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Virtual Slope Control of a Forward Dynamic Bipedal Walker

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Abstract

Active joint torques are the primary source of power and control in dynamic walking motion. However the amplitude, rate, timing and phasic behavior of the joint torques necessary to achieve a natural and stable performance are difficult to establish. The goal of this study was to demonstrate the feasibility and stable behavior of an actively controlled bipedal walking simulation wherein the natural system dynamics were preserved by an active, nonlinear, state-feedback controller patterned after passive downhill walking. A two degree-of-freedom, forward-dynamic simulation was implemented with active joint torques applied at the hip joints and stance leg ankle. Kinematic trajectories produced by the active walker were similar to passive dynamic walking with active joint torques influenced by prescribed walking velocity. The control resulted in stable steady-state gait patterns, i.e. eigenvalue magnitudes of the stride function were less than one. The controller coefficient analogous to the virtual slope was modified to successfully control average walking velocity. Future developments are necessary to expand the range of walking velocities.

Keywords

Gait; Stability; Model; Forward Dynamic

Introduction

Control of bipedal gait is a complex and nonlinear phenomenon. When viewed simply as an actuated mechanical linkage it can be controlled and stabilized by a variety of control methods. Stability is achieved when, “starting from a steady closed phase trajectory, any finite disturbance leads to another nearby trajectory of similar shape” [1]. A closed phase trajectory refers to a phase plane plot of the motion where the initial and final states are equal resulting in a closed loop. In stable gait this implies that for movement trajectories near the limit-cycle, the periodic solution of a system which results in a closed loop phase plane plot [2], will naturally converge toward the steady state trajectories. Active joint torques are the primary source of power and stabilizing control in dynamic walking motion. However the amplitude, rate, timing and phasic behavior of the joint torques necessary to achieve natural and stable performance are difficult to establish. The goal of this simulation was to establish a simple, natural dynamic torque controller to achieve stable bipedal walking with explicit control of walking velocity.

Passive dynamic stability of bipedal walking has been widely demonstrated [3-5]. This describes simulations wherein no active joint torques are required. Instead the walkers derive

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energy input by walking downhill. McGeer [3] employed a two-segment walking pendulum to demonstrate that passive bipedal walking can be intrinsically stable. Later developments included freely swinging knees and confirmed the dynamic stability of higher degree-of-freedom walking systems [6,7]. In those studies the energy gained by walking downhill was balanced by discontinuous energy loss observed at foot strike [3]. Energy loss at foot contact increased with stride-length [3,4]. Likewise, energy gain from downhill walking was related to stride-length and ground slope, i.e. large strides at steep downhill slopes achieved a greater vertical descent per stride than short strides on shallow slopes (Fig. 1). This behavior is a consequence of the governing dynamics associated with swinging pendulums and has been described as natural dynamic walking [3]. Analyses illustrate these natural dynamic behaviors may contribute to limit-cycle stability, i.e. the passive dynamic walker will return to a steady-state equilibrium movement trajectory despite small kinetic or kinematic disturbances. However, energy loss may not be required for stable behavior in passive walking [5] and active controllers may be implemented to contribute to the stability of lossless systems.

The natural dynamic stability of active dynamic walkers has rarely been investigated. Specifically, stable walking on level and uphill ground slopes requires active joint torque and stabilizing controllers. Stable active walking can be achieved using a wide variety of joint torque profiles, amplitudes and timing. These require controllers of various forms and complexity [8]. Control methods based on the impulsive addition of energy were developed and resulted in a stable gait over level ground [6,9]. However, the physiologic validity of high-bandwidth impulsive energy input to represent bipedal animal locomotion is questionable because muscles operate as electromechanical filters [10]. Although the impulsive addition of energy resulted in a stable gait they employed time-based controllers using no state feedback thereby limiting the range of feasible ground slopes for successful walking. Position feedback controllers have also been developed in both linear and nonlinear form [11,12]. These methods, while resulting in stable gait no longer follow the motion trajectories suggested by the natural dynamics of gait. Hence, establishing an input torque profile that is simple, controllable and stable remains unresolved. Recognizing the natural dynamic behavior and intrinsic stability of downhill walking it is logical to apply passive downhill behavior to active bipedal simulations. The motivation for implementing the active virtual-slope controller was to take advantage of natural dynamics of the system while achieving simple stabilizing control.

