Directionally compliant legs influence the intrinsic pitch behaviour of a trotting quadruped

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Limb design is well conserved among quadrupeds, notably, the knees point forward (i.e. cranial inclination of femora) and the elbows point back (i.e. caudal inclination of humeri). This study was undertaken to examine the effects of joint orientation on individual leg forces and centre of mass dynamics. Steady-speed trotting was simulated in two quadrupedal models. Model I had the knee and elbow orientation of a quadruped and model II had a reversed leg configuration in which knees point back and elbows point forward. The model's legs showed directional compliance determined by the orientation of the knee/elbow. In both models, forward pointing knees/elbows produced a propulsive force bias, while rearward pointing knees/elbows produced a braking force bias. Hence, model I showed the same pattern of hind-leg propulsion and fore-leg braking observed in trotting animals. Simulations revealed minimal pitch oscillations during steady-speed trotting of model I, but substantially greater and more irregular pitch oscillations of model II. The reduced pitch oscillation of model I was a result of fore-leg and hind-leg forces that reduced pitching moments during early and late stance, respectively. This passive mechanism for reducing pitch oscillations was an emergent property of directionally compliant legs with the fore–hind configuration of model I. Such intrinsic stability resulting from mechanical design can simplify control tasks and lead to more robust running machines.

Keywords: locomotion; simulation; running; ground reaction force; pitch

1. INTRODUCTION

Running animals have compliant legs that reduce peak ground reaction forces and, in many cases, store and return elastic strain energy within a running step. The multi-segmented legs of animals shorten as their joints flex and lengthen as they extend. Meanwhile, compliant structures acting about those joints tend to resist flexion and aid in extension. Leg compliance has also been exploited in the design of running robots. However, this is typically done by means of prismatic joints that allow the legs to act like pogo sticks. This design restricts compliance to the leg axis (i.e. axial compliance). Although the substrate interactions of multi-segmented and prismatic compliant legs can produce similar centre of mass (CoM) dynamics, it is unclear whether or not the individual legs of animals conform to this construct of simple axial compliance. In a multi-segmented leg, segments may be of different lengths and each articulation has a discrete orientation as well as a rotational compliance determined by the supporting structures about each joint. These design characteristics determine the directional compliance or tendency of the leg's deformation to deflect the ground reaction force vector away from the hip. For example, a directionally compliant leg loaded vertically in a test frame with the foot directly below the hip would exert both vertical and shear components of force, while a leg with simple axial compliance would exert only a

vertical component. During locomotion, a leg's directional compliance might deflect the resultant force vector in front of the hip. Thus, a retracting torque would be required about the hip and forward (propulsive) force would be exerted on the CoM. In this way, leg geometry can influence the intrinsic braking-propulsive function of a leg. Hence, leg design may be instrumental in producing the fore-leg braking and hind-leg propulsive biases reported during steady-speed trotting of quadrupeds (Lee *et al.* 2004). Furthermore, previous trotting simulations suggest a functional advantage to this opposing pattern of ground reaction force, namely, enhancement of pitch stability (Herr & McMahon 2000). Here, we use trotting quadruped models with different joint orientations and fore–hind configurations in order to examine individual leg forces and CoM dynamics.

2. METHODS

Quadrupedal trotting models were constructed and simulated using Working Model 2D 4.0. This software package for dynamic simulations solved the equations of motion using a Kutta–Merson integrator with a time-step of 1 ms. In this section, we provide details of the model's physical properties, mechanical design and feed-forward control.

