SUPERIMPOSITION OF RESPONSE-INDEPENDENT REINFORCEMENT

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Studies that have superimposed response-independent reinforcement (or reinforcers scheduled by contingencies placed on the absence of responding) upon conventional response-dependent schedules are reviewed. In general, providing alternative sources of reinforcement reduced response rates below the levels observed when alternative reinforcement was absent. However, response-rate elevation was sometimes found, particularly when rates of superimposed response-independent reinforcement were low. Superimposition of schedules providing reinforcers contingent on the absence of responding usually produced more severe response-rate decrements than superimposition of response-independent reinforcement. A variant of Herrnstein's equation, which assumes that some of the alternative reinforcers function as if they were delivered by baseline response-dependent source of reinforcement, is in qualitative agreement with the overall body of results obtained, and can predict both increases and decreases in response rate as resulting from superimposed reinforcers.

Key words: response-independent reinforcement, Herrnstein's equation, alternative reinforcement, fixed time, variable time, differential reinforcement of other behavior

The paradigm that is the focus of interest here involves superimposition of response-independent reinforcement—scheduled either periodically, according to a fixed-time (FT) schedule, or aperiodically, according to a variable-time (VT) schedule—onto a schedule of response-dependent reinforcement that remains in force. Such manipulations allow the automatic measurement of conventional operant responses, as contrasted with studies of FT or VT alone, which usually require observational measures of behavior. These methods also allow response-independent reinforcement effects to be studied for long periods of time in steady-state conditions, unlike the studies of transitions from response-dependent to response-independent reinforcement, during which responding sometimes declines almost to zero under the response-independent schedule (e.g., Boakes, 1973). As shown by Rachlin and Baum (1972), superimposition studies can also be easily related to results from response-dependent reinforcement schedules via the medium of Herrnstein's equation (Herrnstein, 1970).

The form of Herrnstein's equation that addresses effects of superimposed response-independent reinforcement is:

$$R = \frac{kr_1}{r_1 + r_2 + r_o} \quad (1)$$

Here, R is the rate of the measured operant, r_1 is the rate of reinforcement that it produces, and r_2 and r_o are the rates of reinforcement available from alternative sources. The first (r_2) is the rate of reinforcement that is arranged by the experimenter independent of responding or for activities other than the measured operant, and r_o is the rate of "extraneous" reinforcers, including those arising from the environment but not arranged by the experimenter, and those, such as grooming, arising from the animal's own maintenance needs. The constant k reflects the asymptotic rate at which the measured operant can be emitted.

The effects of superimposition of response-independent reinforcement often have been compared with those of superimposing a contingency requiring a period of nonresponding for reinforcer delivery (differential reinforcement of other behavior, or DRO). There are two reasons for this comparison. First, DRO schedules might be supposed to reinforce nonresponding, or to enhance the emission of responses incompatible with the measured operant. To the extent that behavioral effects of FT and VT depend on this process, FT and VT should affect behavior in the same way as does DRO. Second, the study by Rachlin and

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Baum (1972) had a seminal influence in this area, and this article drew particular conclusions from a comparison of the effects of a DRO and VT schedule superimposed on a baseline of variable-interval (VI) reinforcement.

Before proceeding with a review of the results of superimposition studies, a note on nomenclature is required, for in this context there have been inconsistent uses of conventional schedule terms. Some writers (e.g., Henton & Iversen, 1978; Rachlin & Baum, 1972) have used the term *concurrent schedule* to apply to those cases in which one schedule is superimposed upon another. Others (e.g., Lattal & Bryan, 1976; Zeiler, 1976) have used the term *conjoint* to describe this same arrangement. To circumvent these difficulties in nomenclature, we will adhere to a terminology that can consistently describe all conditions that have been used. Consider two schedules, A and B (e.g., VI 3 min and VT 3 min). The operation of superimposing schedule B onto schedule A (which remains in effect) will be written A sup B (e.g., VI 3 min sup VT 3 min). The baseline schedule will always be indicated first, the superimposed schedule second. This superimposition description refers to scheduled contingencies and may not represent the actual pattern of reinforcer deliveries.