A virtual slope controller can be implemented to apply active joint torque in a manner that will control bipedal walking on level and uphill ground slopes in a fashion that is representative of stable downhill walking. Pratt [13] introduced the feasibility of virtual active control wherein active joint torque could be used to simulate virtual external support. Similarly, it is reasonable to apply active joint torques during level walking such that the governing equations behave as if the system is walking downhill. This control algorithm and associated active joint torques shall be referred to as a virtual slope controller. During downhill walking the gravity-generated joint torques are related to position or the leg angles. Hence, this input torque scheme can be applied to level and uphill walking as a nonlinear state feedback controller. Since the stride-length is related to the downhill ground slope, controlling the virtual downhill gradient in the active torque controller can permit explicit control of stride-length and associated walking velocity in level or uphill walking. The current study investigated the hypotheses that (1) stability in level bipedal walking can be controlled by applying joint torques that simulate downhill walking and (2) forward velocity of level bipedal walking can be explicitly and simply controlled by the virtual downhill gradient.

Methods

A simulation of bipedal walking was developed including two components: (1) forward-dynamic implementation of a pendulum walker [4] capable of either passive downhill walking

or active driven locomotion, and (2) a nonlinear feedback controller to impose active joint torques. The simulation represents a two-dimensional knee-less walker including two legs of length L and mass M_L , joined by a revolute joint at the point mass of the head-arms-trunk (HAT), M_H (Fig. 2). Leg masses were located at a distance u_{cm} from the hip along a line joining the hip to the point-foot. Anthropomorphic data (Fig. 2) including segment lengths and mass were based upon a 77 kg adult male [14]. The walker moved along a plane of slope γ with respect to horizontal. The walker configuration was represented as a time-dependent vector u_5 $[\theta_S, \theta_N]^T$, where θ_S is the angle of the stance-leg with respect to ground and θ_N is the angle of the nonstance-leg with respect to the stance-leg. Ground clearance of the swing-leg is ignored in this treatment because simple mechanisms such as prismatic joints are readily established to facilitate swing foot clearance that does not influence walker dynamics [4]. The governing equations of motion included differential-equations of movement to represent swing phase dynamics and the conservation of angular momentum to represent foot-strike transitions. These were implemented using classical homogeneous forward-integration techniques (ode45 in MATLAB, Natick, MA).

The equations of movement dynamics were described by classical rigid body mechanics. Because the system was continually in the swing-phase it was represented as a double-pendulum with pivot at the stance foot. The dynamics were determined from two, coupled, nonlinear, second-order differential equations of motion.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta, \gamma) = \tau \quad (1)$$

where θ , $\dot{\theta}$ and $\ddot{\theta}$ are the 1×2 joint angle, velocity and acceleration vectors, M is the 2×2 mass matrix, C is the nonlinear 2×2 velocity matrix and G is a matrix including gravitational terms (Appendix A). The actuation torque vector, τ , included terms describing the joint torques from the ankle and hip, $\tau_5 [\tau_{Ankle}, \tau_{Hip}]^T$. To represent passive walking these joint torques must be zero. An evaluation of the motion equations reveals that the passive movement trajectories are fully determined by the ground slope, γ , the ratio of HAT-mass versus leg mass and the mass-distribution [3-5].

The collision at foot-strike was represented as a plastic collision and the transition stage at foot-strike was assumed instantaneous, i.e. no double-support period [3]. The angle vector configuration after foot-strike θ^+ was related to the pre-impact angles θ^- by an anti-symmetric matrix, J , that establishes the new swing leg at the angle of the previous stance leg and the new stance leg with the configuration of the previous swing leg angle [4-6]. Angular velocities before and after foot strike were related by conservation of angular momentum,

$$Q^+(\theta)\dot{\theta}^+ = Q^-(\theta)\dot{\theta}^- \quad (2)$$

The terms $Q^+(\theta)$ and $Q^-(\theta)$ represent the momentum matrices immediately before and after foot-strike about the forward contact foot. Hence, the state of the system immediately prior to foot contact, $q^- = \begin{bmatrix} \theta^- \\ \dot{\theta}^- \end{bmatrix}$ is related to the state immediately following foot-contact, $q^+ = \begin{bmatrix} \theta^+ \\ \dot{\theta}^+ \end{bmatrix}$, by the relation,

$$q^+ = \begin{bmatrix} J & 0 \\ 0 & \text{inv}(Q^+)Q^- \end{bmatrix} q^- + \begin{bmatrix} -180 \\ 360 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