(a) *Physical properties*

The linear dimensions and mass of our planar quadruped were similar to those of a Labrador retriever dog. Shoulder height and hip height were 0.47 m, the distance from shoulder to hip was 0.50 m, and body mass was 28.6 kg (Lee *et al.* 1999). The whole-body planar moment of inertia

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Figure 1. A single leg of our quadruped model. All legs were identical but were attached to the body with either rearward-pointing or forward-pointing knee orientation. A distal prismatic joint spanned by a spring connects the distal and middle (shaded) leg segments, creating a compliant, telescoping strut. The knee is a pin joint spanned by a spring connecting the middle and proximal leg segments.

was 1.07 kg m^2 . A dimensionless moment of inertia for legged systems; $j = \mathcal{J}/(md^2)$, where $\mathcal{J}$ is the moment of inertia and d is one-half the distance between the hip and shoulder, was initially shown by Murphy (1984) to be of dynamic importance. For our models, this value was 0.60. If the dimensionless moment of inertia is less than 1, upward vertical force on the fore-leg will tend to accelerate the hind-leg downward, whereas if it is greater than 1, the same force will tend to accelerate the hind-leg upward (Murphy 1984; Raibert 1986). For this reason, a dimensionless moment of inertia less than 1 can provide passive pitch stability during bounding (Murphy 1984; Neishtadt & Li 1991). Hence, at $j = 0.60$, our models are expected to adopt nose-down/nose-up pitch oscillations when trotting is disrupted. In order to minimize inertial effects of leg movement, the legs of the model were made extremely light. Together, all four legs (two fore-legs and two hind-legs) represented 0.3% of the total body mass and no more than 1.2% of the total body moment of inertia during trotting simulations.

(b) Mechanical design

Each leg was attached to the body by a pin joint, referred to as a shoulder in the fore-leg and a hip in the hind-leg. Shoulder and hip joints were actuated to produce leg retraction and protraction, as described in the following section. Distal to the hip and shoulder, however, the leg joints were supported passively by linear springs. The leg comprised three segments: a distal segment with a foot, a middle segment and a proximal segment (figure 1). The foot was small (0.05 m in diameter) and circular. The feet collided plastically with the substrate (i.e. their coefficient of restitution was zero) and their coefficient of friction with the substrate was one. A prismatic (i.e. telescoping) joint connected the distal segment to the middle leg segment and a linear spring with stiffness k_{distal} acted across the joint. Together, the distal and middle segments functioned like a pogo stick (figure 1). Such compliant prismatic joints have been used in the legs of most quadruped models that are capable of running (Murphy & Raibert 1985; Herr & McMahon 2000, 2001; Herr *et al.*

2002), as well as in robots (Raibert 1986, 1990; Buehler 2002; Poulakakis *et al.* 2003). Considering only the sagittal (i.e. side-view) plane, these legs rotated about the hip and translated along the leg axis, providing two planar degrees of freedom (DOF). In the legs of our models, however, a pin joint connecting the middle and proximal leg segments (figure 1) provided a third DOF (i.e. pitch axis rotation within the leg). This mid-leg rotating joint is referred to as the elbow in the fore-leg and the knee in the hind-leg and was supported passively by a linear spring with stiffness k_{mid} (figure 1).

There were two principle motivations for including compliant knee and elbow joints in the legs of our quadrupedal models. Our first objective was to explore the effect of different joint orientations on leg force during steady-speed trotting. We predicted that the resultant ground reaction force vector would tend to align with the knee/elbow. In other words, compliant joints would be expected to introduce intrinsic directional compliance. Legs with rearward-pointing joints would tend to exert braking force and legs with forward-pointing joints would tend to exert propulsive force. Examining the extrema of mid-leg spring constants can help to explain the basic mechanism of such directional compliance. A spring constant of zero would produce a resultant force in line with the joint because, in this case, the distal leg segment would behave as a two-force member (i.e. a moment cannot be applied to either end). On the other hand, an infinite spring constant would tend to produce a force in line with the proximal joint (i.e. as if there were no mid-leg joint at all). Two models were tested, differing only in knee and elbow orientation. Model I had rearward-pointing elbows and forward-pointing knees. This pattern was reversed in model II.

