*RELATIVE ALLOCATION ON CONCURRENT SCHEDULES
CAN DEPEND ON SCHEDULE PARAMETERS WHEN
BEHAVIORAL PARAMETERS ARE CONSTANT*

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We develop a simple model of switching between the initial links of a concurrent-chain procedure. Behavior is determined by four parameters μ_1 , μ_2 , q_1 , and q_2 . The first two are the basic rates of switching from Schedule 1 and Schedule 2, respectively. The second two are the probabilities of leaving Schedule 1 and Schedule 2 after the corresponding terminal link has been completed. We show that for fixed values of these four parameters, the relative allocation on the initial links may change as a result of changes in initial-link schedules. The effect can be quite large if the switching rates are low. An implication is that relative allocation is not necessarily a good measure of behavior.

Key words: relative allocation, concurrent chains, switching, molar versus molecular analysis

Choice between concurrent schedules is usually characterized by the relative amount of time spent or number of responses made to a particular schedule during an experimental session. Similarly, the relative allocation of responding on the initial links of concurrent chains has been used as a measure of the subject's valuation of the terminal links (e.g., Green & Snyderman, 1980; Killeen, 1968; Navarick & Fantino, 1976). The delay-reduction hypothesis (e.g., Fantino & Abarca, 1985) predicts that relative allocation in concurrent-chains procedures will depend partly on the initial-link schedules. In this paper we show that, under plausible assumptions about the behavioral mechanisms underlying choice, such a dependence can emerge despite unchanging processes and parameters at the molecular level. Our model is designed to demonstrate this effect, and is not intended to be a complete account of performance on concurrent chains.

THE MODEL

We consider a concurrent-chains procedure in which the initial links are independent

variable-interval schedules. The distribution of schedule intervals is negative exponential, so that there is a constant moment-to-moment probability of a terminal link being available (Fleshler & Hoffman, 1962). The schedules are referred to as Schedule 1 and Schedule 2; Schedule i together with its correlated terminal link is referred to as side i ($i = 1, 2$). Schedule i sets up entries into its terminal link at a rate λ_i ($i = 1, 2$). When the subject is responding on Schedule i , we ignore any delay between a terminal link being made available by Schedule i and that terminal link being produced by a response. Thus, strictly speaking, the initial links modeled are variable-time schedules, but our analysis will hold if the rate of responding while on a schedule is high. When a terminal link is produced on Side i , the subject completes the requirements of this link and then has a probability q_i of switching to the other schedule, and a probability $1 - q_i$ of staying on Side i . In the absence of entries into a terminal link, the subject switches from Side i to the other side at a rate μ_i .

We assume throughout that Side 1 is the "better" side, in that either the terminal links have the same magnitude but Side 1 has the shorter delay, or the terminal links have equal delays but Side 1 has the greater magnitude of reinforcement. (The details of the terminal links are irrelevant, but we wish to motivate a consideration of unequal treatment of the two schedules even when $\lambda_1 = \lambda_2$.)

At a molecular level, the behavior of the subject is specified by the four parameters μ_1 ,

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Fig. 1. The relative time spent on Schedule 1, p , as a function of varying absolute but equal relative rates of reinforcement, $\lambda_1 = \lambda_2$, when $\tau = 0$, $\mu_1 = .025$, and $\mu_2 = .025$ (μ = rate of switching in the absence of reinforcement). The solid curves correspond to various values of q_2 , the probability of switching from Schedule 2 immediately after reinforcement. From top to bottom: $q_2 = 1$; $q_2 = .5$; and $q_2 = .25$, all with $q_1 = 0$. The broken curve is for $q_1 = .25$ and $q_2 = 1$. It can be seen that p deviates from 0.5 as λ increases and that the effect increases as q_2 increases.

