VARIATION IN HERRNSTEIN'S r_o AS A FUNCTION OF ALTERNATIVE REINFORCEMENT RATE

JAMES D. DOUGAN AND FRANCES K. MCSWEENEY

WASHINGTON STATE UNIVERSITY

In a test of Herrnstein's (1970, 1974) equation for simple schedules, 15 pigeons pecked a key that produced food delivered according to variable-interval schedules. One group of birds was water deprived, and food-reinforced key pecking occurred in the presence of free water. Two other groups were not water deprived; water was present for one and absent for the other. As predicted by Herrnstein, the parameter r_o was significantly higher in the water-deprived group than in the two nondeprived groups. Contrary to Herrnstein's interpretation of r_o , the rate of drinking varied across schedules. Herrnstein's interpretation can be salvaged by considering r_o to be an average. However, if r_o is an average, the equation is not a good explanation of behavior because this average is not valid until all schedules have been sampled. In addition, low percentages of variance accounted for suggest that Herrnstein's equation may be of limited usefulness even as a descriptive model for these situations.

Key words: Herrnstein's equation, simple schedules, r_o parameter, water deprivation, key peck, pigeons

Herrnstein (1970, 1974) proposed Equation 1 to describe the relationship between absolute rates of responding and reinforcement delivery on simple and concurrent schedules:

$$P = \frac{kr_1}{\sum_{i=0}^n R_i} \quad (1)$$

P is the rate of instrumental responding on a single alternative, and r_1 is the rate of reinforcement for that instrumental response. The quantity k is a free parameter representing asymptotic rate of response P , and $\sum R$ is the sum of all reinforcers present in the situation. According to Herrnstein, $\sum R$ includes the rate of reinforcement from all scheduled sources ($r_1, r_2, r_3 \dots r_n$) as well as a constant, r_o , interpreted as the rate of reinforcement from unknown (unscheduled) sources.

On simple schedules, the only reinforcers available are those that are programmed (r_1)

and those from other, unknown sources (r_o). Herrnstein's equation for simple schedules, therefore, is easily derived from Equation 1:

$$P = \frac{kr_1}{r_1 + r_o} \quad (2)$$

As in Equation 1, k is a free parameter representing asymptotic response rate. The quantity r_o is a free parameter representing reinforcement from unscheduled sources. Values for k and r_o are estimated from the data. Although, as Herrnstein (1974) states, the properties of k and r_o are subject to empirical study, little research has investigated systematic changes in k and r_o .

McDowell (1982) has recently argued that all of the scheduled sources of reinforcement belong in the denominator of Equation 1 when subjects respond on concurrent schedules. In general, any increase in scheduled reinforcement from alternative sources leads to a decrease in the rate of a given instrumental response (de Villiers, 1977). These alternative sources of reinforcement may relate to distinct alternative responses (e.g., Bradshaw, Szabadi, & Bevan, 1976; Catania, 1963) or they may be "free" reinforcers, delivered independently of responding (Rachlin & Baum, 1972).

The authors wish to thank Jim Whipple, Jay Wright, John Hinson, and Valeri Farmer for their comments on the manuscript. Requests for reprints should be addressed to James D. Dougan, Department of Psychology, Washington State University, Pullman, Washington 99164.

Although the above experiments support the conclusion that scheduled alternative sources of reinforcement will decrease the rate of instrumental responding, they do not provide strong support for Herrnstein's interpretation of r_o . Because r_o is estimated from the data using curve-fitting procedures, it may simply be an error parameter with no relationship to unscheduled reinforcement. Confirmation of Herrnstein's interpretation requires that r_o , as estimated by curve fitting, covary with an independent measure of unscheduled reinforcement. This relationship has not been demonstrated empirically.