The second reason for including compliant knees was to provide adequate DOF for trotting. Simple pogo stick legs can impose kinematic constraints when used in pairs because they have only one DOF below the hip. Because trotting employs a spring-loaded inverted pendulum (SLIP) mechanism (McMahon 1985; Blickhan 1989; McMahon & Cheng 1990), with simultaneous fore-leg and hind-leg contacts (Gray 1968), the contact legs are compressed during the first half of stance and they extend during the second half of stance. Unless the legs are kept parallel to one another or the hip and shoulder rotate freely to adjust the leg angles, the shortening legs will impose a kinematic constraint. For example, a long body with pogo stick legs rigidly fixed at each end will bounce if the legs are parallel. However, if the legs are not parallel (and there is sufficient friction to prevent the feet from sliding), the fore- and hind-leg will not be able to shorten simultaneously and the system will not bounce. In cases where trotting with pogo stick legs has been successful, the legs have either been kept parallel (Raibert 1986, 1990) or a compliant 'back' joint has been added to the body between the shoulder and hip (Herr & McMahon 2000; Herr *et al.* 2002). The latter design innovation by Herr & McMahon (2000) added another DOF to the system by allowing unactuated pitch-axis rotation about the 'back' joint. The compliant mid-leg joints in the legs of our quadruped model provide an alternative solution. They permit a pair of contact legs to shorten and lengthen freely by adding a third DOF to each leg (i.e. unactuated pitch-axis rotation about the elbow or knee).

(c) *Feed-forward control*

During trotting, diagonal pairs of legs are swept forward and backward (i.e. protracted and retracted) in unison. The two diagonal pairs are half a stride cycle out of phase, producing an alternating pattern of diagonal foot contacts. In our simulations, the stride period (i.e. a complete protraction–retraction cycle of a given leg) was constrained to 0.4 s, based upon data from Labrador retriever dogs trotting at 2.8 m s^{-1} (Bertram *et al.* 2000). The retraction–protraction cycles of diagonal legs were precisely in phase with one another and half a cycle out of phase with the opposite pair of diagonal legs. A motor at each shoulder and hip controlled the angular velocity of the legs by exerting the requisite torques about these pin joints. A feed-forward tabular control was implemented to retract the legs at a constant angular velocity of 315° s^{-1} for three-quarters of the stride period. The legs swept an angle of 94.5° (from 47.25° ahead of vertical to 47.25° behind vertical). The retraction period of the simulation was substantially greater than the contact period. The fraction of stride period in which a given foot was in contact with the substrate (i.e. duty factor) averaged 0.48 in our simulations, which fell within the range of 0.40 to 0.51 measured in trotting Labradors (Bertram *et al.* 2000). Hence, leg retraction was designed to precede initial foot contact and persist beyond final contact by a substantial margin. Retracting the legs just prior to foot contact, referred to as ground speed matching (Raibert 1986), is quite common during fast locomotion of mammals (Gray 1968). In addition, this strategy has been successfully implemented in simulations of bipedal (Seyfarth *et al.* 2003) and quadrupedal running (Herr & McMahon 2000, 2001; Herr *et al.* 2002). The continuation of leg retraction after the foot has left the ground is also typical of high-speed locomotion of mammals (Gray 1968). In our simulations, the legs were protracted during just one-quarter of the stride period. This inequity of retraction and protraction period helped prevent the occurrence of leg protraction during foot contact, which would have caused sudden, catastrophic braking. While the hip and shoulder were subject to feed-forward control, the legs themselves behaved passively during retraction and stance. During leg protraction, however, length actuators shortened the distal legs to provide ground clearance and restored the knee angles to that of the resting length of the knee spring. Mid and distal leg spring constants were set to zero during protraction. Just before retraction, the distal leg springs were restored to their resting length. Upon initiation of retraction, the knee and distal leg length actuators were turned off and their spring constants were restored. The action of these length actuators during protraction served only to provide ground clearance and maintain the knee angle prior to the next retraction period. The knee and distal leg were not actuated during retraction and stance. As described above, the models' physical dimensions, stride period, leg angular velocity and leg angular excursion were fixed for all of the trotting simulations reported. Five free variables were adjusted to produce regular CoM oscillations during trotting. These variables included mid and distal leg spring constants, initial forward $v_{x,i}$ and vertical $v_{y,i}$ velocities and initial vertical position y_i .

