Published in final edited form as:

Med Phys. 2005 September ; 32(9): 2862–2869.

Effects of point configuration on the accuracy in 3D reconstruction from biplane images

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Abstract

Two or more angiograms are being used frequently in medical imaging to reconstruct locations in three-dimensional (3D) space, e.g., for reconstruction of 3D vascular trees, implanted electrodes, or patient positioning. A number of techniques have been proposed for this task. In this simulation study, we investigate the effect of the shape of the configuration of the points in 3D (the “cloud” of points) on reconstruction errors for one of these techniques developed in our laboratory. Five types of configurations (a ball, an elongated ellipsoid (cigar), flattened ball (pancake), flattened cigar, and a flattened ball with a single distant point) are used in the evaluations. For each shape, 100 random configurations were generated, with point coordinates chosen from Gaussian distributions having a covariance matrix corresponding to the desired shape. The 3D data were projected into the image planes using a known imaging geometry. Gaussian distributed errors were introduced in the x and y coordinates of these projected points. Gaussian distributed errors were also introduced into the gantry information used to calculate the initial imaging geometry. The imaging geometries and 3D positions were iteratively refined using the enhanced-Metz-Fencil (EMF) technique. The image data were also used to evaluate the feasible R-t solution volume. The 3D errors between the calculated and true positions were determined. The effects of the shape of the configuration, the number of points, the initial geometry error, and the input image error were evaluated. The results for the number of points, initial geometry error, and image error are in agreement with previously reported results, i.e., increasing the number of points and reducing initial geometry and/or image error, improves the accuracy of the reconstructed data. The shape of the 3D configuration of points also affects the error of reconstructed 3D configuration; specifically, errors decrease as the “volume” of the 3D configuration increases, as would be intuitively expected, and shapes with larger spread, such as spherical shapes, yield more accurate reconstructions. These results are in agreement with an analysis of the solution volume of feasible geometries and could be used to guide selection of points for reconstruction of 3D configurations from two views.

I. Introduction

Reconstruction of 3D data from two or more views is useful for patient positioning (1–6) and vessel reconstruction (7–19). To generate the 3D data, the imaging system must be calibrated, i.e., the transformation relating the imaging systems must be determined. While some of the techniques employ calibration models (7–8), many of the techniques (9–19) use a self-calibration approach. In these self-calibrating techniques, no calibration object is used. The Euclidean coordinate system of one of the imaging systems is selected as the reference or “world” system of coordinates. The corresponding points in the images are identified (usually manually), then an initial estimate of the imaging geometry and the 3D data are obtained and iteratively refined by optimizing the alignment of projections of the 3D locations with the corresponding points in the images. The accuracy of the positioning of the center of mass of

the 3D cloud of points can have errors of approximately 0.5 – 2 cm, depending on the number of identified points, the error in the input image data, and the error in the initial estimate of the imaging geometry (19). In addition to the sensitivity to these parameters, preliminary investigations indicated that positioning of points in 3D may influence accuracy and reproducibility of the 3D reconstruction.

From a statistical point of view, optimal positioning of 3D points represents the experimental design problem. It is known that the accuracy of regression predictions depends on the design matrix X (rows represent value of predictors for each item, the 3D coordinates of the configuration points in the present case). The inverse matrix of $X'X$ has the largest effect on the efficiency of the prediction, and typically points in the design space should be spread out so that $\det(X'X)$ is as large as possible; this is the so-called D-optimality criterion (20). Selection of the corresponding points in the 2D images and the number of such pairs of points selected in given case is controlled by the investigator in 3D reconstruction problems, but their 3D location cannot always be controlled. Often, initial imaging geometry errors and 2D errors are beyond the investigator's control. Given this situation, the question arises as to whether the results can be improved by appropriate selection of corresponding points or by proper positioning of the configuration of points chosen or additional markers placed on the patient.

In many respects the problem is similar to the image registration problem and to the study of optimal configuration of fiducial markers (5,24,25). The major difference is that the image registration problem is aligning two 3D (or n-D) sets of points, while the biplane reconstruction problem is reconstructing 3D from 2D data using epipolar constraints. These epipolar constraints result in correlation of the errors. The magnitude of the errors is larger in the direction orthogonal to the image plane of the reference coordinate system.