Several lines of evidence indicate that Herrnstein's interpretation of r_o may be incorrect. For example, Herrnstein and Loveland (1974) have argued that if r_o does represent unscheduled reinforcement, then increasing the subject's deprivation of the programmed reinforcer should increase the size of r_o relative to programmed sources of reinforcement. A study by McSweeney (1975), however, failed to find systematic variation in r_o as a function of body weight when pigeons responded on concurrent schedules of food delivery. In a more recent study, McSweeney (1982) found that for individual subjects r_o changed considerably over time, despite the fact that conditions that would be expected to alter unprogrammed sources of reinforcement were held constant. Finally, in a recent review of the concurrent-schedule literature, McSweeney, Melville, and Whipple (1983) found r_o to be negative in 28% of cases and extremely large (greater than 50 reinforcers per hour) in 47% of cases. Rates of unscheduled reinforcement in excess of 50 reinforcers per hour seem large considering that subjects were deprived of the scheduled reinforcer and that rates of scheduled reinforcement rarely exceeded 200 per hour. A negative value of r_o indicates an implausible inverse relationship between responding and reinforcement, and there is currently no satisfactory interpretation for negative rates of positive reinforcement. Although Herrnstein's equation may not be meant to deal with negative values of r_o , those negative values nevertheless occur in a substantial number of cases in which the equation should apply.

An additional problem with the r_o parameter is more fundamental. Several studies (Dougan & Eacker, 1982; Hinson & Staddon, 1978; Innis, Simmelhag-Grant, & Staddon, 1983), as well as several theoretical positions (Staddon, 1977; Staddon & Simmelhag, 1971), indicate that the rate of reinforcement for alternative (unscheduled) activities is inversely proportional to the rate of reinforcement for the instrumental response. The mathematics of Herrnstein's equation, however, require that alternative reinforcement be constant across rates of instrumental response. If the rate of responding for alternative sources of reinforcement varies as a function of instrumental reinforcement rate, then either the form of Herrnstein's equation (with r_o as a constant) is fundamentally wrong, or Herrnstein's interpretation of r_o is incorrect.

The present experiment, therefore, asked whether a parameter representing unprogrammed reinforcement should be included in Herrnstein's equation. If r_o , as estimated by Herrnstein's equation, could be shown to covary with an independent measure of unprogrammed reinforcement, then Herrnstein's interpretation of the r_o parameter would be strongly supported. The present experiment also asked whether r_o should be considered a constant; in particular, it asked whether the rate of alternative reinforcement varies as a function of the programmed rate of reinforcement.

Three groups of pigeons were used. One group was water deprived, and in the presence of free water, key-peck responses could produce food. The remaining two groups were not water deprived, and responded for food in either the presence or absence of water. If Herrnstein's interpretation of r_o is correct, birds that consume water at a high rate (the independent index of unprogrammed reinforcement) should have a higher value of r_o than birds that consume little or no water. Rates of water drinking were also measured across food-reinforcement schedules. If the rate of drinking changes as a function of food-reinforcement rate, then r_o should not be viewed as a constant.

A group design was used instead of a single-subject design because McSweeney (1982)

found considerable variation in the size of r_o over time in individual subjects. Therefore, a single-subject design might confound changes in r_o over time with changes in r_o as a function of other variables.

METHOD

Subjects

Subjects were 15 pigeons selected from the colony at Washington State University. Of these, 13 had extensive experimental histories and 2 were experimentally naive.

Apparatus

The apparatus consisted of a standard operant-conditioning unit measuring 29 cm in length, 27 cm in height, and 30 cm in width. Three Plexiglas response keys were evenly spaced in the front wall, 5 cm from each other and 5 cm from the ceiling. The two outer keys were 6 cm from the side walls. Only the center key, which required a force of 0.25 N to operate, was used during the experiment. The key was illuminated from behind by a white 5-W bulb. A grain hopper was centered in the front wall, 7.5 cm from the floor, and a drinking spout that protruded 4 cm into the chamber was located in the lower right corner of the front wall, 2 cm from the right wall and 2.5 cm from the floor. Access to water contained in a glass drinking tube was through a hole, 1 cm in diameter, located near the end of the spout. Illumination of the chamber was provided by a single 5-W houselight located in the upper left corner of the chamber. The entire chamber was housed in a sound attenuating box, which included an exhaust fan that provided masking noise. Electromechanical scheduling equipment was located in an adjacent room.

Procedure

All subjects were maintained at approximately 80% of their free-feeding weights. The 2 naive subjects were given five sessions of autoshaping to establish responding on the center key. During autoshaping trials, 4-s access to grain was allowed once every minute, on the average, and was preceded by an 8-s presentation of the center key light. Both naive birds rapidly acquired the key-pecking response.