These conservation laws lead to discontinuous change in segment rotational velocities and associated kinetic energy loss at each foot-strike.

The active joint torque controller was based upon the gravitational term of downhill passive walking. Specifically, the active torques applied to the ankle and hip can be described as,

$$\begin{bmatrix} \tau_{\text{Ankle}} \\ \tau_{\text{Hip}} \end{bmatrix} = - \begin{bmatrix} M_L g (L - u_{CM}) + (M_H + M_L) g L & M_L g u_{CM} \\ 0 & M_L g u_{CM} \end{bmatrix} \times \begin{bmatrix} \cos \theta_S & -\sin \theta_S \\ \cos(\theta_S - \theta_N) & -\sin(\theta_S - \theta_N) \end{bmatrix} \begin{bmatrix} \cos \gamma_V - \cos \gamma \\ \sin \gamma_V - \sin \gamma \end{bmatrix} \quad (4)$$

where g is the acceleration due to gravity, 9.81 m/sec^2 and γ_V is the virtual ground slope. Note that this form is similar to the gravitational term, $G(\theta, \gamma)$ in Eq. (1). In fact, when the virtual slope γ_V is set to 0, then active torques, $[\tau_{\text{Ankle}}, \tau_{\text{Hip}}]^T$ exactly cancel gravitational effects, $G(\theta, \gamma)$. Virtual slope, γ_V , is a constant value throughout each stride. Clearly, the system can be made to appear to walk downhill regardless of the true ground slope, even if the true ground slope γ should represent an uphill incline. Note also that the active joint torques are a function of the leg angle representing a simple nonlinear feedback controller. This controller was implemented without feedback delay as a separate subroutine called by the ode45 solver in MATLAB. Linearized versions of this controller will be examined in future research.

Explicit control of the stride-length should be possible via the virtual slope control. During passive downhill walking the angular excursion of the legs is a function of ground slope as developed by McGeer [3]

$$\theta^3 = 4.11\beta\gamma \quad (5)$$

where θ_0 is the stance leg angle at toe off and β represents the relative mass distribution and pendulum frequencies of the system with respect to the contact foot and hip (Appendix B.). Note that geometric constraints of the two-segment walker require leg angles of the stance leg and swing leg must be equal and opposite at foot strike. Since stride-length (toe-off to heel-strike of the same foot [15]) is equal to $4L\sin\theta_S$, one can readily compute stride-length versus ground slope (Fig. 1). As the HAT mass only moves half the distance of the foot during each swing phase the average walking velocity measured at the HAT is the product of half the stride-length and the swing-frequency.

The model described above was implemented as a totally passive simulation to determine the natural stride-length and the associated joint trajectories. The simulations for each downhill slope ranging from 0.005 deg to 5.5 deg. Equilibrium characteristics were recorded and compared with the stride-length predicted using Eq. (5). Equilibrium characteristics were determined by running the model and allowing it to converge toward steady-state until the state and stride-length was repeatable to a precision of 5 significant digits. Data collected from the passive model simulations were then used as a baseline for comparison with the active virtual slope model. The active model was tested for a range of virtual slope and ground slope conditions and compared with results from the passive model.

