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Normal vibration frequencies of the vocal ligament

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Abstract

The vocal ligament is the tension-bearing element in the vocal folds at high pitches. It has traditionally been treated as a vibrating string, with only length and longitudinal stress governing its normal mode frequencies. Results of this investigation show that, when bending stiffness and variable cross section are included, the lowest normal mode frequency can more than double, depending on the strain of the ligament. This suggests that much higher phonation frequencies may be achievable than heretofore thought for a given vocal fold length (e.g., nearly 1000 Hz at 50% elongation over cadaveric resting length). It also brings back into the discussion the concept of “damping,” an old misnomer for a reduction of the effective length of vibration of the vocal folds by relatively stiff boundary segments known as macula flavae. A formula is given for correcting the ideal string equation for the lowest mode frequency to include bending stiffness and macula flavae effects.

I. INTRODUCTION

The vocal ligament is a portion of the nonmuscular layers of the vocal fold known as the lamina propria (layers in motion). It comprises the intermediate and deep layers of the lamina propria (Hirano, 1975). The ligament is composed of densely packed collagen and elastin fibers, which are aligned in a nearly parallel fashion and course antero-posteriorly between the vocal process of the arytenoid cartilage and the thyroid cartilage (Fig. 1).

From a functional point of view, the vocal ligament limits vocal fold elongation and helps position the vocal fold when the arytenoid cartilage moves. More importantly, for phonation, the vocal ligament supports large tensile stresses for high pitched sounds in singing, as well as for squeals and falsetto productions in spontaneous vocalizations. Because the ligament is thin (a few square mm in cross-section), large tensile stresses can be obtained with moderate muscle forces produced by the intrinsic laryngeal muscles (primarily the cricothyroid muscle). Among mammalian species, some have a highly developed vocal ligament (like the pig) while others have a poorly developed or nonexistent vocal ligament (like the dog). For human phonation, van den Berg (1958) hypothesized that the vocal ligament was critical for vocal registers (falsetto versus chest) and for high-pitched singing.

In previous studies of pitch control (e.g., Titze et al., 1989), our assumption has always been that the ligament vibrates as a string with fixed boundary conditions, a uniform cross-section, uniform tension, and negligible bending stiffness. With these assumptions, the normal mode frequencies were easy to predict using the ideal string law. But recent attempts to reconcile human phonation frequencies with biomechanical data on vocal fold length and stress-strain

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characteristics of the ligament suggest that the ideal string law may underestimate the normal mode frequencies. Other authors (Descout et al., 1980; Perrier, 1982; Perrier et al., 1982; Guérin, 1983) recognized the “beam” nature of the vocal folds early on, predicting some natural frequencies based on bending stiffness. These predictions were not based on measured stress-strain curves of the vocal ligament, but rather on an average human tissue Young's modulus. Guérin also included large-amplitude vibration, for which the natural frequencies were dependent on amplitude of vibration and, hence, lung pressure. Bickley (1987) and Bickley and Brown (1987) claimed that the bending beam model is a better predictor of young children's F_0 in speech than either the ideal string model or the mass-spring model, the former predicting values too high and the latter too low. They stated that non-uniform tissue growth in early development may determine which model fits the anatomy best at any given age.

Forty years ago, Fletcher (1964) had a similar concern about the normal mode frequencies of a stiff piano string. It was known that the ideal string law

$$F = \frac{1}{2L} \sqrt{\frac{\sigma}{\rho}}, \quad (1)$$

where F is the natural frequency of the lowest mode, L is the length of the string, σ is the longitudinal stress, and ρ is the material density, underestimated the vibration frequency of a stiff piano string. The error was small, but when the proper bending stiffness was included in the analysis, the corrections matched with observations. In the case of the vocal ligament, the effect of including bending stiffness (and, in addition, variable cross-section) may be more than a small correction because the endpoints of the vocal ligament (the macula flavae) widen the “string,” possibly causing the boundary conditions to change more dramatically.

The purpose of this paper is to reanalyze the normal mode frequencies of the vocal ligament with the inclusion of bending stiffness and variable cross-section (at the macula flavae) in addition to tensile restoring forces. Both analytical and finite-element computational methods will be employed. The result will be based on actual measurements performed earlier on human vocal ligaments (Min et al., 1995).