Another important aspect of the problem is the choice of appropriate criteria for the measurement of accuracy and reproducibility. Previous investigators used the distances between the projections of the reconstructed 3D points and the indicated image points (10, 13,15). While these may be the only available data in clinical situations, these distances may not reflect the errors in the actual 3D position of the points, specifically in the position along the x-ray-beam axis or equivalently in the magnification. However, in simulation studies, these 3D errors can be easily evaluated. Therefore, in this study, we investigate the effect of shape of the 3D configuration of points on the accuracy and reproducibility of 3D data reconstructed from 2D projection data using self-calibration techniques developed in our labs (11,19) and compare these results with the R-t solution volume (26).

II. Methods

II.A. Determination of 3D from two views

A schematic diagram of a two-projection-view imaging system is shown in Figure 1. The transformation relating the two 3D coordinate systems, xyz and $x'y'z'$, which are defined for each single-plane system, can be expressed as

$$\mathbf{x}'_1 = \mathbf{R}(\mathbf{x}_1 - \mathbf{t}) \quad (1)$$

where $\mathbf{R}$ describes the rigid rotation relating the xyz axes and the $x'y'z'$ axes, and where the vector, $\mathbf{t}$, in the xyz system describes the 3D translation from the origin of the xyz system to the origin of the $x'y'z'$ system (i.e., from the position of the focal spot used for the first view to that used for the second view).

When an image of an object consisting of a number of identifiable points is obtained with this imaging system, the image coordinates (u_i , v_i) of the projection of the i -th point in the image plane is related to the 3D position of that point, $\mathbf{P}_i = (x_i, y_i, z_i)$, by

$$u_i = x_i * D / z_i \quad (2a)$$

and

$$v_i = y_i * D / z_i, \quad (2b)$$

where D is the source-to-image-plane distance (SID) in the unprimed system. Similar equations are used for the primed system. With these equations, the imaging geometry, i.e., $\mathbf{R}$ and $\mathbf{t}$, as well as the 3D positions of the point- or line-like objects, can be determined (7–19).

The method used here is the enhanced Metz-Fencil technique (11–19). Briefly, the gantry information (right anterior-oblique/left-anterior-oblique (RAO/LAO) angles, the cranial-caudal (CC) angles, magnification, and SID of each view) are used to estimate the imaging geometry, i.e., the transformation relating the two imaging systems. Using this imaging geometry and indicated points (e.g., bifurcation points) in the images, a 3D configuration of points is generated. A 3D Procrustes algorithm (21) is employed to translate, rotate, and scale the configuration until it aligns optimally with the set of lines that connects a focal spot with the corresponding set of image points in each view. This alignment procedure is applied independently for each view. The transformation (i.e., rotation and translation) that relate the two aligned data sets are then determined by an additional 3D Procrustes calculation. This transformation is then used along with the indicated points to generate a new set of 3D points, which is then used in the next iteration of the algorithm. This technique has been shown to provide reliable and accurate estimates of the imaging geometry as well as the 3D configuration of points (11,19).

The reconstruction problem can also be considered in two parts, i.e., the determination of the imaging geometry ($\mathbf{R}$ and $\mathbf{t}$) and the reconstruction of the 3D configuration of points using the $\mathbf{R}$, $\mathbf{t}$, and image points. The calculated $\mathbf{R}$ and $\mathbf{t}$ and their errors determine to a large degree the errors of the reconstructed points. Because the image data are used as input both in the calculation of the imaging geometry and the reconstruction, we undertook an investigation to determine the set of $\mathbf{R}$'s and $\mathbf{t}$'s which would be possible (feasible) given the input image data. The method for determination of feasible geometries is relatively straightforward (26). For each image point in view 1, an epipolar line is generated in the image of view 2 using the posited imaging geometry. The minimum distance of this epipolar line from the corresponding image point is calculated. If the distance is less than the assumed input image error, the geometry is considered to be a possible geometry (i.e., feasible) for that image point. Those geometries, which are feasible for all indicated image points, are considered feasible given the error in the image data. By searching the $\mathbf{R}$ - $\mathbf{t}$ parameter space (six dimensions, three rotations and three translations), we can identify those geometries, i.e., $\mathbf{R}$'s and $\mathbf{t}$'s, which are feasible given the image data. We call this volume in the 6D space the feasible solution volume. Note, the feasible solution volume corresponds to all those geometries which are possible for the given input image error. Thus, all self-calibrating two-view techniques which determine $\mathbf{R}$ - $\mathbf{t}$ solutions based on input image error, minimizing the final image error (i.e., the difference between the input image data and 3D data reprojected into the images) will lie in this solution volume if the final image error lies in the input image error.