Analyses were conducted to determine the ability of the control system to achieve dynamic limit-cycle stability. Analysis of limit-cycle stability for the forward-integration model was performed numerically as described by McGeer [6] and Goswami [2]. The initial state of the system at step q_k results from the dynamics following from the initial state at step q_{k-1} ,

$$q_K = f(q_{K-1}) \quad (6)$$

and has been called the “stride function” [6]. If the state is perturbed Δq then a Taylor series representation of the response is described as

$$f(q_{K-1} + \Delta q) = q_K + \Delta q \nabla f \quad (7)$$

where ∇f is the gradient of the stride function with respect to the system states [2,6]. By introducing a perturbation Δq to each of the state variables at q_{k-1} and observing the response q_k a numeric representation of ∇f is achieved [1-3]. With eigenvalue magnitudes of ∇f greater than one the system may be unstable or it may converge to a new steady-state trajectory, whereas eigenvalue magnitudes less than one indicate that the kinematic trajectory will converge toward its initial steady state behavior.

Results

Behavior of the walker when controlled through active joint torques with virtual slope γ_V , was indistinguishable from the natural dynamics of a passive pendulum-walker at an equivalent downhill ground slope. This natural behavior of the actively controlled walker was independent of the actual ground slope; even uphill walking was readily achieved. Symmetric periodic behavior was observable only for shallow ground or virtual slopes, $0.005 < \gamma_V < 4.40$ deg. The movement trajectory of leg angles (Fig. 3) was smooth, symmetric and continuous during the swing phase. At foot-strike the swing-leg instantly becomes the stance-leg and vice versa. At this foot-strike event there was an instantaneous loss of rotational velocity without change to the walker configuration (Eq. 3). Hence, the angle trajectory is not smooth or continuous at foot-strike. The form of the movement trajectories associated with the active-torque virtual-slope walker was identical to the passive downhill results and agreed with published results from passive pendulum-walkers.

At virtual slopes steeper than 4.40 deg the periodic movement converged to an asymmetric gait wherein the steady-state movement was a two-step pattern including a short step followed by a long step. This period doubling was observed again at a slope of 4.9 deg wherein the steady-state behavior was a four-step pattern. Published literature [4,5] suggests that this bifurcation quickly approaches a chaotic gait pattern. This was observed in our simulation thereby limiting the controllable slope range to less than 5.0 deg and stride lengths to less than 139% of leg length. Recall that a stride length is approximately twice the individual step length so equivalent step lengths were approximately 50% leg length. Identical bifurcation behaviors were observed in both passive downhill walking and when using the active virtual slope controller. Similar bifurcations have been discovered at very small slopes, $\gamma < \sim 10^{-3}$ [5]. Our model developed no such bifurcations when tested at the slope of 0.005 deg. Testing at shallower slopes should be investigated in future studies to examine behavior of the controller for low energy expenditure conditions [5].

Explicit control of forward velocity was successfully achieved. Swing frequency was roughly independent of the ground slope or virtual slope actuation 1.12 ± 0.01 swings/second, over the range of ground slopes with a mean value of .005 deg to 4.4 deg. Since swing-frequency was nearly constant the velocity was proportional to stride-length. The virtual slope produced stride-lengths that were in excellent agreement with the theoretical predictions (Fig. 1) and identical to the stride-lengths produced by walking down an equivalent ground slope. A priori prescription of walking velocity at any ground slope, including control of walking speed on downhill, level or uphill grades, was simply achieved by selecting the appropriate virtual slope. However, the range of available virtual slope values was small to avoid chaotic gait patterns. Hence, the range of available velocities was limited to less than 0.72 leg-lengths/s.

Active joint torques were a function of prescribed virtual slope and dynamic leg angle. The torque profile was continuous over the entire gait period save for the instant of foot-strike/toe-off (Figs. 4 and 5). At this foot-strike event there existed a joint torque discontinuity due to the energy loss associated with the inelastic collision of foot strike. Unlike the human bipedal gait, hip joint torque always acted to cause swing leg flexion, i.e. driving the swing leg forward to the direction of motion. Peak hip joint torque for a fast walking velocity of 0.47 leg-lengths/s

ranged from 0 Nm for passive walking on a decline, 3.52 Nm on a level surface, to as much as 7.03 Nm on an uphill slope of 4.0 deg. Ankle joint torque was always in the dorsiflexion direction, i.e. driving the stance leg forward to the direction of motion. Peak ankle joint torque for a fast walking velocity of 0.47 leg-lengths/s ranged from 0 Nm for passive walking on a decline, 42.03 Nm on a level surface, to 84.29 Nm on an uphill slope of 4.0 deg. However, the dynamic range of joint torques was small indicating that the influence of the leg angle was small. The dynamic range of the ankle and hip was less than 9% and 12% of maximum, respectively, at a virtual slope of 4.0 deg.

