Birds' tails do act like delta wings but delta-wing theory does not always predict the forces they generate

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Delta-wing theory, which predicts the aerodynamics of aircraft like the Concorde, is the conventional explanation for the way in which a bird's tail operates in flight. Recently, doubt has been cast on the validity of applying a theory devised for supersonic aircraft to the small tails of slow-flying birds. By testing delta-wing models and birds' tails behind bodies with wings, I empirically show that the tails of birds produce lift in a very similar way to conventional delta-wing models. Both Perspex and birds' tail models produce lift similar to that predicted by delta-wing theory when narrowly spread and at low angles of attack. However, when widely spread and at high angles of attack, both tails and Perspex models produce much less lift than predicted, owing to vortex breakdown after which the assumptions of delta-wing theory are violated. These results indicate that birds' tails can be regarded as delta wings but that the theory predicting the forces produced by delta wings can only be applied within acceptable limits (i.e. tails spread less than 60° and at angles of attack of less than 20°).

Keywords: delta-wing theory; aerodynamics; birds' tails; bird flight

1. INTRODUCTION

The avian tail has many functions: it apparently controls stability, produces lift at low speeds, improves manoeuvrability, reduces body drag and has an ornamental role in some species. However, the precise aerodynamic functions of the tail have been the subject of speculation (Maynard Smith 1952; Pennycuick 1975; Norberg 1990). A bird's tail is a similar shape to the wings of modern high-performance (delta-wing) aircraft like Concorde. Thomas (1993) suggested that a bird's tail could be regarded as a low aspect ratio delta wing; he then used theories developed to predict the lift forces on delta-wing aircraft to predict the aerodynamic forces generated by a bird's tail. Delta-wing theory now provides the conventional explanation for force generation by birds' tails.

Delta wings are sensitive to the phenomenon of vortex breakdown, after which the wing stalls. Vortex breakdown occurs at a critical angle of attack after which the vortices that were previously bound to the upper surface of the wing become detached. Wind-tunnel experiments on delta-wing models have demonstrated that vortex breakdown occurs earlier as the apical angle increases (Ericsson 1984) and minor variations in the apex geometry of a delta wing can produce large effects on vortex breakdown (Lowson & Riley 1995). While a delta-wing aircraft and the tail of a bird are superficially morphologically similar, there are many differences, the most obvious of which is that a bird's tail has a body and wings in front of it which will disrupt the airflow over the tail, particularly in slow flight and during turns. The presence of the bird's body at the apex of the tail also substantially changes the apical geometry compared to the point of a classic delta wing. It has been estimated that the theoretical optimal tail lift-drag ratio for a bird's tail occurs at an apical angle of 120° (Thomas & Balmford 1995): this is much wider than conventional delta wings and vortex breakdown should occur

at a very small angle of attack on such a delta wing. Thus, there are reasons to doubt the applicability of this theory to the tails of birds.

There have been two attempts to provide tests of delta-wing theories as applied to the tails of birds. Maybury *et al.* (2001) showed that the lift produced by a bird's tail does not change as predicted by delta-wing theories when the tail is experimentally spread. However, this experiment was conducted on model birds with the wings removed which means that the interaction between the wings and the rest of the body is missing. Evans *et al.* (2002) examined the flight of live swallows in a wind tunnel and demonstrated that the tail and wing morphology did not vary in the manner predicted if the tail produced lift according to delta-wing theories (Thomas 1996). The predictions used in this experiment would be sensitive to the particular aerodynamic model (in this case Pennycuick's momentum jet model; Pennycuick 1975, 1989) that was used as the basic framework. Therefore, it is possible that the failure of the predictions to be upheld might be owing to factors other than the force generation of the tail.

The objective of the work presented here was to provide a direct, empirical test of the hypothesis that birds' tails produce forces in a similar way to delta wings.

2. MATERIAL AND METHODS

(a) Wind-tunnel design

Measurements were made in an open circuit wind tunnel with a 1 m long closed working section that was 0.94 m × 0.94 m in cross-sectional area. The contraction ratio of the wind tunnel was 10 : 1. Dynamic pressure was measured using a MO521 pitot static tube (Furness Controls Ltd, Bexhill-on-Sea, UK) mounted at the upstream end of the test section. This was connected to a Sen-I-Tran FCO 322 (0–100 Pa) pressure transducer, with a reported accuracy of ±0.01 Pa. The same pitot tube was used to examine spatial variation in airspeed in a plane

Figure 1. Diagram showing the positions and arrangement of the bird and tail during force measurement. Also shown is the design of the balance used to measure lift and drag forces.

parallel to the entrance of the test section along the line of test object insertion. At 10 m s^{-1} the spatial variation in airspeed (taken as the mean of the residuals about the sample mean, as a percentage of the mean), was $\pm 0.5\%$ taken to within 0.04 m of the wall. The airspeed in the wind tunnel was controlled by a 1 m diameter LC1000/4/5A/22.5 fan (London Fan Co., London, UK); a 584SV inverter control mechanism provides 1000 digital settings ranging between 0 m s^{-1} and full speed.