μ_2 , q_1 , q_2 . If no terminal link is obtained on arrival at Schedule i , then the time spent on Schedule i has a negative exponential distribution with parameter $\alpha_i = \mu_i + \lambda_i q_i$. If $q_1 = q_2 = 0$, then the terminal links have no effect on switching and the time between arriving on Schedule i and switching to the other schedule has a negative exponential distribution with mean $1/\mu_i$ (cf. Heyman, 1979). If $q_1 = 0$ and $q_2 = 1$, then the subject always goes to Schedule 1 after a terminal link has been completed. In this case, the expected time on Side 1 is $1/\alpha_1 = 1/\mu_1$, and the expected time on Side 2 is the probability of no initial link being entered on arrival at Side 2 times $1/\alpha_2$ which equals

$$\frac{\mu_1}{\mu_1 + \lambda_2} \cdot \frac{1}{\alpha_2}.$$

Thus the proportion p of the session that is spent on Side 1 is given by the equation

$$p = \frac{\frac{1}{\mu_1}}{\frac{\mu_1}{\alpha_2(\mu_1 + \lambda_2)} + \frac{1}{\mu_1}} = \left[\frac{\mu_1^2}{(\mu_2 + \lambda_2)(\mu_1 + \lambda_2)} + 1 \right]^{-1}. \quad (1)$$

Table 1

Two model-derived measures (p and r), obtained with $q_1 = 0$ and $q_2 = 1$: p is the relative time spent on Schedule 1, r is the proportion of terminal links obtained from Schedule 1, and q_i is the probability of switching away from Schedule i immediately after reinforcement. These measures are based upon the procedures reported by Fantino (1969), for which corresponding data are also presented.

High basic switching rates							
$1/\lambda_1 =$		$\mu_1 = .25$	$\mu_1 = .2$	$\mu_1 = .1$			
$1/\lambda_2$	p	$\mu_2 = .25$	$\mu_2 = .3$	$\mu_2 = .4$	p	r	p
(s)	(data)	p	r	p	r	p	r
40	.95	.548	.503	.646	.512	.842	.544
120	.81	.516	.500	.616	.504	.816	.515
600	.60	.503	.500	.603	.501	.803	.503
Low basic switching rates							
$1/\lambda_1 =$		$\mu_1 = .025$	$\mu_1 = .02$	$\mu_1 = .01$			
$1/\lambda_2$	p	$\mu_2 = .025$	$\mu_2 = .03$	$\mu_2 = .04$	p	r	p
(s)	(data)	p	r	p	r	p	r
40	.95	.800	.609	.861	.647	.958	.758
120	.81	.640	.525	.731	.548	.899	.625
600	.60	.532	.501	.632	.508	.829	.530

It can be seen from Equation 1 that p increases with increasing λ_2 for fixed μ_1 and μ_2 . When both q_1 and q_2 are nonzero, then both λ_1 and λ_2 will influence p . The general equation for p , together with the equation for the proportion r of terminal links that are obtained from Side 1, are derived in the Appendix. That general model includes the possibility of a delay, τ , in switching from side to side. The usual measure of relative allocation on concurrent chains is based on responses rather than time. The Appendix gives an equation for the relative response allocation R . R is equal to p when the response rates are equal and $\tau = 0$.

RESULTS

The basic point that we wish to make is that relative allocation can change as a result of changes in λ_1 or λ_2 , when μ_1 , μ_2 , q_1 , and q_2 remain constant. These four parameters characterize the subject's behavior at the molecular level. It follows that the molar characterization of behavior in terms of relative allocation can change despite constant behavioral parameters at the molecular level. The

effect is illustrated in Figure 1, which shows the relative allocation p as a function of $\lambda = \lambda_2 = \lambda_1$, with $\mu_1 = \mu_2$ throughout. With $q_1 = 0$ we show p for three values of q_2 (solid lines). The broken line gives p for $q_1 = .25$ and $q_2 = 1$. If $q_1 = q_2 = 0$, then $p = .5$ for all values of λ . As q_2 increases, the initial links have a greater effect on p . It can be seen that p tends to 0.5 as λ decreases. This sort of trend has been found on concurrent chains (e.g., Fantino, 1969), and so it is of interest to see if the model can account for the data without any changes in the behavioral rule. Table 1 gives the average results of Fantino ("data"), together with p and r for various values of μ_1 and μ_2 , with $q_1 = 0$ and $q_2 = 1$. (In Fantino's experiment, Schedule 1 led to a VI 30-s terminal link and Schedule 2 led to a VI 90-s terminal link.)