Stable steady-state movement trajectories were achieved on downhill, level and uphill ground slope surfaces using the active virtual-slope controller. Periodic gait is a sign of stable gait (Fig. 3) but periodicity by itself does not prove stability. Each simulation was repeated with small perturbations to each state as described in the Methods section. The range of viable perturbations was found to be $\Delta q = 5e^{-11}$ to $1e^{-2}$ degrees (Fig. 6). If too large a perturbation is used the linear eigenvalue analyses (Eq. (7)) are insufficient for stability assessment. If too small a perturbation is used then numerical error within the computational forward integration becomes an issue. Stability analyses in this study implemented $\Delta q = 1e^{-3}$ because it was in the middle of the range of possible perturbation sizes. Simulations run for a virtual slope of 1.5 deg° resulted in the eigenvalues of [0.0008, 0.2897, 0.2897]. Eigenvalues at virtual slopes from 0.005 deg to 4.4 deg were found to have magnitudes of less than one. Thus for a small perturbation the active model trajectory was stable.

To demonstrate the ability to control walking velocity the model was initialized with steady-state conditions for equilibrium at a virtual slope of 2.0 deg and allowed to walk for 4 strides. At the fourth foot contact the virtual slope within the active controller for subsequent strides was changed to 1.0 deg, 1.5 deg, 2.0 deg, 2.5 deg and 3.0 deg then allowed to continue for a total of 30 strides (Fig. 7). Results demonstrated that the model could control walking velocity in that it can converge from one stride-length and associated walking velocity to another in mid-simulation. The model's ability to converge to the new equilibrium state enables the control of the model's walking velocity.

Discussion

Active joint torques at the ankle and hip of the pendulum-walker were controlled by means of the virtual-slope algorithm. The joint torques attributed to gravitational load during passive downhill walking were expressed as a nonlinear feedback control law, Eq. (4). Hence, the applied torques could be represented as a function of leg angle and ground slope. When walking on level or uphill grades this control law was implemented to generate active joint torques. On level ground the controller could readily generate active torques to simulate any pre-specified downhill virtual slope, $0.005 < \gamma_V < 4.4$ deg. The active virtual-slope controller achieved repeatable and stable gait patterns. Because virtual slope simulates the forces incurred during downhill walking the movement trajectories were identical to those of the passive walker on an equivalent downhill slope.

This controllable virtual slope permitted explicit control of stride-length and forward walking velocity. For forward gait the swing foot must remain above the walking surface, excluding the swing foot scuffing the surface as the two links near vertical. During passive downhill walking once the pendulum has passed vertical the walking surface is lower than the stance foot. This allows the swing foot to travel farther forward before foot strike resulting in a longer stride-length. The virtual-slope actuators generated identical behavior, drawing the swing foot further forward for steeper virtual slopes. The nonbifurcating virtual slopes tested result in available stride lengths ranging from near zero to 1.4 leg lengths, encompassing the normal range of stride lengths for a healthy human [16].

Recall that the pendulum-walker maintains a near constant swing frequency of 1.12 swings/s. Recognizing that walking velocity is a function of stride length and swing frequency then control of walking velocity is inherent when modulating stride length. This resulted in the model being capable of walking at any HAT mass velocity ranging from near zero to 0.72 leg lengths/s. It is immediately apparent that the simulation was limited to slow walking velocities. Alexander [17] concludes the peak walking velocity must be related to the Froude number $v = \{gL\}^{1/2}$ where g is the gravitational constant and L the leg length. Normal data recorded from healthy adults demonstrates typical walking velocity Froude numbers of 0.45. The peak walking velocity of the active virtual-slope pendulum-walker was less than 24% of the calculated Froude number. Therefore, the simulation walked notably slower than an average healthy adult. This is likely related to the inability to control walking cadence. Vaughan [16] finds that this cadence should be scaled by $(g/L)^{1/2}$, which for our model results in a nondimensional value of 0.34. This is distinctly slower than the typical value of 0.60 for healthy adults [16]. McGeer shows stride frequency can be controlled using a simple torsional spring at the hip joint [6]. Future efforts will investigate the efficacy of controlling both stride length and stride frequency.