II. REVIEW OF ANALYTICAL TREATMENT OF LIGAMENT VIBRATION

A theoretical analysis of vibration of beams with longitudinal tension and bending stiffness was given by Morse (1936, reprinted 1976, pp. 166–170). The equation of motion is fourth order,

$$\rho \frac{\partial^2 \xi}{\partial t^2} + R \frac{\partial \xi}{\partial t} = \sigma \frac{\partial^2 \xi}{\partial x^2} - E \kappa^2 \frac{\partial^4 \xi}{\partial x^4}, \quad (2)$$

where ξ is the transverse displacement, ρ is the material density, R is the viscous loss coefficient (force per unit volume per m/s velocity), σ is the longitudinal stress (tension per unit area), E is the Young's modulus, and κ is the radius of gyration (for bending moments in the tissue). The loss term (second on the left side) was not included in Morse's discussion, but was included by Fletcher (1964) in the treatment of a stiff piano string. For a steel wire under high tension, the second term on the right side is small in comparison to the first, but in vocal fold tissue the two terms appear to be of comparable size, as will be seen.

For normal modes, for which the loss term R is neglected, a partial wave solution of the form

$$\xi(x, t) = A e^{i\omega t - \gamma x} \quad (3)$$

satisfies the equations. Substitution of Eq. (3) into Eq. (2) yields the characteristic equation for wave propagation,

$$-\rho \omega^2 = \sigma \gamma^2 - E \kappa^2 \gamma^4, \quad (4)$$

which has four algebraic solutions for the propagation constant γ ,

$$\gamma = \pm \left\{ \frac{\sigma}{2E\kappa^2} \pm \left[\left(\frac{\sigma}{2E\kappa^2} \right)^2 + \frac{\rho\omega^2}{E\kappa^2} \right]^{1/2} \right\}^{1/2}. \quad (5)$$

For a ligament of rectangular cross section, the radius of gyration κ is defined as

$$\kappa = w / \sqrt{12}, \quad (6)$$

where w is the width of the ligament in the direction of transverse vibration (Morse, 1976, p. 153; Gieck and Gieck, 1997). The relative sizes of the terms in Eq. (5) will be discussed later.

III. CLAMPED BOUNDARY CONDITIONS

We assume clamped boundary conditions at the posterior ($x=0$) and the anterior ($x=L$) endpoints. This implies that both the displacement $\xi(x)$ and the derivative $\partial\xi/\partial x$ need to vanish at these boundaries,

$$\xi(0, t) = \xi(L, t) = 0, \quad (7)$$

$$\frac{\partial \xi}{\partial x}(0, t) = \frac{\partial \xi}{\partial x}(L, t) = 0. \quad (8)$$

With these boundary conditions, a complete solution to the fourth-order differential equation is written according to Morse (1936) or Fletcher (1964) as

$$\xi(x, t) = [A_1 \cos \gamma_1 x + B_1 \sin \gamma_1 x + A_2 \cosh \gamma_2 x + B_2 \sinh \gamma_2 x] e^{i\omega t}, \quad (9)$$

where

$$\gamma_1 = -i \left\{ \frac{\sigma}{2E\kappa^2} - \left[\left(\frac{\sigma}{2E\kappa^2} \right)^2 + \frac{\rho\omega^2}{E\kappa^2} \right]^{1/2} \right\}^{1/2}, \quad (10)$$

$$\gamma_2 = \left(\frac{\sigma}{E\kappa^2} + \gamma_1^2 \right)^{1/2} \quad (11)$$

from Eq. (5). Both γ_1 and γ_2 are written to be positive and real, their imaginary and negative counterparts in Eq. (5) having been taken into consideration by choosing both trigonometric and hyperbolic functions in Eq. (9). Furthermore, since all four partial functions in brackets in Eq. (9) are linearly independent of each other, and since four arbitrary constants have been introduced, the solution is guaranteed to be complete and able to accommodate the four boundary conditions [Eqs. (7) and (8)].