3. RESULTS AND DISCUSSION

(a) *Spring characteristics*

The rotational mid-leg joints (i.e. the elbow and knee) and the prismatic distal joints performed mechanically distinct functions during periodic, simulated trotting. The mid-leg spring constant was 75 kN m^{-1} and the distal leg spring constant was 3 kN m^{-1} . These spring constants are not directly comparable because the mid-leg spring spans the knee/elbow rotary joint, whereas the distal spring determines the axial compliance of the distal leg. Most of the compliant behaviour of the leg was owing to the distal spring. This is in qualitative agreement with kinematic measurements from trotting dogs, in which angular excursions of the ankle were substantially greater than those of the knee (Carrier *et al.* 1998), suggesting greater compliance in the distal limb. The combined stiffness of a diagonal limb pair in our simulation was also similar to the predicted virtual leg stiffness of a running mammal. Based upon the empirical power function $k_{\text{leg}} = 0.715m^{0.67}$ (Farley *et al.* 1993), a 28.6 kg mammalian runner is predicted to have a virtual leg stiffness of 6.76 kN m^{-1} , compared with 6.0 kN m^{-1} ; the sum of two distal leg springs (i.e. of a fore-leg and hind-leg). This good agreement of virtual spring properties emerged when regular CoM oscillations were achieved in the trotting simulations. Although their effect on overall leg compliance was negligible, the stiff mid-leg joint springs served an important function in minimizing pitch oscillations during simulated trotting. If this spring were made too compliant, the angular velocity of the body would become uncoupled from the step cycle and its magnitude would increase over time. On the other hand, if this spring were made too stiff, the angular velocity of the body would be tightly coupled to the step cycle, but would have a much greater magnitude. The final section of this report discusses the mechanical link between pitch behaviour and the directional compliance imparted by compliant mid-leg joints.

(b) *Trotting characteristics*

The initial forward velocity $v_{x,i}$ of 2.97 m s^{-1} was slightly higher than the mean forward velocity (2.90 m s^{-1}). This was expected because the forward velocity of a SLIP reaches a maximum during the flight period when the CoM is at its highest point (Blickhan 1989; McMahon & Cheng 1990). Accordingly, the model was dropped from an initial CoM height y_i of 0.452 m, corresponding to the apex of the flight trajectory during periodic trotting (figure 2). Initial vertical velocity $v_{y,i}$ was zero, as expected at the apex of the flight trajectory where the CoM velocity changes direction from upward to downward. During simulated trotting, the CoM height reached a maximum during flight and a minimum during stance (figure 2), such that its vertical velocity was zero at the beginning of the step and at mid-stance. Forward CoM velocity also fluctuated during simulated trotting, reaching a minimum at mid-stance and a maximum during the flight period. These dynamics emerged from simple, feed-forward control (i.e. constant leg retraction velocity and unactuated mid and distal leg joints during stance). Foot velocity did not require explicit control. In contrast to these general characteristics of a SLIP, the observed differences

Figure 2. Vertical and pitch oscillations during trotting of model I (dark lines) and model II (light lines) during 10 complete stride cycles (20 trotting steps). Positive angles indicate nose-up pitching. Please see the accompanying Quicktime movies of model I and model II trotting simulations in the Electronic Appendix.

in pitch behaviour between model I and model II were unexpected (figure 2). Beginning with the second trotting step, nose-down pitch velocity was apparent during the flight period of each step (i.e. negative slopes at 0.2, 0.4, 0.6 s...; figure 2). Conversely, nose-up pitch velocity was apparent during mid-stance (i.e. positive slopes at 0.1, 0.3, 0.5 s...; figure 2). This pattern is consistent with bound-like pitching tendencies predicted for a legged system with dimensionless moment of inertia below one. Observed nose-up pitching moments about the CoM during the first half of stance and nose-down pitching moments during the second half of stance are directly related to this pitching pattern, as described in the final section of this report. In comparison to available data from animal dynamics, this pattern closely resembles the pitching behaviour and the CoM moments of trotting cockroaches (Ting *et al.* 1994).