SUPERIMPOSITION OF RESPONSE-INDEPENDENT AND DRO SCHEDULES UPON RESPONSE-DEPENDENT SCHEDULES

Rachlin and Baum (1972) initially trained pigeons on conjoint VI 3-min VI 3-min schedules in which availability of reinforcers from one of the VI 3-min schedules was signaled; they also exposed the pigeons to either VI 3 min sup (tandem VT 3 min DRO 2 s), under which a reinforcer was scheduled on average every 3 min and delivered when a subsequent 2-s period of nonresponding occurred, or VI 3 min sup VT 3 min. Rachlin and Baum reported that the rate of key pecking declined ordinarily as the amount or rate of non-VI reinforcers increased; they concluded that the source of the alternative reinforcement was unimportant, because in their experiment the superimposed VT and DRO produced very similar effects.

Two questions that arise from this study

Fig. 1. Effect of FT superimposition on responding maintained by fixed-ratio schedules of reinforcement (data from Zeiler, 1979). The baseline schedule was either FR 180 (left column) or FR 300 (right column), and rates of reinforcement delivered by superimposed FT schedules varied from 0 to 360 reinforcers per hour.

provide a framework for discussing other results. First, what effects does superimposition of a DRO or a response-independent schedule have on response rate or patterning maintained by a response-dependent baseline schedule? Second, does the scheduling of the superimposed reinforcement play some critical role, or is the influential factor merely its rate of delivery?

As for the first question, the overall pattern of results provides considerable support for Rachlin and Baum's contention that increasing rates of superimposed "alternative" reinforcement produce progressively greater and greater decrements in response rate, relative to situations without arranged sources of alternative reinforcement. In a few cases, low rates of superimposed response-independent reinforcement appear to have rate-enhancing effects, but in almost all studies some rate of superimposed response-independent or DRO reinforcement can be found that maintains a lower response rate than the baseline schedule alone.

For example, Edwards, Peek, and Wolfe (1970) studied the effects of fixed-ratio (FR)

sup FT and typically found response-rate decrements when the rate of FT reinforcement was substantially greater than that produced under the FR schedule alone. Similarly, Lattal (1974) varied in an ordinary VI the percentage of reinforcers that were delivered independently of responding, and found that higher percentages led to lower response rates. Note, however, that although this procedure holds overall reinforcement rate constant, it confounds two operations—reducing the rate of response-dependent reinforcement and increasing the rate of response-independent reinforcement—both of which might be expected (after Herrnstein, 1970, and Rachlin & Baum, 1972) to produce decreases in response rate.

Zeiler (1976) studied FI 3-min sup FT and FI 3-min sup DRO schedules. Parameter values for the superimposed schedules were varied over a wide range (10 to 100 s). Both the superimposed FT and DRO schedules generally reduced response rates below those obtaining under FI alone, with higher rates of superimposed reinforcement usually producing lower response rates. Zeiler (1977) found similar results under variable-ratio (VR) 100 and VI 100-s schedules with and without various FT and DRO superimpositions. In a later study, Zeiler (1979) superimposed various FT schedules upon FR 180 and 300, and various DRO schedules upon FR 60, 180, and 300. The results of these manipulations were more complex than in earlier studies. Figure 1, drawn from a table in Zeiler's 1979 study, shows the effect of those FT superimpositions.

Although high FT rates totally suppressed responding for all subjects, low FT reinforcement rates (e.g., FT 80 and 120-s schedules) reliably produced increases in response rate above that prevailing under FR. The only exception occurred in data from Bird 158 under FR 300. Zeiler did not provide details regarding delivery rates of DRO reinforcement, but, in general, DRO superimposition tended to produce response suppression. Responding on the FR-300 schedule was eliminated by DRO values less than 120 s; with FR 180, DRO values of 80, 40, 20, and 10 s produced total response suppression; with FR 60, only DRO 10 s had this effect.

Lattal and Bryan (1976) found similar results when superimposing various FT and VT schedules on FI baselines. Response-rate decrements were common, although in some cases

the superimposition resulted in rate increases (e.g., when FI 5 min was changed to FI 5 min sup VT 2.5 min). Lattal and Boyer (1980) also obtained reliable rate increases by what was, in effect, a VT schedule superimposed on FI 150 s. However, instituting various DRO requirements (with the DRO being arranged according to a VI schedule) usually produced marked decreases in response rates, with rates generally declining ordinally with increases in the rate of DRO reinforcers delivered.