II.B. Shape

According to Kendall (22), “shape is all the geometrical information that remains when location, scale, and rotational effects are filtered out from the object”. There is a long history of shape analysis in different types of applications; for an excellent review consult Dryden and Mardia (23). Here, we are mainly interested in certain shape aspects of the imaged 3D “cloud” of configuration points. We generate the configuration of points from a 3D (multivariate) normal distribution. Different shapes correspond to different relative values of the eigenvalues of covariance matrix of the Gaussian distribution. If a large number of points is simulated from such a distribution, the sample covariance matrix will approach the parent covariance matrix, and the ratios of the three eigenvalues of the covariance matrix of the parent distribution will represent different configuration shapes.

Among the shapes that are encountered clinically are those of the heart and the neurovasculature. The left and right coronary systems in the heart (7,8,10,13,16–19) and the internal carotids (27) and/or the cranial vessels distal to the Circle of Willis (28) are reconstructed using two views. Thus, we chose to generate from four distributions to represent the configurations reconstructed clinically. Data which use a single coronary vessel on the surface of the heart are similar to an elongated but relatively flat ellipse, like a flattened cigar. The distribution of vessel points along the carotid is similar to an elongated ellipse, like a cigar. The distribution of vessel points on the surface of the heart, say the right or left coronary systems, is like a disk on a relatively flat or spherical surface. The distribution of vessel points for the cranial vessels, or which includes both right and left coronary vessels, is similar to a ball or sphere. To obtain distributions that were directly related to clinical data, five bifurcation points were indicated in a clinical biplane image sequence (10 images) of the right coronary artery system yielding a total of 50 pairs of corresponding points. The enhanced Metz-Fencil technique (11) was used to reconstruct the 3D configuration of points corresponding to these 50 pairs of image points. From this 3D configuration, the covariance matrix was estimated, and three eigenvalues (5.02, 0.71, 0.04) were calculated. Different combinations of the square roots of these eigenvalues were used to construct covariance matrices for simulating configuration points with desired distributional shapes of parental distribution. The combination of square roots of eigenvalues together with the natural logarithm of their product (log-volume, LV, which is proportional to the volume of the ellipsoid) are provided in Table 1. A typical configuration for each of the shapes is presented using 3D scatterplots (Figure 3).

II.C Simulation Studies

To investigate the effect of shape on the accuracy of the reconstructed 3D data, two types of simulation studies were performed. The first simulation study was designed to study the effect of a global change in shape/volume. The second simulation study was designed to study the effect of the distance of a single (“traveling”) point external to the pancake shape. This approach was chosen to represent the scenario when there is a point available external to the internal organ to be reconstructed. Besides this shape/distance design variable, the effect of the other design variables (e.g., 2D noise, gantry error, and number of points in the configuration) was evaluated. The 3D coordinates of the specified number of points were generated from the appropriate Gaussian distribution. The number of points, N , per configuration was 5, 10, or 50. The errors in the imaging geometry were generated by introducing errors into the gantry angles (RAO/LAO and CC angles) and errors in the magnification. The errors in the x and y coordinates of the image data, the errors in the gantry angles, and the errors in the magnification were chosen independently from Gaussian distributions with specified variances. Three replicates of 100 configurations were analyzed to obtain estimates of standard errors of response variables.

In preliminary investigations, we determined that the errors in x and y were smaller (by a factor of approximately 10) than those in z. These x-y errors are smaller due to the fact that the reconstructed points are constrained to lie near the projection lines (lines from the focal spot to the respective image points) for those techniques which minimize the distance of the projected reconstructed points and the original image points (7–9,11,13). Errors in the geometry were found to be correlated with the z error. Therefore, the response variables chosen were the error in the z center of mass of the reconstructed configuration (B_z) and the square root of the largest eigenvalue of the covariance matrix of errors (L_1). The second response variable L_1 is close to the standard deviation of the z errors if the direction of the largest principal component of errors is close to the z direction. All configuration simulations were performed using SAS/IML 9.1 package, while imaging modeling was performed using C programming.