Eigenvalue analyses of the stride function [4,6] demonstrated that the active pendulum-walker successfully achieved stable trajectories during uphill and downhill walking and throughout a range of walking velocities (Fig. 6). The stability was limited to shallow virtual slopes. At virtual slopes greater than 5.0 deg the gait pattern became chaotic. This agrees with the results of Goswami [2] who observed chaotic movement behavior during passive downhill walking at slopes of 5.04 deg. Characterizing the stability behavior of multi-stride periodicity and chaotic gait patterns will require further analyses. Clearly, there exists an upper limit to ground slope or virtual slope for which stable pendulum-walking can be achieved. Beyond this slope limit more advanced controllers must be implemented requiring more degrees-of-freedom than the simple two-segment pendulum-walker. Nonetheless, we agree with others [6,18] who point out that the natural dynamics of the system contributes markedly to the stabilizing behavior of walking and suggest the advantages of this approach can be implemented in active control of walking.

The active simulation developed here also displayed period doubling bifurcation as the virtual slope became steeper, eventually leading to chaotic gait patterns similar to the results published in passive pendulum-walking [4,5]. The primary motivation for implementing the active virtual-slope controller was the inherent stability of passive downhill walking and to take advantage of natural dynamic characteristics of the system. The feedback controller metered active joint torques to provide more input energy at longer stride lengths; equivalent to greater vertical descent down steep virtual slopes. This achieved an energy balance representative of negative feedback control.

Previously published active pendulum-walkers have employed leg-length cycling and impulsive force actuation to achieve bipedal movement. Cyclically changing the leg length has been used to facilitate powered walking along level surfaces [6,9]. This approach modulates the swing dynamics of the model resulting in a joint angle profile contrasting with that of the passive or virtual-slope walkers. McGeer [6] achieved stable walking by periodically inputting impulsive forces to modify kinetic energy. These short bursts of energy are representative of the power generated during ankle push-off measured in human data but is in contrast to the smooth and continuous nature of the in-vivo power curves [19]. Both of these control systems use a time based application of energy to the system. The stability of time-dependent control is limited as it represents an open loop control. However, the pendulum-walker demonstrates nearly constant stride frequency indicating that angle and time are highly correlated for a given slope. This time-versus-angle relationship may allow the time-dependent control to suffice as state feedback. The advantages of a simple state-feedback controller are manifest when

attempting to achieve dynamic stability suggesting the virtual-slope controller as a feasible approach.

Position feedback control has been implemented to control bipedal stability. Successful pendulum-walking was implemented by Ono [11], achieving autonomous stability using linear feedback of the hip joint only. Feedback gain in that simulation was such that torque changed dramatically throughout the gait cycle unlike the joint torques generated by virtual-slope controller wherein dynamic range of torque was small (Figures 4-5). Asano [12] described a control system for both the hip and ankle torque similar to the virtual-slope controller. This method simulated the effect of gravity on both the ankle and hip joint in the tangential direction only. By ignoring the normal component of gravity that model lacked the ability to accurately control stride length and velocity.

The two link walker model was used to develop this control algorithm because it is the simplest model which can be used to show stability. McGeer developed a four-link kneed model to compare to the two-link model and found that the four-link model actually exhibited greater stability [6]. The addition of the knees essentially creates a method of toe clearance while adding little to the overall dynamic response of the walking model. Others add ankles or extensible legs to provide a means to input energy into the system for the active control of walking [9,20]. It is apparent that the two-link walker provides a simplified interpretation of dynamics and control but also has limitations. One drawback of this method is that it is difficult to match the kinematics of the model to those of a healthy subject as the degrees of freedom are different. Yet many have shown that such simple models are applicable to predicting trends in human kinematics [5,6,21].