Thus, the finding of increased response rates resulting from superimposition of response-independent reinforcement is not unusual, although it may be less common than a rate decrement of the type reported by Rachlin and Baum (1972). The reasons that both effects might occur are discussed further below.

Evidence bearing on the second question posed earlier provides only partial support for Rachlin and Baum's (1972) contention that the source of superimposed reinforcement has little influence over and above its overall rate of delivery. For example, Zeiler summarized the results of his 1977 study as follows:

Even at low time requirements where the schedules operate most similarly, the DRO is likely to establish the lower rate. With still longer times, DRO consistently generates less responding. Then, with the longest times, FT may establish lower rates than DRO, because the DRO requirements are rarely if ever met. (p. 31)

Other differences between a DRO and FT superimposition were found by Zeiler (1979). As mentioned earlier, when the DRO contingency was met, and DRO reinforcers were delivered, a degree of response-rate decrement with respect to the FR baseline was usually noted. Low rates of FT reinforcement, on the other hand, consistently elevated response rates when superimposed above the FR schedules used. In addition, Lattal and Bryan (1976) found differences in response patterning contingent on superimposing VT and FT on FI, and Lattal and Boyer (1980) found different effects of a DRO and VT schedule on responding maintained by FI 150 s.

All these results suggest, of course, that the source of alternative reinforcement can matter. At the same time, it should be emphasized that these findings do not constitute a direct contradiction of, or failure to replicate, Rachlin and Baum's (1972) results, because most

of the studies employed schedules very different from those used by Rachlin and Baum. One particularly striking difference between Rachlin and Baum's procedures and those of Zeiler (1976, 1979), Lattal and Bryan (1976), and Lattal and Boyer (1980) is that Rachlin and Baum's study employed a baseline schedule (VI) and superimposed schedules (VT and tandem VT DRO) under which reinforcer deliveries were aperiodic, whereas the other authors employed baseline conditions (FI and FR), or superimposed schedules (FT or simple DRO) under which reinforcer delivery was periodic, or followed some distinct event such as a substantial period of nonresponding.

The elevations of response rate above baseline levels produced by superimposed response-independent reinforcement noted here have usually accompanied changes in characteristic response patterns generated by the baseline schedules, particularly a marked decrease in postreinforcement pausing. For example, Lattal and Boyer (1980) found that the VT superimposition that produced elevated response rates was generally correlated with disruption of the typical FI pattern of responding and decreases in proportion of total time spent pausing. Zeiler (1979) also noted changes in response patterning occasioned by superimposed FT schedules, and examination of the cumulative records he presented suggests that reduction or elimination of the postreinforcement pause observed under FR was a common effect of the superimposed schedules.

A study by Henton and Iversen (1978) also illustrates the importance of interactions between the events arranged by the superimposed schedule and the pattern of behavior maintained by the baseline schedule. These workers varied the superimposition conditions imposed on a VI baseline using rats as subjects. Their requirements that responses be present or that responses be absent in a 1-s period before superimposed reinforcer delivery had marked effects on response probability: The requirement of no responding during the 1 s before the reinforcer (similar to a DRO requirement) maintained less responding than the requirement that a response be present (similar to a variable delay-of-reinforcement requirement). A simple VT schedule markedly decreased lever-press probability, but observation of the animals revealed that this

probability was related to the frequency with which previous food deliveries had been contiguous with some behavior other than lever pressing. In general, the outcome of a superimposition condition seemed to depend in complex ways on the contiguities between lever pressing, other behavior, and food delivery.

This type of finding suggests that the outcome of any particular superimposition experiment may be predictable only from a complex combination of factors. The pattern of behavior generated by the baseline schedule and the requirements of the superimposed condition may be foremost, but the details of the interactions of the two may also be very influential. On the other hand, when a baseline schedule generates responding that is approximately uniform over time (such as a VI schedule), and when superimposed schedules are aperiodic and impose minimal or zero response requirements (such as VT or a brief DRO arranged in tandem with VT), then the rate of reinforcement delivered by the superimposed schedule may be a dominant predictor of response rate. Such conditions were those used by Rachlin and Baum (1972).