II.C.1 Design variables related to shape—The square roots of the eigenvalues of the covariance matrix of the Gaussian distributions, used to simulate the configurations, are proportional to the half-axes of the ellipsoid of constant density of the parent distribution. The real data described in II.B resulted in the following lengths of the half-axes: 2.24 cm, 0.84 cm and 0.19 cm. These original values correspond to a flattened cigar shape. The shape, which we call a cigar, corresponds to half-axis lengths of 2.24 cm, 0.84 cm, and 0.84 cm. The thin-disk or pancake distribution had half-axis lengths of 2.24 cm, 2.24 cm, and 0.19 cm. The ball-shaped distribution had a radius of 2.24 cm. We chose to investigate the effect of global changes in shape, prior to investigating small perturbations of the shapes.

Experiment 2 was designed to evaluate the effect of additional point on 3D reconstruction accuracy. Such a marker can be positioned on the surface of the patient body. In this experiment, only the pancake shape was selected for further investigation because this shape is most similar to the distribution of the bifurcation points of the left or right coronary vessel system. One configuration point was located at the distance $R = 0, 5, 10, 15$ and 20 cm from the pancake configuration. This “traveling” point was moved along the normal to the pancake (positioned at the center of the region) toward the x-ray source along the z axis of the reference coordinate system (the z axis is given by the line between the source and the center of the image of view 1, i.e., the reference coordinate system).

II.C.2 Design variables related to imaging—Two-dimensional input error (σ), gantry error (g), and the number of points (N) in the configuration are expected to influence the accuracy of the 3D reconstruction (11,19). The 2D input error was selected from normal distributions with full-widths at half maximum (FWHM) of 0.01 cm, 0.025 cm, 0.05 cm. These FWHM were chosen as representative of the range of the various levels of error which might be introduced due to automated calculations (a pixel or less) or user indication (a pixel or so). The gantry error was also considered at three different levels ($g = 0, 1, 3$). For the gantry error, normally distributed error was added to each gantry angle ($0^\circ, 1^\circ$, and 3° , respectively) and to the magnification (0, 1, and 3%, respectively) of the second view. All simulations were repeated for $N = 5, 10$ and 50 configuration points.

II.C.3 Response variables—For given values of the design variables (II.C.1-II.C.2), three replicates of 100 configurations are simulated. For each generated configuration, errors (reconstruction-true) for N configuration points are computed. The errors (3D vectors) in the reconstructed configuration points are averaged, and the resulting vector can be thought of as an error of the center of mass of the configuration. Also it is worth noting that the reconstruction errors for a given cloud of points do not cancel out, i.e., the errors of the individual points are similar in magnitude and direction. One hundred simulations are used to create 100 such error vectors. From previous preliminary investigations in our group, it was known that these error vectors form a “cloud” of points, looking like an elongated ellipsoid (or cigar), with the major

axis to within $\sim 2^\circ$ of the z axis (the line perpendicular to projection plane and passing through the x-ray source for the reference coordinate system). There are two important characteristics of this error cloud of points: the location of center of the cloud and the length of the longest and dominating half-axis. For an accurate method, a center should be close to the zero vector, because a non-zero translation would correspond to systematic error (bias) in the reconstructed 3D data. If the cloud of error points is elongated exactly along the z direction (of the reference coordinate system), then the longest half-axis of ellipsoidal cloud of vectors would be the same as the standard deviation of z component. Because the long axis was not exactly along the z axis, the longest half-axis (L1) of this “cloud” (estimated as a square root of the largest eigenvalue of sample covariance matrix of these 100 center of mass error vectors) was selected as a characteristic of interest. The z component of bias (B_z) of the error of the center of mass was selected as another characteristic of interest in the study. All other eigenvalues and components of bias were negligible in comparison with those selected.