Turbulence intensity was measured as the root-mean-squared fluctuations in the longitudinal velocity expressed as a percentage of the mean velocity. These measurements were taken in a plane parallel to the entrance of the wind tunnel along the line of object insertion. At 10 m s^{-1} , turbulence intensity was less than 0.75% over the whole cross section to within 0.03 m of the walls. At airspeeds between 2 and 15 m s^{-1} the turbulence intensity was less than 0.5% in the centre of the working section. Turbulence intensity was measured using a hot-wire probe (Dantec Electronics Ltd, Bristol, UK), a constant temperature single wire anemometer that measures velocity within a two-dimensional plane.

(b) Force measurement

Force measurements were made using a strain gauge system designed to measure forces along two axes. The test object was mounted on a cylindrical steel sting $0.47 \text{ m} \times 0.0033 \text{ m}$, which was attached to a machined steel block (figure 1). This block had two perpendicular flats machined on it, onto which electronic strain gauges (RS 308-102, reported accuracy 0.5%) were glued using cyanoacrylic 'superglue' (Loctite Ltd). The strain gauges were wired into one arm of a Wheatstone bridge circuit with other arms maintained by high-stability resistors; the resulting signal was amplified using a strain gauge amplifier (RS Components Ltd, Corby, UK) and fed into a LabView (National Instruments, Austin, TX, USA) data acquisition program which made 1000 measurements of the voltage at 500 Hz and calculated the mean and standard deviation of this sample. The standard deviation therefore reflects true sampling error. The balance extended through the sidewall of the wind tunnel

and was mounted on a solid pillar standing independently of the wind tunnel. The balance was mounted so that one flat was vertical and one horizontally positioned.

Force calibration was conducted before every data collection session. The calibration was conducted using weights and pulleys arranged to exert force either vertically or horizontally on the sting at the same point at which the test objects were mounted. This calibration allowed voltage changes in the strain gauge system to be converted into force. Cross talk between the lift and drag channels was determined by measuring the lift output while loading only drag and vice versa; cross talk was measured as being less than 1% and was corrected for during data processing. Blockage corrections were neglected as the model areas were small relative to the cross section of the wind tunnel. The resonant frequency of this system was measured as 8 Hz when unloaded.

Measurements were made in sets of three as follows.

- (i) Measurements of both lift and drag were made with the fan stationary (power supply on); this corresponds to the forces exerted by the weight of the model in stationary air.
- (ii) Measurements were made with the fan running and producing the required air speed. These measurements correspond to the forces (lift, drag and weight) exerted on the model in moving air.
- (iii) The final set of measurements was made in stationary air, identical to the first set.

Lift and drag forces were calculated from these measurements by subtracting the mean of the first and third measurements from the second measurements. Data presented here are therefore corrected for the mass of the model.

(c) Flow visualization

The smoke-wire technique was used to visualize airflow around the models during these experiments (Batill & Mueller 1981). A vertical nickel-chromium wire that was 0.37 mm in diameter, was stretched upstream across the airflow, and coated with oil (Power Fog, Optical Smoke System, The Smoke Co. Stockport, UK); this produced short bursts of smoke when the wire was heated resistively. The airflow was illuminated by a 150 W tungsten-halogen light shining vertically down through a narrow transparent slit in the otherwise opaque upper surface of the wind tunnel. The airflow was filmed using a Sony Digital Handycam Video Camera (DCR-VX1000E). The video was digitized using the miroMOTION Dc20 digitizer on an Apple Macintosh 9500. This method for flow visualization only worked well at airspeeds below 7 m s^{-1} ; therefore flow visualization was only conducted on models at 2.5 and 5 m s^{-1} .

(d) Experimental design

This experiment was conducted using three species of birds—woodpigeons (*Columba livia*), starlings (*Sturnus vulgaris*) and house sparrows (*Passer domesticus*). Experiments were conducted at a range of airspeeds (starling and house sparrow, 5 and 10 m s^{-1} ; woodpigeon, 2.5 and 5 m s^{-1}). These speeds were chosen to encompass the range of speeds at which small- to medium-sized birds typically fly and spread their tail in level flight.