Results from this model suggest that most of the torque necessary for walking was applied at the ankle joint. Measured human data demonstrated ankle push-off is a primary energy source for walking [22]. However, because there are neither feet nor extensible legs the model predicted a maximum dorsiflexion moment of 92.80 Nm. This is unrealistic for a human gait since an average foot can only support a dorsal moment of 1.5 Nm before lifting the toes [14]. During human walking the push-off power is generated during double stance phase of walking. Rigid-body dynamics of the pendulum-walker necessitate instantaneous load transfer, i.e. no double stance phase. Therefore, a comparison of the pendulum-walker should not be expected to simulate push-off dynamics of human walking unless leg-length cycling is permitted [6,9]. To permit improved realism future studies will expand the model to include actively controlled knee and ankle joints thereby permitting the double-stance phase of walking.

The idea of this natural gait trajectory controlled by a simple feedback control is related to neurologic central pattern generation (CPG) as described by Thelen [23]. CPG is a theory wherein the basic pattern of motion is instinctual and controlled by the spinal cord, using the higher brain centers merely for occasional control correction. Output is influenced by neuromuscular sensory input similar to a feedback controller. Virtual-slope control for pendulum-walking fits the description of a CPG. There is a basic motion pattern that is controlled by simulating gravity using joint angle as feedback. Linearizing the controller allows the model to be simulated as a linear feedback system simplifying the control algorithm. By adjusting the gain (virtual gravity magnitude and direction) walking performance, i.e. velocity, can be explicitly controlled. This creates a computationally simple control system analogous to the spinal cord and CPG.

The development and use of the virtual slope control algorithm enables the simple bipedal gait model to better simulate human locomotion. As the model becomes more realistic it will be used to model pathologic gait patterns. Through the addition of knees, ankles and muscle-

specific actuators new elements of gait will become apparent, i.e. double stance phase, the transition from walking to running, and pathologic based constraints and their effects.

Appendix A

Coefficients of equation of motion:

$$M = \begin{bmatrix} M_2 L_1^2 + 2M_2 L_1 u_2 \cos \theta_2 + I_1 + I_H + I_2 & I_2 + M_2 L_1 u_2 \cos \theta_2 \\ I_2 + M_2 L_1 u_2 \cos \theta_2 & I_2 \end{bmatrix}$$

$$C = \begin{bmatrix} -2M_2 L_1 u_2 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 - M_2 L_1 u_2 \dot{\theta}_2^2 \sin \theta_2 \\ M_2 L_1 u_2 \dot{\theta}_1^2 \sin \theta_2 \end{bmatrix}$$

$$G = - \begin{bmatrix} (M_1(L_1 - u_1) + M_H L_1 + M_2 L_1) (-g_x \sin \theta_1 + g_y \cos \theta_1) + M_2 u_2 (-g_x \sin(\theta_1 + \theta_2) + g_y \cos(\theta_1 + \theta_2)) \\ M_2 u_2 (-g_x \sin(\theta_1 + \theta_2) + g_y \cos(\theta_1 + \theta_2)) \end{bmatrix}$$

Appendix B

The stride length of passive dynamic walking is monotonically related to ground slope [3]. Since in the modeled system energy during the swing phase is conserved then the energy at foot-strike, $E_f(k)$, is equal to the energy at toe-off, $E_i(k)$, for an arbitrary step k . A simplified system wherein the HAT mass, M , is much larger than the leg mass [5] the energy of the system is approximated as the sum of kinetic and potential energies of the HAT mass.

$$E_f(k) = 1/2 M (1 - r_{gyr}^2) L^2 \dot{\theta}_f^2(k) \quad (B1)$$

$$E_i(k) = 1/2 M (1 - r_{gyr}^2) L^2 \dot{\theta}_i^2(k) + 2MgL \sin \theta_i \sin \gamma$$