II.D Statistical analysis

For both experiments and for each response variable (L1 and B_z), similar statistical analyses were performed. A multiple regression model was used separately for each response variable. The relationship between the average response and the design variables was typically non-linear. Statistical models representing the main dependencies on the design variables (variables of interest) and the selected interactions (products of variables of interest) were fitted. The Bayesian information criterion (BIC) (29) was used to select the final model. This criterion, similar in spirit to the Akaike information criterion (AIC) (30), allows comparison of models with different numbers p of independent variables. In the AIC, the penalty depends only on p , while in BIC, the penalty term is also dependent on the sample size. In all models, the 2D noise, the gantry error, and the number of points were the dominant factors affecting accuracy. For ease of interpretation, the results are presented and discussed using analysis with fixed levels of the three dominant factors (stratified analysis) instead of the final model analysis. Such an approach allows for easier understanding of the dependencies on the volume/shape. For a fixed number of points in the configuration, fixed 2D noise level, and fixed gantry error, the mean response may potentially depend only on the shape/volume of the configuration in first experiment, and on the distance of the “traveling” point from the “pancake” in the second experiment. The “volume” can be measured by computing the determinant of $X'X$, where X is the $N \times 3$ matrix of the coordinates of the configuration points.

III. Results

III.A Effect of shape

The data were fit using regression models relating one of the response variables (L1, B_z) and the design parameters (N - number of points in the configuration, σ - standard deviation of the distribution of 2D errors in mm, g - standard deviation of the error in the gantry angles in degrees and in magnification in percentages, and LV - the natural logarithm of the product of the eigenvalues of the distribution used to generate the configurations). The L1 was found to depend not only on the linear terms in N , σ , g and LV (main effects), but also on selected products of the design variables: $g*LV$, $\sigma*LV$, $g*\sigma$, $N*g$, $N*\sigma$, $N*\sigma*LV$ (interaction terms), $p < 0.0001$. The result of the analysis is in agreement with what is known from the literature, i.e., the eigenvalue L1 increases with increasing σ and g , and decreases with an increasing number of points. The new and interesting result of this analysis is that LV or the volume of the configuration also affects the quality of the reconstructed 3D configuration (i.e., as the 3D volume of configuration of points increases the error magnitude (L1) decreases, which seems intuitively reasonable). The importance of the interaction terms indicates that various combinations of the design characteristics (variables) affect the error measure in different ways. To explain the role of the significant interactions, separate analysis was performed which

concentrated on the role of LV, while stratifying the other design variables. Table 2 presents the results of this stratified analysis, indicating when the effect of LV was statistically significant ($p < 0.05$).

These results generally do not improve with change of shape when a large number of points ($N=50$) is used. Primarily because the results are good to start with, and an increase of LV results in a small, non-significant improvement. For situations with fewer points (e.g., 5 and 10), LV can improve the selected measure of the 3D reconstruction error for specific combinations of gantry error and 2D error, in particular for larger 2D errors.

In Table 3, we present the means of the L1 error for realistic values of the other design parameters, $\sigma=0.025$ cm and gantry=1 deg and 1% magnification error. For a given shape (LV), L1 decreases as the number of points in the configuration increases as would be expected. An additional small study was performed to investigate the dependence of L1 on N. The pancake-shape distribution was used; 3x100 simulations were performed for $N=5, 10, 15, \dots, 45, 50$, $\sigma=0.025$ and $g=1$. The dependence of L1 on N is presented in Figure 4. The regression of L1 on N was performed using natural logarithms of both variables. For all N, the power of dependence was -1.11 ($p<0.0001$, 95% confidence interval (CI): $-1.29, -0.93$). Considering only large N ($N \geq 25$), power of dependence was -0.65 ($p=0.057$, 95% CI: $-1.31, 0.02$), so that -0.5 corresponding to $1/\sqrt{N}$, suggested by the central limit theorem is inside this confidence interval. These results are similar to the dependence on N in registration problems. Fitzpatrick et al. (24), showed that the fiducial registration error (FRE) has a $1/\sqrt{N}$ dependence for image registration problems with fiducial homologous markers. But it must be remembered that the propagation of errors in the self-calibrating two-view reconstruction techniques is different from the registration problem, specifically because of the anisotropy introduced by the epipolar constraints which cause the errors to propagate primarily along the z direction. Thus, more careful analysis would be required to understand this dependence better. For fixed, small to moderate N, the configuration having a larger volume and/or filling the space more homogeneously reduces the error by 60% for $N=5$, and approximately by 50% for $N=10$.