HERRNSTEIN'S EQUATION

The present section seeks to use Herrnstein's equation (both in its original form and in a variant developed below) as a framework for discussing the overall pattern of results obtained when response-independent and DRO schedules are superimposed upon maintained operant baselines.

Herrnstein's equation readily encompasses the finding that a superimposed response-independent schedule, particularly one delivering a high rate of reinforcement, can produce a decrease in responding maintained by a response-dependent schedule that is held constant. An alternative source of reinforcement would contribute to r_2 in Equation 1, so for a constant r_1 (i.e., a constant rate of reinforcement contingent upon the measured operant), increases in the rate of superimposed reinforcement would produce greater and greater response decrements. This is obviously true of all variants of Herrnstein's equation (e.g., McLean & White, 1983; Wearden, 1981) in which alternative sources of reinforcement contribute only to the denominator of the equation.

Fig. 2. Effects of superimposed response-independent reinforcement predicted by Equation 2. The dotted line shows response rate in the absence of superimposed reinforcement. The function with $p = 0$ is equivalent to the prediction of Herrnstein's equation (Equation 1).

Herrnstein's equation fares less well, however, with several other findings. One of these is the result that low rates of superimposed VT or FT apparently can elevate response rates. Equation 1 also fails to address the difference usually found between DRO and VT (or FT) conditions when these are superimposed upon a simple response-dependent schedule. The equation takes into account only the overall rate of reinforcement delivered, thus predicting that DRO and a response-independent schedule would have identical effects.

It is possible to modify Herrnstein's equation to better accommodate at least some of these findings. We begin by considering the notion of alternative reinforcement. In Equation 1, there are two sources of alternative reinforcement: r_2 , which is provided by the experimenter but superimposed upon the reinforcement schedule for the measured operant, and r_o , which is presumed to arise from various other sources. Herrnstein's equation implicitly assumes that when any particular reinforcer occurs, it unerringly functions as a member of the appropriate class (r_1 , r_2 , or r_o). r_o reinforcers might remain functionally distinct from those produced by measured operant responses, inasmuch as they may be qualitatively different (e.g., food vs. "leisure," or food vs. the reinforcing consequences of some maintenance activity such as grooming), but

food reinforcers delivered according to superimposed schedules might not always be functionally separable from those delivered by the superimposed schedule. For example, in the case of VI being succeeded by VI sup VT, Equation 1 requires that a subject's behavior be controlled independently by r_1 and r_2 . Under VI, of course, all food reinforcers operate as r_1 . Before considering the question of the basis on which functional separation of reinforcers may occur, we shall consider some formal consequences of failure of this separation.

Assume that a certain proportion (p) of the r_2 reinforcers function as r_1 . Assume further that, because of a previous training history on some conventional response-dependent schedule, all scheduled r_1 reinforcers function as r_1 . Then the effective rate of reinforcement for r_1 becomes $r_1 + pr_2$, and the effective value of r_2 becomes $(1 - p)r_2$. Assuming that r_o remains constant, Herrnstein's equation is thus modified to produce:

$$R = \frac{k(r_1 + pr_2)}{r_1 + pr_2 + (1 - p)r_2 + r_o} \quad (2)$$

In Equation 2, all terms remain as in Equation 1 except for the addition of p , a variable expressing the proportion of r_2 reinforcers functioning as r_1 , and thus taking values from 0 to 1. p might thus be interpreted as the degree of "generalization" from r_2 to r_1 , or the degree of "confusion" between r_2 and r_1 (although it should be noted that subjects are assumed never to be "confused" about the source of r_1 reinforcers). When $p = 0$, Equation 2 obviously reduces to Equation 1; when $p = 1$, all food reinforcers function as r_1 , regardless of source.

Figure 2 illustrates several properties of Equation 2. For the purposes of calculation, response rates are expressed as a fraction of the asymptote expressed as k in Herrnstein's equation. The parameter r_o is maintained at a constant value of $0.5r_1$, and r_1 is held constant at 1 to simulate the conditions in which the rate of reinforcement produced by the measured operant remains constant. The parameter r_2 is varied across a range from $0.1r_1$ to $10r_1$ and p is also varied.