$$E_i(k+1) = 1/2 M (1 - r_{gyr}^2) L^2 \dot{\theta}_i^2(k+1) \quad (B2)$$

where $E_f(k)$ and $E_i(k+1)$ are the energy at foot-strike of step k and ensuing energy of the subsequent toe-off of step $k+1$. In these equations, g is the gravitational constant, L is leg length and r_{gyr} the radius of gyration of the leg normalized to leg length, γ the ground slope and θ_i , $\dot{\theta}_i$ are the angle and angular velocities of the stance leg at toe-off and $\dot{\theta}_f$ the angular velocity of the stance leg at foot-strike. The conservation of angular momentum dictates that initial angular velocity of the stance leg at toe-off of step $k+1$, $\dot{\theta}_i(k+1)$ is related to the velocity of the stance leg during step k at foot-strike, $\dot{\theta}_f(k)$ by the relation,

$$\dot{\theta}_i(k+1) = \dot{\theta}_f(k) (r_{gyr}^2 + \cos(2\theta_i)) (1 + r_{gyr}^2)^{-1}. \quad (B3)$$

For a period-one system trajectory the values of initial angle and velocity are similar for step k and $k+1$. Substituting Eqs. (B1) and (B2) into Eq. (B3),

$$1/2 M (1 - r_{gyr}^2) L^2 \dot{\theta}_i^2 = \left\{ 1/2 M (1 - r_{gyr}^2) L^2 \dot{\theta}_i^2 + 2MgL \sin \theta_i \sin \gamma \right\} \times (r_{gyr}^2 + \cos(2\theta_i))^2 (1 + r_{gyr}^2)^{-2} \quad (B4)$$

where step arguments k and $k+1$ are ignored due to periodicity.

By inspection of the stance leg trajectory the velocity and angle range of motion are related as,

$$\dot{\theta}_i = 2\theta_i T^{-1} \quad (B5)$$

where T is the stance phase period. This period must be equal to the period of the swing leg as it moves from θ_i to $-\theta_i$, with natural frequency ω_{sw} ,

$$-\theta_i = \theta_i \cos(\omega_{sw} T) + \frac{\dot{\theta}_i}{\omega_{sw}} \sin(\omega_{sw} T) \quad (B6)$$

with solutions at $\omega_{sw} T = 3.15$ and 4.05 . The lower value represents short-step gait wherein foot contact occurs with angular velocity in the forward direction while the larger value represents a long-step gait with foot contact occurring while the swing leg is rotating posteriorly. Garcia [5] indicates that the short-step may be unstable, so we focus on the long-step behavior, i.e. $\omega_{sw} T = 4.05$. The stance leg velocity can therefore be represented as

$$\dot{\theta}_i = \theta_i \omega_{sw} / 2.025. \quad (B7)$$

Combining Eqs. (B4) and (B7) then applying small angle approximations to represent the trigonometric functions, the initial leg angle, θ_i , can be represented as a function of ground slope, γ , for equilibrium conditions

$$\theta_i^3 = 4.11\beta\gamma, \quad (B8)$$

where the scalar term β represents the ratio of natural frequencies of the stance and swing legs, $\beta = (1 + r_{gr}^2)(\omega_{st}/\omega_{sw})^2$ with ω_{st}^2 . This simplified relation between the ground slope and the steady-state toe-off angle is illustrated in Fig. 1 and agrees with the form of similar analyses by Garcia [5].

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Fig. 1.

A comparison of actual and predicted stride lengths, initial stance angle, and average HAT velocity at various slopes while walking along a level surface

Fig. 2.
Diagram and anthropometry of walking model

Fig. 3. Hip and ankle joint angles associated with a virtual slope of 1.5 deg along a level surface. The piecewise resetting of angles at foot-strike is shown. Four strides are shown in the figure.

Fig. 4. Hip joint torque normalized to max torque, one stride illustrated. Walking simulated along the level surface with various virtual slopes.

Fig. 5. Ankle joint torque normalized to maximum torque. One stride illustrated. Walking simulated along a level surface with the active virtual slope labeled.

Fig. 6. Maximum eigenvalues calculated for various perturbation magnitudes applied to the walker simulating a virtual slope of 1.5 deg.

Fig. 7.

Stride length and associated walking velocity can be controlled by the virtual slope coefficient of the active controller. Starting from a steady state walking pattern with a virtual slope of 2.0, the stride length can be adjusted (at foot-strike of arbitrary step 4) and the simulation converges on a new stride length.