved detector arrays. The number of bins per view is 888 with 984 views evenly spanned on a circular orbit of 360° . The detector arrays are on an arc concentric to the x-ray source with a distance of 949 mm. The distance from the rotation center to the curved detector band is 408.075 mm. The detector cell spacing is 1.0239 mm. Figure 3(a) shows the reconstructed image from the original sinogram by the standard FBP with the Ramp filter at 100% Nyquist frequency. For the purpose of comparison, Figure 3(b) shows the reconstructed image by the conventional FBP using the spatially-invariant low-pass Hanning filter with cutoff at 80% Nyquist frequency. Significant noise-induced streak artifacts can be observed in these FBP reconstructed images. Figure 3(c) shows the de-noised image from Figure 3(a) again using the soft-threshold method with db4 wavelets. Figure 3(d) shows the standard FBP reconstruction from the de-noised sinogram by the soft-threshold method with db4 wavelets. The threshold-

based wavelet de-noising strategies also can not remove the noise and artifacts effectively. The standard FBP (with Ramp filter at the Nyquist frequency cutoff) reconstructed image from the single-scale PWLS approach is shown in Figure 3(e). The standard FBP reconstructed image from the wavelet-based PWLS method is shown in Figure 3(f). Both the FBP reconstructions from PWLS restored sinograms in single- and multiscale domains removed the noise-induced streak artifacts satisfactorily as shown by Figure 3(e) and Figure 3(f). Manipulating the cutoff frequency of the Hanning filter in a conventional FBP reconstruction could not reach a comparable noise-resolution tradeoff as that of these PWLS approaches. At a higher cutoff frequency, the Hanning filter produced a more noisy result, while at a lower cutoff frequency, more edge details were smoothed out. Manipulating the thresholds in the soft-threshold wavelet method also could not reach a comparable noise-resolution tradeoff as that of the PWLS approaches. In contrary, excellent noise reduction with satisfactory resolution preservation by both PWLS sinogram-smoothing methods is seen. A better performance of the multiscale method over the single-scale approach is again observed in the zoomed region of interest of Figure 3(e) and Figure 3(f), see the area indicated by the arrows in Figure 3(g). The edge of region I in Figure 3(g) is more blurred than the corresponding area in Figure 3(h), where the small separation between the two bony materials is not visible in Figure 3(g). The streaking artifacts in region II in Figure 3(h) is noticeably reduced in the corresponding area in Figure 3(g). To further quantitatively evaluate these two PWLS approaches, we performed noise-resolution tradeoff studies as described in the following section.

C. Noise-Resolution Tradeoff Measure

The noise-resolution tradeoffs of the multiscale PWLS method and the single-scale PWLS approach were computed by computer simulations using an ellipse digital phantom, as shown in Figure 4, which is similar to the one used in [11]. The hot disks are assumed to be bone and they are surrounded by tissue equivalent materials. The detector array and X-ray source configuration is exactly the same as the commercial CT scanner described previously. The ellipse phantom was sampled by the detector system on a circular orbit. The noise-free sinogram was computed based on the known densities and intersection lengths of the projection rays with the geometric shapes of the objects in the phantom. Noisy sinograms were generated by adding signal-dependent Gaussian noise according to equation (14), simulating the low-mAs CT data acquisition protocol. Image reconstruction was performed on a 512×512 array size in the image domain.

The reconstructed image resolution was analyzed by the edge spread function (ESF) along the central vertical profiles on the left disk and the center disk in the reconstructed ellipse-phantom image, which is similar to that used in [11]. The ESF is a measure of the broadening of a step edge. Assume that the broadening kernel is a Gaussian function with standard deviation σ_b ; the ESF can be described by an error function parameterized by σ_b . By fitting the vertical profiles through the centers of the left disk and the center disk to an error function, we can obtain the parameter σ_b . The obtained parameter σ_b reflects the broadening Gaussian function and, therefore, the reconstructed image resolution. Figure 5 shows, for example, a profile along the center of the left disk in a reconstructed image after the sinogram was processed by the multiscale PWLS method. The profile was fitted by the error function and the broadening of the edge reflects the resolution loss of the noise reduction method.