Figure 2 shows that Equation 2 behaves in several distinctly different ways depending on p value. When p is large, increasing r_2 consistently increases response rates above the

Fig. 3. Effects of superimposed response-independent reinforcement predicted by Equation 2 when p (filled squares) is varied as rate of superimposed reinforcement varies. The dotted line shows response rate in the absence of superimposed reinforcement.

level obtaining when r_2 is absent. Low values of p , on the other hand, produce consistently declining values of response rate as r_2 increases, and this obviously also applies when $p = 0$, and Equation 2 reduces to Equation 1.

The Appendix shows that variation in r_2 produces consistently increasing or consistently decreasing response rates except when $p = r_1/(r_1 + r_0)$, in which case response rate remains constant as r_2 changes. Thus, the processes embodied in Equation 2 permit increases, decreases, or no change in response rate with r_2 variation, depending on p . They do not, however, permit certain r_2 values to increase response rate, and others to decrease it, if p remains constant.

The question of what factors might influence p is considered in greater detail below, but to further develop the present analysis we propose that one possible influence is rate of alternative reinforcement itself. If this is low relative to the baseline r_1 rate, then superimposition of response-independent reinforcement produces only a slight change in the overall rate of reinforcement. It also may produce very little disturbance to other aspects of the schedule, such as the pattern of contiguities between measured operant responses, other types of behavior, and food delivery. On the other hand, with higher rates of superimposed reinforcers, overall rates of reinforcement may change markedly from the response-dependent phase, and substantial numbers of reinforcers may be contiguous with behavior other

than the measured operant. Such differences between high and low rates of superimposed reinforcement may tend to make reinforcer "misfunction" (i.e., r_2 functioning as r_1) much more likely when superimposed reinforcers are delivered at lower rates. Thus, we might suppose that p and r_2 are negatively correlated with low r_2 rates accompanying high values of p , and high r_2 rates accompanying low values of p . The consequences of such a development are illustrated in Figure 3.

In the calculations for Figure 3, p progressively decreases as r_2 increases. This results in low r_2 values reliably increasing response rates above the level obtained when r_2 is zero, whereas high r_2 rates reliably produce response decrements. This pattern of results is rather similar to that obtained by Zeiler (1979) and by Lattal and Boyer (1980).

Consideration of the factors determining whether food reinforcers function as r_1 or r_2 also enables this modified version of Herrnstein's equation to deal, at least qualitatively, with the effects of DRO and VT superimposition. Reinforcers delivered according to simple response-independent schedules like VT are delivered without regard for responses, and can thus follow measured operant responses in the way that reinforcers scheduled by the prevailing response-dependent schedule can. Reinforcers provided by DRO schedules, on the other hand, can be delivered only when at least a minimum period of nonresponding has occurred (a condition that never arises under a simple response-dependent schedule), and this requirement may make them less liable to misfunction than VT reinforcers. In general, then, in terms of Equation 2, DRO reinforcers may have a lower p value than VT reinforcers, and as a consequence may produce greater response suppression for a given r_2 level. This is the result obtained by Zeiler (1979) and by Lattal and Boyer (1980). On the other hand, when a small DRO requirement is scheduled aperiodically, and the comparison response-independent schedule is also arranged aperiodically, p values for DRO and VT reinforcers may be very similar, with consequently similar effects on prevailing response rate (e.g., Rachlin & Baum, 1972).

DISCUSSION

In a consideration of this modification of Herrnstein's equation, a few general points

should be made. First, the modification employed here is not the only one possible. For example, it might seem more logically consistent to assume that reinforcers misfunction in both directions (i.e., r_1 sometimes functions as r_2 , as well as r_2 functioning as r_1 , as in Equation 2). However, our calculations show that such cross-misfunction with constant p in both directions produces predictions very much at variance with data. It seems necessary to assume that a higher proportion of r_2 reinforcers function as r_1 than the other way around. This assumption is to some extent arbitrary, but it may be justified by prior history of training. If subjects have had long experience with a single response-dependent schedule such as VI, then any delivered food reinforcers may have a tendency to function as r_1 , regardless of whether or not they are response-dependent.