As is to be expected from Figure 1, the model produces the same trend as the data. When the switching rate is high, the change in p is much smaller than that of the data, but for low switching rates, the model comes close to the data. The switching rate can be summarized by the switching index I introduced by Heyman (1979). I is half the harmonic mean of the mean visit lengths. When $q_1 = q_2 = 0$, all switches are determined by μ_1 and μ_2 and $I = 1/(\mu_1 + \mu_2)$. When either q_i is non-zero, I will be less than $1/(\mu_1 + \mu_2)$ and will decrease as λ_i increases. For example, in our low-switching-rate condition, $1/(\mu_1 + \mu_2) = 20$ s, but when $\lambda_1 = \lambda_2 = 1/40$, I is just under 6 s. On concurrent variable-interval schedules with no changeover delay, I seems to lie between 1 s and 10 s (Houston, 1983). We do not know of any relevant data for concurrent chains.

DISCUSSION

The effect that we have described arises from what is known in statistical theory as "censorship." In the absence of entries into the terminal links, the mean visit duration on Schedule i is $1/\mu_i$ and the proportion of time spent on Schedule 1 is $\mu_2/(\mu_1 + \mu_2)$. Because the subject may switch after completing a terminal link, the entries into the terminal links curtail or censor the distribution of visit times. The same sort of effect can be seen in models of foraging in which the animal leaves a foraging area either as soon as it obtains a food

item or after a time T has elapsed without finding any food. If food items are found at a constant rate λ , then the expected time before leaving is $1/\lambda[1 - \exp(-\lambda T)]$ (McNamara & Houston, 1985).

We have made no attempt to develop a complete account of behavior on concurrent chains. Such an account would have to relate behavioral parameters to the obtained reinforcements. Thus in general, μ_1 , μ_2 , q_1 , and q_2 will vary with the schedule parameters and relative allocation will be determined by two effects: the way in which the parameters depend on experience, and the censorship effect of q_1 and q_2 . It has been our aim in this paper to isolate the second effect, and to this end we have kept μ_1 , μ_2 , q_1 , and q_2 fixed while the schedule parameters varied. The censorship effect may occur on the concurrent variable-interval variable-interval procedure, but in this case we feel that it is necessary to incorporate the effect of the schedule intervals on the behavioral parameters. (The results of Heyman, 1979, indicate that switching on concurrent VIs does not depend on reinforcements, but Heyman's position has not gone unchallenged; see, for example, Hinson & Staddon, 1983; Shimp, 1982.)

The regularities in molar behavior, as expressed by the matching law (Baum, 1974; Herrnstein, 1961) and the delay-reduction hypothesis (Fantino & Abarca, 1985) are impressive, but it must not be forgotten that a large amount of information about behavior is lost in the move from the molecular to the molar level. It is clear from the models of Heyman and Luce (1979) and Houston and McNamara (1981) that many different molecular patterns of behavior can result in the same behavior at the molar level. In this paper we have shown that molar behavior may change when the parameters that characterize behavior remain constant. We hope that this result will stimulate research into the molecular details of choice on concurrent chains, and especially into the influence of the terminal links on switching.

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APPENDIX

We consider a subject that switches between two independent variable-interval schedules. Schedule i sets up entries into its terminal link at a rate λ_i ($i = 1, 2$). In the absence of entries into a terminal link, the subject switches from Side 1 to Side 2 at a rate μ_1 and switches from Side 2 to Side 1 at a rate μ_2 . After completing the requirements of a terminal link on Side i , there is a probability q_i that the subject does not return to Schedule i .

A time τ is required to move from one schedule to the other. We assume that the rate of responding is high, so that when the subject is on Schedule i there is no delay between a terminal link becoming available through Schedule i and a response producing that terminal link.

The subject leaves a schedule either as a result of a switch in the absence of a terminal link or as a result of a switch following the completion of a terminal link. A visit to a schedule starts when the subject switches to it from the other schedule ("arrives") and ends when it switches away in either of the above ways. Thus a visit to a schedule may be interrupted by entries into the terminal link, but not by visits to the other schedule.

We begin by calculating the proportion of times that an arrival on Schedule i results in a terminal link being obtained. This proportion is denoted by c_i ($i = 1, 2$). To calculate c_i , we define two further terms, which we now specify for the case of an arrival on Schedule 1. Consider the visit to Schedule 2 that preceded this arrival. The arrival that began this visit either resulted in a terminal link being obtained or did not.