(c) Ground reaction force

In both of our models, the total fore-aft force–time curves showed braking in the first half of stance followed by equal propulsion in the second half of stance (figure 3). This pattern, typical of a SLIP (Blickhan 1989; McMahon & Cheng 1990), resulted in a mean fore-aft force of zero during stance. The total braking and propulsive force peaks were reduced in model II with respect to model I. To a lesser extent, this was also true of individual limb fore-aft force. We expected that directional compliance and a constant retraction velocity would produce net braking force in legs with rearward-pointing joints and net propulsive force in legs with forward-pointing joints. This was the case in both models, regardless of the fore–hind configuration of the legs (i.e. whether the leg was used as a fore- or hind-leg). Nonetheless, the fore–hind configurations of models I and II produced subtle differences in ground reaction force magnitude and relative phase of braking and propulsive peaks. In model I, the fore-leg with a rearward-pointing joint exerted a substantial braking force followed by a smaller propulsive force. When the same leg was used as a hind-leg in model II, it produced a similar force–time curve, but with a reduced and delayed peak braking force (figure 3). In model I, the hind-leg with

a forward-pointing joint exerted a small braking force followed by a greater propulsive force. When the same leg was used as a fore-leg in model II, it produced a similar force–time curve, but with an earlier and reduced peak propulsive force (figure 3). In both models, directional compliance appears to dictate the braking–propulsive force pattern of a given leg, while the leg’s position as a fore-leg or hind-leg only modulates this basic pattern. The fore-aft force–time curves of model I closely resemble those of trotting dogs (Lee *et al.* 2004).

Unlike total fore-aft force, total vertical force was nearly the same in the two models. Surprisingly, this similarity resulted from the summation of substantially different fore-leg and hind-leg vertical forces in model II. In model I, the rearward-pointing joint of the fore-leg and the forward-pointing joint of the hind-leg exerted roughly the same vertical force throughout stance but slight shifts in the time of peak force were evident. The hind-leg peak was just before mid-stance and the fore-leg peak was just after mid-stance (figure 3). Similar shifts in force–time peaks have been observed in fore- and hind-legs of trotting dogs (Lee *et al.* 2002, 2004). The fore-legs and hind-legs of model II exerted roughly equal vertical force, but showed pronounced differences in the time of peak force. In contrast to model I, the fore-leg peak was well before mid-stance and the hind-leg peak was well after mid-stance (figure 3). Because the leg configuration is reversed in model II, the earlier peak still occurred in the leg with a forward-pointing elbow/knee and the later peak, in the leg with a rearward-pointing elbow/knee. The only consistent interpretation, therefore, is to regard these shifts in peak force as a feature of the directional compliance of a given leg. For example, a leg with a forward-pointing knee (i.e. propulsive directional compliance) produces a greater vertical force component and a smaller fore-aft force component in the first half of stance than in the second half. This effect was intensified in model II and minimized in model I by nose-down pitch velocity in the beginning of stance and nose-up pitch velocity at mid-stance (figure 2).

(d) Pitch stability

As illustrated by the resultant force vectors of figure 3, time-variant moments are exerted about the CoM during simulated trotting. During the first half of stance ($T/4$), a net nose-up moment was exerted about the CoM in both models. At mid-stance ($T/2$), no net moment was exerted in either model. During the second half of stance ($T/4$), a nose down moment was exerted about the CoM in both models. This switching pattern of CoM moments (nose-up then nose-down) was conserved across the two models, and is even apparent in running animals for which it has been measured (Ting *et al.* 1994). Although simple, the mechanism behind this pattern has not been described previously. As illustrated in figure 3, the contact legs are placed further forward (i.e. in a protracted position) during the first half of stance and they are placed further back (i.e. in a retracted position) during the second half of stance. If equal vertical forces were exerted on the feet at either of these times, a substantial moment would be exerted about the CoM by the foot farthest from the CoM (i.e. the fore-leg during

Figure 3. Patterns of fore-leg (f), hind-leg (h) and total (dashed lines) ground reaction force during a trotting step. Upward vertical force and propulsive fore-aft force are positive. Model I data are shown in the top panels and model II data in the bottom panels. Resultant ground reaction force vectors are shown within a trotting step at times $T/4$, $T/2$ and $T3/4$, where T is the step period. Large, open arrows indicate the direction of travel. Please see the accompanying Quicktime movies of model I and model II trotting simulations in the Electronic Appendix.