We did find a slight (0.02 cm) bias along the z direction. While this is negligible, the general linear model indicated that the number of points, σ , and LV do significantly ($p < 0.05$) affect the bias both as linear terms and as interaction terms ($N*\sigma$, $N*LV$). Stratified analysis indicated significance of shape only for small N and large σ .

These analyses indicate that the investigator who wants to reduce error of the 3D reconstructed configuration should first concentrate on increasing the number of points in the configuration and in doing so should consider distributions of points for which the LV is increased. However, this may not be possible or may be restricted, therefore, we investigated the effect of adding on an additional point distant from the primary configuration.

III.B Effect of a distant point

Since it is not always possible to select a configuration which will optimize the LV, we investigated the effect of positioning a single configuration point away from the primary cloud of points. A pancake shape was selected as the primary shape distribution in this investigation, since such a shape is most similar to a portion of the surface of the heart as was found from actual 3D data of the left coronary system. The distances $R = 0, 5, 10, 15$, and 20 cm were considered. General linear modeling of the effect of the design variables on the 3D reconstruction error was performed. Apart from the main effects (N, σ, g, R), selected interactions ($g*R, \sigma*R, \sigma*g, N*g, N*\sigma*R$) also appeared to be important in explaining changes in L1 ($p<0.0001$). Since a dependence on interactions was detected, stratified analysis was performed to elucidate the dependencies. Table 4 presents combinations of N, σ , and gantry

error, g , which resulted in statistically significant dependence of the 3D error on the distance of the traveling point.

The situation here is very similar to the shape effect: for a small number of points and relatively large 2D error (σ), the addition of a single point at a distance 5 – 20 cm from the original configuration can significantly reduce the 3D reconstruction error. The results for 5 – 20 cm did not differ significantly however. Typical results are shown in Table 5. Errors are reduced by approximately 50% by adding this one outside point for configurations of 5 and 10 points. When a large number of points is used (i.e., 50), the single outside point does not appear to improve the results and may degrade the results slightly for certain combinations of σ and g , but the errors are consistently less than 1 mm in all these situations and thus negligible.

We did find a slight (0.03 cm) bias along the z direction. While this is negligible, the general linear model ($p < 0.0001$) indicated that the N , σ , and R do significantly affect the bias both as linear terms and as interaction terms ($N \cdot \sigma$, $\sigma \cdot R$). Stratified analysis indicated significance of R only for small N and large σ .

In the investigations of the feasible solution volume, we found that it exhibited a dependence on the configuration similar to that seen for the EMF (see Figure 5), both for configuration shape and for the “moving” point. Because the feasible solution volume corresponds to all those geometries which will yield solutions with final image errors less than the input image error and because most if not all self-calibrating techniques will calculate geometries satisfying this condition, we believe that the dependencies of the feasible solution volume, in particular its dependence on the configuration, represent those of self-calibrating two-view techniques.

Summary and Discussion

In this study, we investigated the effect of the shape of a cloud of configuration points on the error in the 3D reconstruction of that cloud when using a self-calibrating two-view technique. We found that

1. the number of points in the configuration, the 2D error, and the gantry error had significant, dominant effects on the error in the 3D reconstruction;
2. for a large number of configuration points ($N = 50$), the results are accurate, and no significant improvement can be made by changing the configuration of points;
3. for a small number of points ($N \leq 10$), the configurations with larger volumes and isotropic shapes (ball) should be selected;
4. if an optimal selection is not possible, an additional configuration point at the distance ($R \geq 5\text{cm}$) will improve the accuracy of the 3D reconstruction;

The accuracy improvement with increasing number of points in the configuration on selected measures of accuracy was expected, and thus, we recommend using as large a number of configuration points as possible. If a large number of points is not possible, the configuration of points having a larger volume (i.e., filling the space more completely) should be used, e.g., in cardiac vessel reconstruction incorporating both left and right ventricular systems. If this configuration choice is not available, adding one point outside the body should improve the accuracy. The dominating error in the z direction (with respect to reference system of coordinates) can be approximately cut in half by the appropriate choice of the configuration of points. Also, it seems advisable not to use fewer than 10 points, since for small number of points, errors along the z axis of reference coordinate system may be larger than 1 mm.