Solution of the above equations for the constants A_1 , B_1 , A_2 , and B_2 is a matter of algebra. Substituting Eq. (9) into Eqs. (7) and (8) yields four equations in four unknowns. The results are written here for ease of sequential numerical solution. Assuming A_1 to be an overall scale factor,

$$A_2 = -A_1, \quad (12)$$

$$B_1 = \frac{-\gamma_1 \sin \gamma_1 L - \gamma_2 \sinh \gamma_2 L}{\gamma_1 \cosh \gamma_2 L - \gamma_1 \cos \gamma_1 L} A_1, \quad (13)$$

$$B_2 = -(\gamma_1 / \gamma_2) B_1, \quad (14)$$

$$2\gamma_1 \gamma_2 (\cos \gamma_1 L \cosh \gamma_2 L - 1) = (\gamma_2^2 - \gamma_1^2) \sin \gamma_1 L \sinh \gamma_2 L. \quad (15)$$

A_1 cannot be determined because the four algebraic equations [Eqs. (7) and (8)] are homogeneous, as they always are for normal mode solutions. Instead, a condition is imposed on the propagation constants γ_1 and γ_2 via Eq. (15). Unfortunately, this equation is transcendental and can only be solved numerically to a high degree of accuracy. For this purpose, we assume that

$$\gamma_1 = \gamma_{1n} = \frac{n\pi}{L} + \delta_n, \quad (16)$$

where n is a mode integer, $n\pi/L$ is the wave number for a classical (nonbending) string, and δ_n is a correction factor. Equation (15) can be solved by computer, with repeated trials of δ_n , realizing that γ_2 is related to γ_1 by Eq. (11). Numerical results will be given below.

As a final step in the boundary value problem, the normal mode frequencies are calculated from Eq. (10) by solving for ω ,

$$\omega_n = \left(\frac{\sigma}{\rho}\right)^{1/2} \left[\gamma_{1n}^2 + \frac{E\kappa^2}{\sigma\rho} \gamma_{1n}^4 \right]^{1/2}. \quad (17)$$

If the material were linearly elastic under longitudinal tension, then

$$\sigma = eE, \quad (18)$$

and E/σ in Eq. (17) could be replaced by the inverse of the longitudinal strain, $1/\epsilon$. It is readily seen, then, that the bending stiffness term (second term in brackets) is inversely proportional to ϵ . The lower the strain, the more the bending stiffness will dominate relative to the tension term.

By substituting Eq. (16) into Eq. (17), making a first-order expansion of the square root term, and retaining only first power terms of δ_n , Morse (1936) approximated the normal mode frequencies for a beam under tension to be

$$\omega_n \approx \frac{n\pi}{L} \left(\frac{\sigma}{\rho}\right)^{1/2} \left[1 + \frac{2}{\pi} B^{1/2} + \left(\frac{4}{\pi^2} + \frac{n^2}{2} \right) B \right], \quad (19)$$

where

$$B = \frac{\pi^2 E \kappa^2}{\sigma L^2} = \frac{\pi^2 \kappa^2}{e L^2}. \quad (20)$$

This approximation will be compared to the numerical solution for ω_n . Note that the value of B decreases with increasing strain.

Figure 2 shows the displacement function $\xi(x)$ for the first four modes. The mode amplitudes are all normalized to show the detail of the derivative at the endpoints. With bending stiffness (solid lines), the derivative goes to zero at $x=0$ and $x=L$, whereas it retains a finite value for the classical string. Nodes and antinodes are shifted toward the center unless they occur at the center. In general, the modal patterns are not dramatically different. This is perhaps the reason why bending stiffness has generally been neglected; the differences do not show up on stroboscopic viewing.