Second, we have not attempted to fit our Equation 2 to any actual results obtained, because no study has provided data in an appropriate form. As shown here, however, Equation 2 is qualitatively consistent with many trends in obtained data, and deals better than Equation 1 with data such as those shown in Figure 1.

Third, the use in our model of a parameter that represents the extent to which reinforcers misfunction in terms of their source is not a radical step, and is consistent with several recent arguments about reinforcement-schedule performance. There appears to be growing interest in the notion that such performance might be explained by imperfect adherence to some simple process such as the matching law (Wearden, 1983). Our Equation 2 similarly describes the effect of imperfect adherence to Herrnstein's original equation (Equation 1), but portrays the "imperfection" as systematic departure.

Fourth, apart from some discussion of the way differences in the events arranged by DRO and response-independent schedules might be manifested in terms of our Equation 2, we have not examined here the general question of how patterns of contiguities between responses and reinforcers (or other local events such as changes in reinforcer periodicities) should be interpreted in terms of Herrnstein's equation. The relation between molar accounts of behavior, which emphasize overall response and reinforcement rates over substantial periods of time, and local events is a difficult and controversial subject which

cannot be fully treated here. (See Baum, 1981, and Thomas, 1983, for two contrasting discussions.) However, it should be noted that a formulation such as Equation 2 may not be able to account for detailed local patterns of performance when response-independent reinforcers are superimposed (e.g., Henton & Iversen, 1978), although it may deal adequately with overall rates of measured operations under such conditions.

What factors might determine the value of the reinforcement misfunction parameter (p in Equation 2) in some particular study? One possibility is that a reinforcer misfunctions in terms of source to the extent that it is not distinguishable from reinforcers delivered by other sources. Operations that might tend to make the food reinforcers procedurally arranged as r_1 and r_2 discriminably different (e.g., arranging that the baseline and superimposed reinforcers differ in some way, by being of different type or magnitude, being delivered in different locations, or being differentially signaled) might thus tend to alter p in predictable ways.

Even in cases in which the baseline and superimposed reinforcers are not procedurally distinct, it is possible that certain features of the operant contingencies may determine the functional nature of reinforcers. For example, reinforcement misfunction may be less common during VI sup VT—under which the reinforcers provided by VI must immediately follow responses whereas the VT reinforcers need not—than when VT is superimposed upon response-dependent schedules that themselves permit variable relations between responses and food delivery (e.g., see Burgess & Wearden, 1981).

Techniques such as that of Killeen (1978) may prove useful for distinguishing and characterizing functions of reinforcement sources. In his procedure, the basis for reinforcement of one of two responses was whether a prior event was response-dependent or response-independent. The present Equation 2 addresses similar discriminations, in which the response dependency to be discriminated relates to reinforcement itself.

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APPENDIX

From Equation 2,

$$R = \frac{k(r_1 + pr_2)}{r_1 + pr_2 + (1 - p)r_2 + r_o}$$

or

$$R = \frac{k(r_1 + pr_2)}{r_1 + r_2 + r_o}.$$

The slope of the function produced by plotting response rate (R) against r_2 (dR/dr_2) is thus, by the quotient rule,

$$\frac{dR}{dr_2} = \frac{k[(r_1 + r_2 + r_o)p - (r_1 + pr_2)]}{(r_1 + r_2 + r_o)^2}.$$

The denominator must always have a positive sign, because it is a square, so the sign for the expression dR/dr_2 is determined by the numerator alone. dR/dr_2 is therefore positive when

$$pr_1 + pr_o - r_1 > 0,$$

or when

$$p > \frac{r_1}{r_1 + r_o}.$$

Similar reasoning shows that dR/dr_2 is negative when

$$p < \frac{r_1}{r_1 + r_o},$$

and is zero when

$$p = \frac{r_1}{r_1 + r_o}.$$

The expression for the numerator of dR/dr_2 does not contain any powers greater than one and can therefore only be monotonically increasing, monotonically decreasing, or zero as r_2 is varied with the other terms constant.