The reconstructed image noise was characterized by the standard deviation of a uniform region around the left disk and the center disk in the reconstructed ellipse-phantom image. By varying the penalty parameter β for the multiscale PWLS method and the direct or single-scale PWLS approach, we obtained the noise-resolution tradeoff curve for each approach. The noise-resolution tradeoff curves are shown in Figure 6, where the KL-domain PWLS [13] is included for a reference as suggested during the review process. At both the left disk and the center disk,

the proposed multiscale PWLS method shows a better performance than the single-scale PWLS approach in all the resolution range. The gain of the multiscale PWLS method over the single-scale PWLS approach can be explained by the fact that in the proposed multiscale method the PWLS criterion is applied mostly on the noise-dominant wavelet coefficients of the multi-resolution representation of the original sinogram. There is no modification on the signal-dominant coarsest approximation image $S_{2^J} f(x, y)$. Additionally, the penalty parameter in

the presented wavelet-based PWLS method is adaptive to different scales. These considerations lead the proposed method to preserve signal as much as possible and smooth only those noise-dominant components. Indeed, this study concurs with the previous work that a multi-resolution Bayesian tomographic reconstruction outperforms the commonly-used fixed resolution Bayesian methods [35].

In addition to the noise-resolution trade-off studies described above, we also calculated the FWHM for a point-like source. The point-like source is presented at pixel (256, 51) in Figure 4 and its radius is 1 mm. The noise-free sinogram was calculated in the same procedure as described in the noise-resolution trade-off study above. The noisy sinogram was generated according to the signal-dependent Gaussian noise model of equation (14). The noisy sinogram was then processed by both the single-scale and the multiscale PWLS algorithms. The smoothing parameter is chosen such that the noise level (measured by the variance of the uniform ellipse area in Figure 4) is the same in both reconstructed images from the single-scale and multiscale PWLS processed sinograms. The vertical profiles through the center of the point source are shown in Figure 7. It can be observed that the profile from the multiscale PWLS method is narrower than the single-scale PWLS approach. The FWHM of the profile is 2.32mm for the single-scale PWLS approach and 2.20mm for the multiscale PWLS algorithm. This quantitative measurement is consistent with the noise-resolution trade-off studies described above.

The presented multiscale PWLS method is computationally efficient. Its computation complexity is $O(N^2 \log(N))$ for an $N \times N$ array size [27]. It took only 20 seconds on a PC with 2.4 GHz CPU to process an 888×984 array size sinogram without any optimization on the computer code. The multiscale PWLS algorithm can be implemented in a parallel fashion on the multiscales and this could further improve the computational efficiency. For simplicity, this work is presented in two dimensions. The principles remain the same when the presented method is extended to three dimensions.

IV. Discussion and Conclusion

Inverting the Radon transform by FBP algorithm for tomographic image reconstruction has shown advantages in multiscale domain over in single-scale or the original sinogram space [23-26]. The advantages could be further realized if noise reduction is performed in the multiscale domain [35]. Wavelet-based multiscale de-noising methods have been extensively studied and applied in medical imaging processing in recent years. Most of them are based on thresholds, where the wavelet coefficients are discarded if they are less than some thresholds. A moderate improvement has been observed in this study concurring with others [14-17]. Further improvement has been seen using minimum mean-square error (MMSE) estimation, a typical example of soft-threshold restoration approach, to restore the wavelet coefficients [18-20,36-38], where the variance of the wavelet coefficients were estimated from the local statistics among the nearby pixels. Given the relationship of the variance and the mean of low-dose CT sinogram [3,9], the presented multiscale PWLS cost function is a better choice than the MMSE.

The presented multiscale PWLS method decomposes the low-dose CT sinogram to different resolution levels using the Mallat-Zhong's wavelet transform and applies the PWLS criterion

to restore the wavelet coefficients at each decomposition level. In the Mallat-Zhong's dyadic wavelet transform, there is no down-sampling operation during the decomposition and thus the translation invariance property of the transform is preserved. The translation invariance is important in noise reduction to preserve all the information of the signal in the original sinogram image [39]. The weights in the PWLS cost function are updated at each decomposition level and the penalty parameter is adaptive to each different scale. Because of a more accurate cost function for the sinogram noise and the unique properties of translation invariance and no down-sampling, the effectiveness in noise reduction and the efficiency in computation of the presented multiscale PWLS method are expected. The wavelet transform provides a means to separate signal and noise, i.e., the signal-dominant approximation and the noise-dominant wavelet coefficients. After the wavelet transform, the noise is dominant in the wavelet coefficients at lower scales. The penalty strength is adaptive to the noise level at different resolution scales during the PWLS processing. This novel method could facilitate further development of the multi-resolution tomographic image reconstruction framework [23-26] toward low-dose CT application.