Figure 3 shows the normal mode frequencies $\omega_n/2\pi$ plotted as a function of mode number n for several conditions. The following constants were chosen:

$$\begin{aligned} E &= 91.7 \text{ kPa}, & \rho &= 1040 \text{ kg / m}^3, & \sigma &= 18.3 \text{ kPa}, \\ L &= 0.0136 \text{ m}. \end{aligned}$$

These constants were selected from data by Min et al. (1995) and our own anatomical measures from Hirano and Sato's (1993) histology. They would represent a medium-pitched phonatory

adjustment for speech (20% elongation) if the ligament were the only tissue in vibration. Three of the curves in Fig. 3 are for solutions given so far. They are represented by the unfilled data symbols. Note that the classical string model (lowest curve—open circles) greatly underestimates the normal mode frequencies for a string with bending stiffness (exact solution in open diamonds). For mode 1, neglect of bending stiffness lowers the frequency from 170 to 130 Hz (see expanded view in upper right corner). For mode 4, neglect of bending stiffness lowers the frequency from 1030 to 500 Hz. Morse's approximation is shown with dotted diamonds. This approximation [Eq. (19)] is a good match to the exact solution up to mode 3. At mode 4, an 11% error is seen.

IV. VARIABLE CROSS-SECTION (MACULA FLAVAE)

A second step in the development of normal mode frequencies of the vocal ligament was the inclusion of the macula flavae, which are gradual widenings of the cross-sectional areas at the anterior and posterior endpoints (recall Fig. 1). It is believed that these macula flavae are nature's way of reinforcing the ligament at the boundaries in order to accommodate the vibrational stresses (both bending and shear). Using histological sections published by Hirano and Sato (1993) and Sato et al. (2003), an average resting length of 13.6 mm, and a cross section of 7.54 mm^2 as reported by Min et al. (1995), the geometry of the ligament was simplified to two cases with identical volume, as shown in Fig. 4: a rectangular parallelepiped (solid lines) and a parallelepiped with trapezoidal ends (shaded area). These geometries were implemented with 100 finite elements using ANSYS for a structural beam (elements LINK10 and BEAM54), where the cross section at any point was a perfect square.

The uniform geometry incorporated both bending stiffness (BEAM 54) and no bending stiffness (LINK10), but ANSYS did not provide a nonbending option for the trapezoidal geometry (BEAM54). One finite element case (uniform geometry) could thus be checked against both the string equation and the exact analytical solution, and the second case was used to extend the theory beyond the analytical solution.

Returning to Fig. 3, the finite element results for modal frequencies are shown together with the analytical results (again, 20% strain, as in the analytical solution, with $E = 91.7 \text{ kPa}$). The uniform cross section with no bending stiffness was indistinguishable from the string equation; hence, no new data points are shown. The FEM solution with bending stiffness (closed diamonds) matched the analytical calculations exactly only for the lowest mode (open triangles). This match is also seen better in a zoomed-in version of the graph in the upper right. There is a discrepancy, however, for the higher modes. At modes 3 and 4, the finite element solution predicts lower frequencies than the exact analytical solution. This difference is attributed to the nature of the beam element equations when higher order shear effects are neglected. In other words, for a thick string with sizable cross section, bending and shear will generally interact, requiring another degree of freedom. But shear was not part of the overall analysis here because the differential equation was assumed to be one-dimensional [Eq. (2)]. Hence, an inconsistency exists in the FE analysis for higher modes where shear is set to zero but the cross section remains finite.

By adding the macula flavae, an additional increase in the normal mode frequencies (on the order of 30% at the lowest mode) is observed at this ligament length (tapered area—closed squares). For higher modes, the percent difference is diminished, but the absolute difference remains about the same.

V. NONLINEAR ELASTIC PROPERTIES OF THE VOCAL LIGAMENT

Min et al. (1995) quantified the nonlinear elastic properties of the human vocal ligament by mathematically fitting the stretching portion of a sinusoidal stress-release cycle with an exponential function:

$$\sigma = A(e^{Be} - 1). \quad (21)$$

The A and B parameters were reported for each of eight ligaments with an average of $A = 1.4$ kPa and $B = 9.6$.