In conclusion, the results of the study indicate that the dominant reconstruction errors can be cut in half by appropriately selecting 2D image points which correspond to the set of 3D

configuration points which occupy larger volumes, when only a small number of points can be selected. When large numbers of configuration points can be selected, the accuracy of reconstruction is very good regardless of the design parameters.

Acknowledgements

This work was supported by NIH grants HL52567 and EB02916, and Toshiba Medical Systems Corporation.

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Figure 1.
Schematic example of image and object coordinate system for biplane projection views with arbitrary relative orientation.

Figure 2.

Two views of the left coronary system. Bifurcations are indicated by the white disks. Ten frames were used, and the positions of the bifurcation points were used to estimate the covariance matrix of the “flattened cigar” distribution.

Figure 3.

Typical 3D configurations shown as 3D scatterplots of 50 points: flattened cigar (upper left), pancake (upper right), cigar (lower left), ball (lower right).

Figure 4.
Empirical dependence of L1 on the number of points in the configuration for the pancake shape

Figure 5.

Dependence of L1 and the volume of the feasible solution space on configuration shape (N=10). Configuration numbers correspond to (1) flattened cigar, (2) cigar, (3) pancake, (4) ball, (5) pancake with remote point 5 cm distant, (6) pancake with remote point 10 cm distant, (7) pancake with remote point 15 cm distant, (8) pancake with remote point 30 cm distant. Trends for the configurations in the EMF method are similar to those for the solution volume, except for the pancake shape (3), i.e., errors decrease as the volume of the 3D configuration increase.

Table 1

Correspondence between shapes and half-axes; LV=natural logarithm of product of half-axes

Shape	Half-axes (cm)	LV
Flattened Cigar	2.24, 0.84, 0.19	-1.03
Pancake	2.24, 2.24, 0.19	0.05
Cigar	2.24, 0.84, 0.84	0.46
Ball	2.24, 2.24, 2.24	2.42

Table 2

Presentation of stratified analyses for L1; X indicates case when LV was significant ($p < 0.05$)

		N=5			N=10			N=50		
					2D error level					
		.001	.025	.050	.001	.025	.050	.001	.025	.050
Gantry	0	X	X	X	X	X	X		X	
	1		X	X		X	X			
	3			X		X	X			

Table 3

Mean L1 values for $\sigma = 0.025$ cm and gantry=1 deg for various numbers of points and shapes. Entries in the table are average maximal error eigenvalue L1 from three replications and standard deviation of this estimate.

N	Flattened cigar	Pancake	Cigar	Ball
5	0.84 (0.09)	0.57 (0.00)	0.46 (0.04)	0.34 (0.03)
10	0.40 (0.13)	0.32 (0.08)	0.20 (0.06)	0.19 (0.06)
50	0.06 (0.07)	0.02 (0.02)	0.03 (0.03)	0.07 (0.03)

Table 4

Results of stratified analysis - distance of the “traveling point” significantly affects the value of L1. The symbol X, indicates a significant distance effect ($p < 0.05$).

		N=5			N=10			N=50		
					2D error level					
		.001	.025	.050	.001	.025	.050	.001	.025	.050
Gantry	0	X	X	X	X	X	X		X	X
	1		X	X		X	X			
	3		X	X						X

Table 5

L1 as a function of the number of points in the configuration and the distance of a single point from the bulk of the configuration for $\sigma = 0.25$ cm and $g = 1$.

N	Pancake				
	R=0	R=5	R=10	R=15	R=20
5	0.57 (0.00)	0.28 (0.02)	0.23 (0.02)	0.24 (0.03)	0.28 (0.10)
10	0.32 (0.08)	0.19 (0.03)	0.15 (0.04)	0.14 (0.04)	0.18 (0.04)
50	0.02 (0.02)	0.02 (0.01)	0.01 (0.02)	0.03 (0.03)	0.04 (0.04)