In this study, we mainly focus on the comparison between multiscale and single-scale PWLS approaches. The gain by the multiscale PWLS approach is demonstrated by computer simulations, phantom experiments, and noise-resolution tradeoff measurements. In our previous study [13], we compared the single-scale PWLS sinogram noise reduction with the conventional low-pass linear and non-linear noise filters, such as Hanning and Butterworth and specifically the adaptive trimmed mean (ATM) filter which is well known in CT field. The gain by the single-scale PWLS approach was documented by both noise-resolution trade-off measurements and computer observer studies. Compare this study with [13], we can make a conjecture that multiscale PWLS approach will outperform those linear and non-linear noise filters for low-dose CT application.

Inverting the Radon transform by FBP method in the Karhunen-Loève (KL) domain is another interesting topic, where the KL transform can de-correlate the sinogram data and rearrange the sinogram noise along the KL principal components in the KL domain. In [13], we utilized the KL transform to consider the angular correlation of tomographic data and applied the single-scale PWLS approach to adaptively treat the noise of the principal components in the KL domain. For reference, we performed a comparison study between the presented multiscale PWLS and the single-scale PWLS in KL domain. Both approaches have a similar performance in terms of noise-resolution tradeoff, as shown by Figure 6. It is clear that adapting the multiscale PWLS to treat the KL principal components would further improve the noise reduction toward low-dose CT application. This is worth for further investigation.

The presented multiscale PWLS cost function is based on the noise properties of the log-transformed and system calibrated sinogram in the Radon space, where FBP inversion of the Radon transform has been extensively investigated. Therefore, both simulation and experimental studies were performed in the Radon space. Alternatively, the studies would start from the raw data space prior to the log transform operation. The raw data can be modeled accurately as compound Poisson distribution (to consider the polyenergetic nature of the X-rays) plus electronic noise background [22]. The modeled cost function could be minimized for image reconstruction with some approximations. So far this alternative approach remains in the single-scale raw data domain and its implementation is relied on an iterative algorithm because of the non-linear log transform between the raw data space and the Radon space [10, 11]. On the other hand, theory has been established linking the Poisson model of X-rays generation and the approximated Gaussian model of the sinogram data [7]. The polyenergetic nature of the X-rays imposes a convolution operation on the Poisson model [22], resulting in a variation in the variance of the approximated Gaussian model. The relationship between the variance and the mean (i.e., the first and second statistical moments) of the approximated

Gaussian model has been established from repeated experiments for the log-transformed and calibrated sinogram [3,9] and is fully utilized by the presented PWLS method. Although higher order statistical moments could be utilized by the more accurate cost function in the raw data space, minimizing the cost function in the raw data space is much more challenging than the numerical treatment of the approximated Gaussian functional in the Radon space. Considering the system calibration, the challenging becomes more severe. Based on the approximated Gaussian model, inverting the Radon transform in different domains becomes possible and the associated numerical optimization for image reconstruction is not only tractable, but also simple and efficient. It is worth for further research effort to explore various approaches in either the raw data space or the Radon space. It should be noted that the PWLS approach is not necessarily depending on the Gaussian noise model, but is strongly depending on the mean-variance relationship. At very low count levels, the Gaussian noise model may no longer hold but the PWLS approach remains valid once the mean-variance relation holds.

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Bio and Photos:

Jing Wang received the B.S. degree in Material Physics from University of Science and Technology, China, in 2001 and the M.A. and Ph.D. degrees in Physics from State University of New York at Stony Brook (SUNY-SB) in 2003 and 2006 respectively. During his Ph.D. studies, he worked as a graduate research assistant in the Laboratory for Imaging and Informatics of the Department of Radiology at SUNY-SB. He is currently a Postdoctoral Fellow in the Department of Radiation Oncology at Stanford University. His research interests include tomographic image reconstruction, medical image processing and image-guided radiation therapy.