Table 1

Sequence of reinforcement schedules, mean number of sessions per schedule, and number of reinforcers per session on each schedule.

<i>Schedule</i>	<i>Number of Sessions</i>	<i>Reinf./Session</i>
VI 30 s	30	40
VI 15 s	28	50
VI 180 s	24	10
VI 90 s	27	15
VI 720 s	24	3
VI 1080 s	19	2

Subjects were divided into 3 groups of 5, with the stipulation that birds with similar reinforcement histories were assigned to different groups. One of the 3 groups (Deprivation/Water) received water only while in the experimental chamber; no water was available in the home cage. A second group (No Deprivation/No Water) had free access to water in the home cage but not in the experimental chamber. The third group (No Deprivation/Water) had free access to water in both the home cage and the experimental chamber. For both groups that had water available in the experimental chamber, the drinking tube was filled with 100 ml of cold tap water immediately prior to the session.

Pecks on the center key produced food according to six different variable-interval (VI) schedules of reinforcement. Table 1 presents the sequence in which the schedules were presented, the number of reinforcers available per session for each schedule, and the average number of sessions conducted under each schedule.

All schedules were constructed according to the method suggested by Catania and Reynolds (1968). At high rates of reinforcement, special precautions were taken to ensure that the electromechanical tape readers did not distort the Catania and Reynolds series. These precautions included the use of high-speed tape drives, restarting the tape as soon as a reinforcer began, and frequent repair and replacement of tapes. Reinforcement consisted of 4-s access to mixed grain. Sessions were terminated on the basis of number of reinforcers; this number was varied among schedules in an attempt to keep session times approximately equal. Schedules were changed when all birds

Table 2
Mean rates of drinking (ml/min) during all schedules by animals in the Deprivation/
Water and No Deprivation/Water groups.

Bird	Programmed Reinforcers per Hour						MEAN
	3	5	20	90	120	240	
Deprivation/Water group							
101	2.00	2.98	0.94	0.95	1.11	1.49	1.57
8	1.85	1.04	0.61	0.50	0.51	1.19	0.95
7	1.12	0.96	0.91	0.97	0.93	1.14	1.00
7642	2.99	2.29	0.81	0.95	0.84	1.26	1.52
5258	1.93	1.31	0.86	0.65	1.72	3.79	1.70
MEAN	1.97	1.71	0.82	0.80	1.02	1.77	1.34
No Deprivation/Water group							
12	0.03	0.07	0.06	0.05	0.15	0.25	0.10
2455	0.05	0.11	0.05	0.08	0.13	0.11	0.09
2560	0.05	0.06	0.10	0.07	0.18	0.14	0.10
11	0.04	0.12	0.08	0.07	0.14	0.12	0.10
103	0.04	0.06	0.05	0.04	0.08	0.17	0.07
MEAN	0.04	0.08	0.07	0.06	0.14	0.15	0.09

in a group had reached a stability criterion of five consecutive sessions in which neither a new high nor a new low in response rate occurred. Sessions were conducted 5 to 6 days per week.

Immediately following each session, the amount of water consumed by subjects in the Deprivation/Water and No Deprivation/Water groups was measured by subtracting the quantity of water remaining in the drinking tube from the 100 ml that was present at the beginning of the session. On the several occasions that a bird consumed all of the water in the tube, the quantity was recorded as 100 ml.

RESULTS

Rates of water consumption for the Deprivation/Water and No Deprivation/Water groups were calculated by dividing the quantity of water consumed in each session by the session time, corrected by subtracting from the total time of the session the number of seconds the food magazine was present. Drinking rates under each of the six VI schedules for each of the animals in these two groups are presented in Table 2. Over the six schedules, the Deprivation/Water group consumed water at a mean rate of 1.34 ml/min, as compared to a rate of 0.09 ml/min in the No Deprivation/Water group. A *t* test showed these differences

to be significant ($t(8) = 7.29$; $p < .001$). In fact, water consumption was so low in the No Deprivation/Water group that it was not significantly higher than the amount of water that could be attributed to spillage during water measurement. In addition, direct observations indicated that the birds in the No Deprivation/Water group did not drink from the spout. Because the No Deprivation/Water group did not drink water and was therefore functionally identical to the No Deprivation/No Water group, data from these two groups have been combined, except where otherwise indicated.