Let

a_1 = Probability of entering a terminal link on arrival at Schedule 1, given that no terminal link was obtained on the last arrival at Schedule 2.

b_1 = Probability of entering a terminal link on arrival at Schedule 1, given that a terminal link was obtained on the last arrival at Schedule 2.

a_2 and b_2 are the corresponding terms for Schedule 2.

From these definitions,

$$c_1 = c_2 b_1 + (1 - c_2) a_1$$

and

$$c_2 = c_1 b_2 + (1 - c_1) a_2;$$

hence,

$$c_1 = \frac{a_1 + a_2 b_1 - a_1 a_2}{1 - (a_1 - b_1)(a_2 - b_2)}$$

and

$$c_2 = \frac{a_2 + a_1 b_2 - a_1 a_2}{1 - (a_1 - b_1)(a_2 - b_2)}.$$

CALCULATION OF a_1 AND a_2

If no terminal link is obtained on arrival at Schedule i , then the time X_i spent on Schedule i has a negative exponential distribution with parameter $\mu_i + \lambda_i q_i = \alpha_i$. $a_1 = P$ (terminal link on arrival at 1/no link on arrival at 2), and so,

$$\begin{aligned} a_1 &= E\{1 - \exp[-\lambda_1(X_2 + 2\tau)]\} \\ &= 1 - \exp(-2\lambda_1\tau)E[\exp(-\lambda_1X_2)]. \end{aligned}$$

Now

$$\begin{aligned} E[\exp(-\lambda_1X_2)] &= \int_0^\infty \alpha_2 \exp(-\alpha_2x) \exp(-\lambda_1x) dx \\ &= \frac{\alpha_2}{\alpha_2 + \lambda_1}, \end{aligned}$$

and hence,

$$a_1 = 1 - \frac{\alpha_2}{\alpha_2 + \lambda_1} \exp(-2\lambda_1\tau).$$

By symmetry,

$$a_2 = 1 - \frac{\alpha_1}{\alpha_1 + \lambda_2} \exp(-2\lambda_2\tau).$$

CALCULATION OF B_1 AND B_2

$$\begin{aligned} b_1 &= P \text{ (terminal link on arrival at 1/terminal link on arrival at 2)} \\ &= P \text{ (terminal link on arrival at 1/switch from 2 after terminal link)} \\ &\quad q_2 + P \text{ (terminal link on arrival at 1/do not switch from 2 after terminal link)} (1 - q_2) \\ &= (1 - \exp(-2\lambda_1\tau))q_2 + a_1(1 - q_2). \end{aligned}$$

By symmetry,

$$b_2 = (1 - \exp(-2\lambda_2\tau))q_1 + a_2(1 - q_1).$$

RELATIVE ALLOCATION

Consider a random visit to Schedule 1. Let T_1 be the time on this schedule, not including the switching time τ . From our definitions,

$$P(T_1 = 0) = c_1 q_1$$

and

$$P(T_1 > t | T_1 > 0) = \exp(-\alpha_1 t),$$

and so,

$$E(T_1) = c_1 q_1 0 + (1 - c_1 q_1)/\alpha_1;$$

thus,

$$E(T_1) = (1 - c_1 q_1)/\alpha_1.$$

Let

$$\begin{aligned} N_1 &= \text{number of entries into a terminal link during a visit to Schedule 1;} \\ E(N_1) &= c_1[1 + (1 - q_1)E(\tilde{N}_1)] \\ &\quad + (1 - c_1)E(\tilde{N}_1), \end{aligned}$$

where $E(\tilde{N}_1)$ is the expected number of entries given that the subject does not switch immediately after arrival. As entries occur at rate λ_1 and the expected time is $1/\alpha_1$,

$$E(\tilde{N}_1) = \lambda_1/\alpha_1.$$

The equations for $E(T_2)$ and $E(N_2)$ can be obtained by symmetry. The relative allocation, p , is given by the equation

$$p = \frac{E(T_1) + \tau}{E(T_1) + E(T_2) + 2\tau}.$$

If the subject responds at a rate R_i on Schedule i , then the relative allocation of responding, R , will be given by

$$R = \frac{E(T_1)R_1}{E(T_1)R_1 + E(T_2)R_2}.$$

The proportion r of terminal links obtained from Schedule 1 is given by the equation

$$r = \frac{E(N_1)}{E(N_1) + E(N_2)}.$$