Hongbing Lu received her Ph.D. degree in Biomedical Engineering from Tsinghua University of China in 1998. After graduation, she had been a Postdoctoral Researcher in Radiology Department at SUNY-SB for three years. She joined the Fourth Military Medical University in 1991 and currently is a Professor and Director of the Computer Application Department in the Biomedical Engineering School. Her research interests focus on medical image reconstruction, noise reduction, medical application of image processing and visualization methods, and modeling and simulation of physiology system. She has authored more than 50 scientific publications and is the principal investigator of several projects funded by the National Science Foundation, the Ministry of Education, and the Department of Defense of China.

Junhai Wen received the Ph.D. degree in Biomedical Engineering from Xi'an Jiaotong University of China in 1999. Thereafter, he had been a Postdoctoral Fellow and Research Associate in University of Hawaii and SUNY-SB, respectively. He joined Beijing Institute of Technology in 2004 and currently is an Associate Professor and Vice-director of Biomedical Engineering Department. His research interests focus on medical image processing, reconstruction and visualization. He has authored about 50 scientific publications.

Zhengrong Liang gained a Ph.D. degree in Physics from City University of New York in 1987, followed by one year Research Fellow in Nuclear Medicine and Radiation Oncology at Albert Einstein College of Medicine. He had been a Research Associate and Assistant Professor in Radiology at Duke University Medical Center. He joined SUNY-SB in 1992 and holds a Professorship in the Departments of Radiology, Physics and Astronomy, Computer Science, and Biomedical Engineering since 2000. He was a co-founder of the Program in Biomedical Engineering at SUNY-SB. His primary research interests in medical imaging include data acquisition geometry, image formation and processing methodology, and feature-based visualization and computer-aided detection. He has authored more than 100 scientific publications and serves on the Editorial Board of the *IEEE Transactions on Medical Imaging* since 1999. He holds several patents in medical imaging applications, leading to a startup company to commercialize virtual colonoscopy for colon cancer screening.

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Figure 1.

Flowchart of the presented multiscale PWLS sinogram restoration for low-dose CT. (a) Mallat-Zhong's dyadic wavelet transform and (b) inverse wavelet transform for the restored low-dose CT sinogram from the PWLS-processed wavelet coefficients, where $\hat{W}$ denotes the processed data.

Figure 2.

Simulation results from the modified Shepp-Logan phantom: (a) noise free image or phantom; (b) standard FBP reconstruction with Ramp filter at the Nyquist frequency cutoff; (c) conventional FBP reconstruction with Hanning filter at 80% Nyquist frequency cutoff; (d) de-noised result from image (b) by soft-threshold method with db4 wavelets; (e) standard FBP reconstruction from de-noised sinogram by the soft-threshold method with db4 wavelets; (f) standard FBP reconstruction from the single-scale PWLS smoothed sinogram with penalty parameter $\beta = 1 \times 10^{-8}$; and (g) standard FBP reconstruction from the multiscale PWLS restored sinogram with penalty parameter $\beta = 2 \times 10^{-7}$. The display window is [550, 850].

Figure 3.

Experimental results from the 10 mAs shoulder phantom data: (a) standard FBP reconstruction with Ramp filter at 100% Nyquist frequency cutoff; (b) conventional FBP reconstruction with Hanning filter at 80% Nyquist frequency cutoff; (c) de-noised result from image (a) by soft-threshold method with db4 wavelets; (d) standard FBP reconstruction from de-noised sinogram by the soft-threshold method with db4 wavelets; (e) standard FBP reconstruction from the single-scale PWLS smoothed sinogram with penalty parameter $\beta = 1 \times 10^{-8}$; (f) standard FBP reconstruction from the multiscale PWLS smoothed sinogram with penalty parameter $\beta = 2 \times 10^{-7}$; (g) and (h) are zoomed region of interest (ROI) of (e) and (f) correspondingly. The display window is [200, 600].

Figure 4.
The phantom used for the noise-resolution tradeoff studies.

Figure 5.
Profiles along the center of the left disk of the phantom in Figure 4.

Figure 6.

The noise-resolution tradeoff curves for the presented multiscale PWLS, single-scale PWLS algorithms and KL domain PWLS approach: (a) left disk and (b) center disk.

Figure 7.

Profile through the point source presented in the ellipse phantom in Figure 4. The dotted line shows the profile of the phantom reconstructed from the noise-free sinogram using FBP. The profile from the multiscale PWLS method is sharper than the one from the single-scale PWLS approach.