For each species, two types of models were constructed as follows.

- (i) Perspex delta-wing models constructed from 10 mm Perspex sheet, with the leading edges cut at an angle of 45° .

These models had the shape of an isosceles triangle with the lengths of the sides equal to the length of the outer tail feathers of the bird. These models were designed to be similar to models used in studies of delta-wing aerodynamics (Lee & Ho 1990).

- (ii) Bird-tail models were made with an epoxy resin tip, which was cast from a mould of the corresponding delta-wing model. This tip extended 20% of the way down the side of the model: tail feathers were pushed into the resin before it set and arranged to lie as they would in a live bird; this created a model that had the same dimensions as the equivalent Perspex model. A specimen of each species was frozen in gliding posture with the tail removed. The feet of the bird were held up as high as possible into the bird's body; in all cases they were partly but not entirely covered by the body feathers. The wings were spread into a gliding position as judged by comparison with photographs of gliding birds. Once frozen the wing muscles were rigid and inflexible (wing span: woodpigeon, 0.54 m; starling, 0.288 m; house sparrow, 0.159 m). A 0.003 m diameter cylindrical rod was pushed into the bird's abdomen extending 0.3–0.4 m back and down from the bird to provide a mounting point. This rod was clamped at the bottom of the test section of the wind tunnel. The body feathers were treated with a light, even coating of hair wax so that feathers retained their position but some compliance remained in the integument (Pennycuik *et al.* 1988). The frozen bird was held by the abdominal rod upstream of the tail model such that the epoxy tip of the tail was within the tail coverts of the bird's body; the epoxy tip was as close to the bird's body as possible consistent with the need for the tail to move in the airflow. The position of the bird was adjusted so that the tail lay in an appropriate position after the airflow had been started. To reduce any problems created by the bird defrosting while the measurements were being taken, the bird bodies were removed from the freezer for just 15 min per day. No loss in the stiffness of the wings or body was noticed during the experiments.

Four tail spreads—40°, 60°, 90° and 120°—were used for every model. Therefore, in this experiment a total of 24 models were employed (three species, two model types, four tail spreads). Each model was mounted in the wind tunnel at a range of angles of attack (0–60° in 5° steps; figure 1). The airflow in the test section was horizontal and so the angle of attack of the tail was measured relative to horizontal. Lift force measurements are expressed as lift coefficients, defined as

$$C_l = \frac{L}{0.5\rho S_t V^2},$$

where V is the airspeed and S_t is the tail planform area and the air density ρ was taken as 1.225 kg m⁻³.

(e) Theoretical predictions

There are several theoretical models available to predict the lift from a delta wing or a bird's tail. I generated lift predictions from three models that make different assumptions about the nature of the airflow.

(i) Slender wing

This is a linearized theory appropriate to thin, flat wings, which assumes that vortices are shed from the widest point of

the wing. It is only appropriate to slender wings (i.e. spread less than 30°) and low angles of attack (Jones 1990). The coefficient of lift is given by

$$C_l = \frac{\pi b_{\max}^2 \alpha}{2S_t},$$

where $b_{\max}$ is maximum tail span and α is the angle of attack relative to the airflow in which the tail is operating.

(ii) Leading edge vortex

This is a conventional delta-wing theory and assumes that vortices are shed from the leading edge of the wing and are bound to the upper surface (Polhamus 1971). Lift is modelled as the sum of the potential and vortex components, the latter being the additional lift generated by the suction of the bound vortices. This model has been shown to have much wider applicability than slender wing theory (i.e. to wider spread wings and higher angles of attack). The coefficient of lift is given by

$$C_l = k_p \sin \alpha \cos^2 \alpha + k_v \sin^2 \alpha \cos \alpha,$$

where k_p and k_v are the potential and vortex force constants, respectively. These were determined from the aspect ratio using eqns (6) and (7) in Maybury *et al.* (2001).

(iii) Lifting line

This model assumes that vortices are shed from the trailing edge of the wing; it is appropriate to wings in which the wing-span is greater than a chord. The lift coefficient is calculated as

$$C_l = \frac{\alpha m A}{((1 + \tau)m/\pi) + A}$$

(Von Kármán & Burgers 1935), where m is a constant determined by the wing profile (taken as 2π as the tail approximates to a thin, flat wing), A is the wing aspect ratio and τ is determined by the wing planform shape; this was computed using a program provided by Professor J. M. V. Rayner which follows the method of Glauert (1947).