Obtained rates of key pecking and reinforcement were calculated by dividing the number of key pecks per session and the number of magazine operations per session by session time, corrected by subtracting the time during which the magazine was available. These rates of response and reinforcement were used to calculate values of k , r_0 , and the percentage of variance accounted for by Herrnstein's equation (Equation 2). All parameter estimates were made using McDowell's (1981) technique. Response rates, obtained rates of reinforcement, and parameter estimates are presented in Table 3.

As seen in Table 3, the median k value was lowest in the No Deprivation/No Water group (54.08) and highest in the No Deprivation/

Table 3

Pecks per minute (upper row) and obtained reinforcers per hour (lower row) with estimated values of k , r_o , and the percentage of variance explained by Herrnstein's equation.

Bird	Programmed Reinforcers per hour						k	r _o	% Var.
	3	5	20	40	120	240			
Deprivation/Water Group									
101	19.42	31.31	37.88	54.01	62.69	31.89	50.97	4.48	49.74
	3.15	8.86	18.31	27.72	92.40	202.20			
8	27.58	31.20	56.76	45.46	45.40	44.68	61.09	2.65	61.01
	4.18	5.70	18.06	28.46	92.40	208.20			
7	9.74	22.80	29.42	46.47	50.82	52.47	54.32	8.19	91.22
	2.37	3.55	18.34	28.86	97.80	208.32			
7642	31.48	41.79	58.10	61.33	67.30	53.35	64.18	3.18	81.75
	3.07	6.02	18.72	26.10	94.80	206.40			
5258	6.81	19.58	57.67	70.22	50.89	57.47	65.84	7.19	71.62
	3.61	5.23	20.44	26.19	98.85	200.92			
MEDIAN	19.42	31.20	56.56	54.01	50.82	52.47	61.09	4.48	71.62
	3.61	5.70	18.34	27.72	94.80	206.40			
No Deprivation/No Water Group									
13	87.33	112.41	111.36	126.50	122.70	90.37	113.48	0.45	12.60
	3.98	4.03	18.62	27.34	93.00	212.40			
5345	32.56	40.69	46.21	54.47	47.08	55.84	54.08	2.52	81.83
	4.40	6.40	19.14	28.92	93.00	205.80			
2454	46.75	46.09	63.08	48.58	59.51	54.77	57.08	0.86	39.52
	3.10	6.80	20.83	27.34	94.80	209.40			
2457	7.61	17.80	23.55	33.10	32.33	54.69	51.04	17.65	85.94
	3.32	7.20	16.29	26.58	91.80	211.80			
4404	35.58	36.32	50.01	58.62	28.40	46.40	45.81	0.74	12.77
	2.80	4.82	16.92	27.80	99.82	195.00			
MEDIAN	35.58	40.69	50.01	54.47	47.08	54.77	54.08	0.86	39.52
	3.32	6.40	19.14	27.34	93.00	209.40			
No Deprivation/Water Group									
12	60.45	72.72	107.14	101.81	136.82	102.54	119.32	2.47	80.31
	2.66	3.72	16.56	28.00	92.40	199.80			
2455	0.76	29.02	31.98	36.47	37.46	64.93	59.87	17.10	79.87
	3.20	8.38	19.74	25.48	91.20	208.80			
2560	74.96	69.07	66.19	92.39	69.15	79.09	76.98	0.25	4.97
	3.69	4.47	22.14	28.78	93.60	208.80			
11	58.66	86.72	83.05	97.77	95.29	114.18	105.27	2.63	80.83
	3.60	7.80	20.88	28.39	93.00	208.20			
103	60.10	65.89	74.76	67.98	55.90	89.10	72.93	0.53	17.96
	2.46	5.20	17.77	27.10	101.41	203.48			
MEDIAN	60.10	69.07	74.76	93.39	69.15	89.10	76.98	2.47	79.87
	3.20	5.20	19.74	28.00	93.00	208.20			
Combined No Deprivation Groups									
MEDIAN	52.70	55.99	64.83	63.30	57.70	72.01	61.35	1.16	59.64
	3.20	5.80	18.88	27.92	93.00	208.50			

Water group (76.98). The Deprivation/Water group had a median k value of 61.09. The k values for the Deprivation/Water group were not significantly different than those for the combined No Deprivation groups (Mann-Whitney $U = 19$; $p > .05$).