In the finite element solutions described above, four strains (1%, 10%, 20%, 30 and 40%) were used to solve for σ , E , and the first ($n = 1$) modal frequency. The tangent Young's modulus $\partial\sigma/\partial\epsilon$ was used instead of the secant Young's modulus σ/ϵ because it is more appropriate when the stress-strain curve is nonlinear and there is a bias strain around which vibration takes place. Figure 5 shows how the lowest normal mode frequency changes with increasing ligament strain. The data points for 20% elongation are identical to those of Fig. 3 for mode 1. For 1% elongation, the contribution of bending stiffness and macula flavae raises the frequency from about 10 to 100 Hz, a profound difference. Zero strain would imply a zero string mode frequency because the tension would vanish. The Young's modulus, however, can remain finite due to nonfibrous (ground substance) tissue components. At 50% elongation, the inclusion of bending stiffness and macula flavae raises the mode 1 frequency from 680 to 770 Hz. The addition of the macula flavae (closed squares) raises the frequency by another 110 Hz. On a percentage basis, the bending stiffness contribution reduces with increasing strain, whereas the macula flavae contribution remains roughly constant (about 20%–30%).

VI. DISCUSSION AND CONCLUSIONS

Bending stiffness and nonuniform cross-sectional area of the vocal ligament appear to have an impact on the normal mode frequencies of the vocal ligament. With the mode shapes appearing visually similar to those of a simple string, it is understandable that the ideal string law has been the standard model. At low strains, the ideal string law underestimates the lowest mode frequency by an order of magnitude if bending stiffness is excluded. Asymptotically, as strain goes to zero, bending moments alone can account for the restoring force of the tissue. In previous modeling (Titze and Story, 2002), a ground substance Young's modulus was used to account for this restoring force. At higher strains, bending moments become less significant as tension takes over. Macula flavae contributions remain relatively constant at about 30% increase in mode 1 frequency. Implementing the full mathematical complexity of bending stiffness and non-uniform area may not be suitable for simple models of vocal fold vibration. A rule is therefore adopted for the lowest string mode frequency between 10% and 50% elongation:

$$f = \frac{1}{2L} \left(\frac{\sigma}{\rho} \right)^{1/2} (1 - 0.45 \ln(e)), \quad (22)$$

which corrects for bending stiffness and the macula flavae. For higher modes it is necessary to go back to Morse's approximation, which is only slightly more complicated.

The above results bring into discussion the old concept of “damping,” a misnomer for the reduction of the effective length of vibration of the vocal folds. We quote from an often-used textbook in speech and hearing science (Zemlin, 1997, pp. 167–168):

“Farnsworth (1940), Brodnitz (1959), Pressman (1942), and Pressman and Keleman (1955) attribute falsetto to a similar mechanism. That is, when the vocal folds have been tensed and lengthened as much as possible, further increases in pitch must be accompanied by a different mechanism, namely *damping*. The posterior portions of the vocal folds, in

the region of the vocal processes, are firmly approximated and do not enter into vibration. As a result, *the length of the vibrating glottis is shortened considerably.*" (Italics added by current authors.)

If vibration is "damped" at the endpoints, the vocal fold is effectively shortened, as if pressing a finger near the endpoints of a violin or guitar string. Perhaps a better term would be "extended clamping" because of the universal usage of damping for energy dissipation. In part, the macula flavae may provide this extended clamping with the addition of tapered endpoints. Thus, it may not require much adductory "pressing" to shorten the effective length. In fact, it is generally understood that falsetto is anything but "pressing." Adduction is usually thought to be rather loose.

For low pitches and in modal voice productions, the vocal fold length is mostly below the cadaveric resting length. Ligament tension does not play much of a role. The stiffness of the thyroarytenoid muscle fibers then dominate in F_0 control, rather than the ligament. It is not yet clear if bending stiffness plays a significant role in thyroarytenoid muscle vibration.

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FIG. 1.
Histological section through a male adult vocal fold showing vocal ligament in dard stain (after Hirano and Sato, 1993).

FIG. 2.
Mode patterns for a stiff ligament (solid lines) and a classical “string” ligament (dashed lines) for the first four modes.

FIG. 3.
Normal mode frequencies plotted against mode number (20% strain, $E = 91.7$ kPa).

FIG. 4. Finite-element shapes of the ligament, uniform beam (solid lines) and a tapered beam with macula flavae (shaded area). Both have identical volume.

FIG. 5. Changes in the lowest normal mode frequency as a function of ligament strain, based on measured nonlinear stress-strain characteristics of human ligaments.