3. RESULTS

The coefficients of lift of the slender (40°) Perspex delta wings were similar to those found by other studies (Lee & Ho 1990). At low angles of attack, the coefficients of lift of these models were similar to those predicted by delta-wing theory (figure 2), with the coefficient of lift increasing to a maximum at 35–40° angle of attack. Beyond the maximum, the coefficient of lift dropped below the predicted lift. Flow visualization revealed that this drop in lift coincided with vortex breakdown. Performance was similar at the two airspeeds and for the three species. The substantial overlap between the standard deviations for observations at the two airspeeds shows that the coefficient of lift at the different airspeeds would not be significantly different. Again, this is similar to published results showing that delta-wing aerodynamics are insensitive to changes in Reynold's number: the range of the chord Reynold's numbers used here was 47 000–7000.

Models with spreads that were wider than *ca.* 60° are not usually regarded as showing delta-wing aerodynamics. I was unable to locate results for delta wings greater than 90° in the literature. However, the optimal lift–drag ratio for a bird's tail has been estimated as occurring at a spread

Figure 2. The coefficient of lift changes with the angle of attack between the model and the airflow and with the amount by which the tail is spread. These results are from (a) woodpigeon-sized models and (b) starling-sized models at two tail spread angles (i) 40° and (ii) 120°. For comparison the lift coefficients predicted by leading edge vortex (solid line), slender wing (dashed line) and lifting line (dotted line) models are shown. The coefficient of lift of the Perspex delta-wing model is given by squares and the bird's tail by circles. Experiments were conducted at two air speeds as described in § 2d; for each species the higher speed is given by the black symbols and the lower speed by the white symbols. Some jitter has been added to the x -axis to increase the legibility of the symbols.

angle of 120° (Thomas & Balmford 1995). In my experiments the Perspex delta-wing models with a 120° spread had a coefficient of lift that was relatively constant across all angles of attack comparable to that predicted at low angles of attack (figure 2). Consistent with this, leading edge vortices formed for only a small range of angles of attack (*ca.* 5°) over the 120° Perspex models. The coefficients of lift for these models were significantly lower than predicted for most angles of attack (figure 2).

The presence of the body in front of the tail was expected to substantially reduce lift compared to the delta-wing model and to predictions. However, it is clear that lift production of the tail was generally similar to lift production of the Perspex delta-wing model (figure 2), especially for narrow tails at low angles of attack. Calculation of the 95% confidence intervals ($\text{s.d.}/n^{0.5} \times 1.96$) shows that the coefficient of lift of the Perspex models was not significantly different from that of the bird's tail models at low angles of attack for all tail shapes, but was significantly different at higher angles of attack for the narrowly spread tails. Widely spread tails had coefficients of lift that were not significantly different from Perspex models with similar configurations at most angles of attack.

The widely spread Perspex models show large standard deviations around the mean measurement at low angles of

attack. This reflects the instability of the lift produced by the models at these angles of attack when the model could be seen moving substantially in the vertical plane in the wind tunnel. It is noticeable that lift produced by the tail models did not show this effect and had less variance in lift than the Perspex models with similar configurations.

The three theoretical models used here were all reasonable predictors of the lift produced by a bird's tail at low angles of attack and unsurprisingly were better predictors for the 40° spread tail than for more widely spread tails (figure 2). The results for the 40° tails suggested that all three models accurately predicted the lift produced by the 40° birds' tail models until moderate angles of attack (up to *ca.* 20°). The lift produced by the tails spread by 60° or more was not well predicted by theory and generally there was significant departure from theory by *ca.* 10° angle of attack for these tails.

Flow visualization revealed that consistent with other published work, vortices developed over narrowly spread Perspex delta-wing models (figure 3): these vortices are initially small and expand as angle of attack increases until a critical angle of attack after which they detach from the surface of the wing. This angle of attack gets progressively shallower as the tails become broader (vortices were bound to the upper surface of the 40° model until an angle

Figure 3. Side views of the flow over models showing the development of vortices and wake, visualized by the smoke-wire technique. All images show woodpigeon-sized models. (a) 40° Perspex model at a 10° angle of attack—note the bound vortices on the dorsal surface; (b) 40° Perspex model at a 50° angle of attack—showing flow detached from the dorsal surface; (c) 120° Perspex model at a 5° angle of attack; (d) 120° Perspex model at a 15° angle of attack—flow already detached from the dorsal surface; (e–h) as above but with bird model. Scale bars, 5 cm.

of attack of 45°, to the upper surface of the 60° model until an angle of attack of 25°, to the upper surface of the 90° model until 15°, while they were not detected on the 120° model). The flow was similar at the two airspeeds (for woodpigeon-sized models) and in the three sizes of models. The presence of the bird's body in front of the tail in the bird models disrupted the airflow over the tail and made it difficult to detect fine structure in the airflow over the tail, especially at low angles of attack when the tail lay within the wake of the body. However, the flow was clearly detached from the upper surface of the tail at

similar angles of attack to that seen in the Perspex models (figure 3).