The median value for r_o in the Deprivation/

Water group was 4.48, whereas values of 2.47 and 0.86 were found in the No Deprivation/Water and No Deprivation/No Water groups, respectively. A one-tailed Mann-Whitney U test showed that the Deprivation/Water group had a significantly higher r_o value than the combined No Deprivation groups ($U = 10$; $p < .05$).

Equation 2 accounted for a median of 71.62% of the variance in the Deprivation/Water group, 39.52% in the No Deprivation/No Water group, and 79.87% in the No Deprivation/Water group. These percentages of variance were not significantly different for the Deprivation/Water group and the combined No Deprivation groups ($U = 18$; $p > .05$).

The upper panel of Figure 1 plots the mean rate of water intake for the Deprivation/Water group as a function of scheduled rate of food reinforcement. The bottom panel plots the rate of pecking for the Deprivation/Water and for the combined No Deprivation groups over the same rates of reinforcement. Examination of Figure 1 indicates a clear inverse relationship between the rate of pecking and the rate of drinking in the Deprivation/Water group. A Pearson product-moment correlation test showed this inverse relationship to be significant ($r = -.86$; $p < .05$).

Figure 1 also provides a visual confirmation of the finding that r_0 was higher in the Deprivation/Water group than in the combined No Deprivation groups. Rates of responding plotted in Figure 1 increase to asymptote much more slowly in the Deprivation/Water group than in the combined No Deprivation groups. The slower increase in response rate indicates a higher value of r_0 . In addition, Figure 1 suggests that the combined No Deprivation groups reached a higher asymptotic rate of response than the Deprivation/Water group. However, this difference was not statistically significant.

DISCUSSION

In the present study, the r_0 parameter of Herrnstein's equation (Equation 2) was found to covary with quantity of water consumed, an independent measure of unscheduled reinforcement. The r_0 parameter was significantly higher in the Deprivation/Water group, which consumed a relatively large quantity of unprogrammed reinforcers, than in the combined No Deprivation groups, which consumed relatively few, if any, unscheduled reinforcers. This result is consistent with Herrnstein's (1970, 1974) interpretation of the r_0 parameter as reinforcement from unscheduled sources. In

Fig. 1. Upper Panel: Mean rate of water intake for the Deprivation/Water (D/W) group plotted over scheduled rate of reinforcement. Lower Panel: Mean rate of key pecking for the Deprivation/Water group (D/W) and for the combined No Deprivation groups (ND) plotted over scheduled rate of reinforcement.

addition, the values of the k parameter were not significantly different across groups. This supports Herrnstein's contention that k is a constant that does not change across experimental conditions (Herrnstein, 1974; but see McDowell & Wood, 1984).

Although the present data support Herrnstein's interpretation of r_0 in a general way, they question the idea that reinforcement from alternative sources is constant across schedules. Assuming that changes in the rate of drinking are proportional to changes in the rate of other (unmeasured) alternative activities, the significant inverse relationship between rate of drinking and rate of pecking suggests that reinforcement from other sources is not a constant. If this is true, the mathematical form of Herrnstein's equation is fundamentally wrong. Note that a failure to assume that changes in drinking rate are proportional to

changes in other unmeasured activities makes Herrnstein's interpretation of r_o virtually untestable.

One way to salvage the equation would be to consider r_o as an average rate of alternative reinforcement across schedules. This would explain why the r_o value was higher in the Deprivation/Water group in the present study, despite the changes in rate of water consumption across schedules. This idea also has some mathematical merit. When Herrnstein's equation is plotted across a narrow range of reinforcement rates at the low end of the scale, the least-squares estimate of r_o is quite high. This is the exact range of reinforcement rates at which the rate of drinking is highest. Likewise, if Herrnstein's equation is plotted along a narrow range of reinforcement rates at the high end of the scale, r_o is very small. The rate of drinking over these same reinforcement values is also relatively low. The r_o parameter, therefore, may represent the average rate of alternative reinforcement across sampled schedules.