the first half of stance and the hind-leg during the second half). The foot closer to the CoM would exert less of a moment due to its shorter lever arm (i.e. distance from the CoM). Therefore, the net moment would be nose-up when the fore-leg is farthest from the CoM and nose-down when the hind-leg is farthest from the CoM. This seems to be a basic feature of multi-legged trotting, present both in our models as well as in cockroaches (Ting *et al.* 1994).

Our simulations show that directionally compliant legs can reduce CoM moments during trotting. Model I provides a good example of this and model II, a counter-example. Owing to directional compliance, the fore-leg of model I exerted greater braking force in the first half of stance, which reduced its moment about the CoM (e.g. $T/4$ in figure 3). Likewise, the hind-leg of model I exerted greater propulsive force in the second half of stance (e.g. $T3/4$ in figure 3). The resulting pitch reduction in model I with respect to model II (figure 2) is an emergent property of directionally compliant legs with the fore-hind configuration of model I. Directional compliance is not the only means of biasing legs toward braking or propulsion. Pitch stability from fore-leg braking and hind-leg propulsion has been successfully exploited in mammalian trotting and galloping models (Herr & McMahon 2000, 2001; Herr *et al.* 2002). In these quadrupedal running simulations, braking and propulsive forces were produced by exerting protracting moments at the shoulder and retracting moments at the hip. These moments were a response to a control system, which explicitly set target retraction velocities that were greater than required at the hip and less than required at the shoulder. Although the leg moments and forces were not intrinsic to their mechanical design (each leg had only a compliant prismatic joint below the hip or shoulder), the models of Herr and co-workers were able to achieve excellent pitch stability by controlling hip and shoulder retraction—and, thus, moments. In contrast, by using directionally compliant legs in the appropriate configuration, our trotting quadruped is able to reduce pitch oscillations without feedback control of any sort. That is, the design is made more robust by the addition of directionally compliant legs. In trotting animals, it is likely that directionally compliant legs and explicit control of hip

and shoulder moments work synergistically to provide pitch stability during steady and non-steady locomotion over various terrains. Furthermore, quadrupedal robots are likely to benefit from robust leg designs and configurations, while still requiring active hip and shoulder control to deal with the real world demands of acceleration and accommodating steep or rough terrain.

(e) *Evolutionary implications*

The limbs of quadrupeds resemble the legs of model I in that the knees point forward and the elbows point rearward. Most quadrupeds capable of running have this knee and elbow orientation but, surprisingly, this does not appear to be the ancestral condition for tetrapods. Early embryonic development indicates no inherent bias in elbow or knee orientation, since the limb buds of tetrapods begin as lateral projections with dorsal extensor muscle masses and ventral flexor muscle masses. Only later in development do the embryos of mammals and birds show the limb rotations characteristic of adult limb design. The fossil record suggests that knee-forward/elbow-rearward limb design appeared in association with parasagittal limb posture separately in the group that gave rise to mammals (*Therapsida*) and the group that gave rise to crocodiles and dinosaurs (*Thecodontia*; Carroll 1988). In light of this history, a functional explanation for knee-forward/elbow-rearward limb design is plausible. Our trotting simulations suggest one such explanation—quadruped limbs are designed for directional compliance that passively reduces pitch moments, as described for model I. Of course, this basic design argument ignores the effects of other joints on the overall limb mechanics. Joint orientations and leg segment proportions, as well as moments exerted by muscle-tendon systems about individual joints, will influence the directional compliance of biological limbs. In order to better understand whole-limb directional compliance in the complex, multi-segmented joints of living quadrupeds, joint dynamics will need to be examined across the entire limb. For example, studies reporting both knee and ankle dynamics, as reported recently for human running (Arampatzis *et al.* 1999; Gunther & Blickhan 2002), will be required to elaborate the role of directional compliance in quadrupedal running.

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