4. DISCUSSION

The results presented here show that birds' tails generally produce lift like delta-wing models, thus validating the assumptions made by Thomas (1993). Similarly there was a good match between the theoretical predictions and the observed lift produced by tails, for certain tail configurations. Clearly, for all types of model there was an upper

angle of attack beyond which theory was no longer a good predictor of lift produced; this coincided with vortex breakdown over the highly inclined wing. The fact that after vortex breakdown, delta-wing theory no longer applies has been widely reported in the aerospace literature (for review see Lee & Ho 1990). This critical angle of attack varies with the extent to which the wing is spread, and wings with greater than *ca.* 60° spread are not usually regarded as demonstrating delta-wing aerodynamics. However, the results reported here show that delta-wing theories can predict the force generated by widely spread tails but only at small angles of attack. A rather obvious conclusion from the work presented here is therefore that delta-wing theories can be applied to birds' tails but a certain amount of discrimination needs to be applied. Thomas & Balmford (1995) predicted that the optimal tail shape for birds would be one that had a triangular plan-form at 120°. My results suggest that when spread to this extent, a tail would produce lift but that this would be substantially lower than that predicted from delta-wing theory for angles of attack over 10–15°.

In some circumstances the lift produced by a tail can be substantial—the maximum lift achieved by the woodpigeon tail was 0.56 ± 0.02 N (11% body weight), the starling tail was 0.34 ± 0.03 N (45% body weight) and the house sparrow tail was 0.22 ± 0.03 N (73% body weight) (all at the higher of the two airspeeds, 120° spread and angle of attack of 35–45°). Recent experiments using model starlings reported substantially less tail lift than that found here (Maybury *et al.* 2001). It is possible that this may be because the models of Maybury *et al.* (2001) had no wings while those used here were intact. Also, in the previous experiments tail lift was calculated by measuring the lift of the whole animal and subtracting the lift of the same body with the tail at an angle of attack of 0°, whereas in my experiments tail lift was measured directly.

The results presented here were obtained with models in which the wings were set in a gliding posture, and assume that the bird would spread its tail in level flight at these airspeeds. It remains a possibility that lift generation would differ when the wings are flapping owing to disruption of the airflow over the tail by the flapping wings. Against this it has been found that in level flight the wakes of flapping and gliding kestrels (*Falco tinnunculus*) are similar (Spedding 1987*a,b*), which would presumably mean that the state of the airflow over the tail would be similar in these two modes of flight. Tails are typically widely spread during slow flight, take-off, landing and during manoeuvres. Although the lowest airspeeds used in my experiments are similar to those that would be used during slow flight, take-off and landing, the impact of changing body angle of attack or bank angle has not been considered here. The presence of the wings on the bird models would mean that the airflow over the tail was deflected downwards by the wings, and the wingtip vortices should have been outside the tail even at maximum tail spread (Spedding 1987*a*). This would mean that the angle of attack relative to the air moving over the tail behind the bird would be slightly less than that recorded here, which assessed angle of attack in relation to the horizon.

The morphology of birds' tails can be modified substantially. The ability to generate highly variable amounts of

force by changing spread and angle of attack and to present this force at variable angles to the direction of movement will be a major advantage to a bird faced with the problems of manoeuvring in crowded environments or maintaining stability.

In conclusion, it would appear from the results presented here that the tails of birds do function like delta wings. Delta-wing theory would appear to be a useful starting point to predict the forces produced by a bird's tail but it cannot be universally applied to all tails. The data presented here suggest that narrowly spread tails (40°) are similar to delta-wing theories up to an angle of attack of *ca.* 20°; however, widely spread tails (60–120°) do not conform to delta-wing theories beyond an angle of attack of 10°. At high angles of attack and for widely spread tails the theory will not be a useful predictive tool. My data suggest that tails continue to produce lift in these configurations but much less than would be predicted by any current theory. Attention needs to be paid to the bounds that should be placed on the application of these theories: within reasonable limits the theory is a useful predictive tool, outside them it is not. As the tails of birds can change rapidly in shape, this will complicate the application of delta-wing theories.

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