If r_o is considered to be an average, however, it constitutes a serious problem for Herrnstein's equation. If the equation is to be considered a theory of behavior, the assumptions of the model must have both theoretical and empirical validity. Otherwise, the equation provides a mathematical description of behavior but does not explain behavior—its assumptions and parameters do not have realistic empirical reference (Timberlake, 1982). This would be the case if r_o were considered to be an average. Animals obviously cannot react to an average rate of unprogrammed reinforcement in performing on individual schedules, because the validity of averaging requires that all schedules have been sampled. Thus, Herrnstein's equation may provide a mathematical description of behavior but not provide an explanation of behavior.

Even if r_o is an average, the present data suggest that Herrnstein's equation may be an inadequate descriptive model. The percentage of variance accounted for by Equation 2 ranged from 4.1% to 91.2%, with a mean of 56.7%. These values are low compared to most published accounts, which report values between

80% and 99% (de Villiers, 1977; de Villiers & Herrnstein, 1976; McDowell, 1982). However, many of these published accounts report the percentage by using data averaged across subjects. Recently, Warren-Boulton, Silberberg, Gray, and Ollom (1985) have shown that this averaging procedure substantially increases the percentage of variance accounted for by Herrnstein's equation. For example, a reanalysis of de Villiers and Herrnstein (1976) indicated that only 78% of the variance was accounted for when these averaging procedures were eliminated, as opposed to the 94% originally reported by those authors. In addition, McSweeney et al. (1983) found very low percentages of variance accounted for in a number of situations when single-subject data were analyzed. The present values, therefore, may be well within the normal range.

It is possible that the low percentage of variance accounted for in the present experiment was due to a failure of some animals to respond to changes in reinforcement rate. Herrnstein's equation did very poorly for 5 animals in particular (13, 4404, 2560, 2454, and 103). Interestingly, these animals also had extremely small r_o values (below one), indicating little change in response rate as a function of reinforcement (see Table 3). In a sense, Herrnstein's equation cannot closely fit these data because most points lie near asymptote, whereas the equation is hyperbolic. For the remaining animals, the equation accounts for 76.39% of the variance, which presents a somewhat better picture. The proper criterion for evaluating percentage of variance accounted for, of course, is unknown (Anderson, 1978). Intuitively, a good two-parameter equation should handle almost all variance, which Herrnstein's equation does not do, at least for the present data (see also McSweeney et al., 1983; Warren-Boulton et al., 1985).

Another aspect of the present data that may have reduced the fit of Equation 2 is the decrease in response rates seen at high reinforcement rates in some animals. Ten of the 15 animals had maximum response rates at points other than that of maximum reinforcement rate. A hyperbolic equation cannot account for such decreases in responding at high reinforcement

ment rates. This decrease in response rate at high reinforcement rates has been predicted by other models (Baum, 1981; Staddon, 1979). Although the present data do not allow evaluation of those alternative models, they do suggest that models predicting a decrease in response rate at high reinforcement rate deserve attention.

Finally, the fit of Equation 2 may have been reduced by the differences in the number of reinforcers presented across schedules. At high rates of reinforcement (VI 15 s), 50 reinforcers were presented per session. At low rates of reinforcement (VI 1080 s), only two reinforcers were presented per session. This wide variation in the number of reinforcers presented may have led to differences in satiation across schedules, and thus increased variability in the data. However casual observation of subjects on all schedules gave no indications of satiation. In addition, the procedure of varying the number of reinforcers presented across schedules is a common solution to the problem of controlling both session time and the number of reinforcers presented, in studies which vary rate of reinforcement. Therefore, it is doubtful that the procedure used in the present study would produce results substantially different from those of other published studies.

The present data do contradict earlier studies that used concurrently available food and water reinforcers. Wood, Martinez, and Willis (1975) failed to find any interaction between schedules on a concurrent fixed-interval fixed-ratio schedule (concurrent FI FR) when food was presented by the FI schedule and water was presented by the FR schedule. The significant change in r_o as a function of water deprivation found in the present study, however, shows that food and water reinforcement may interact under some conditions. Other studies (e.g., Hursh, 1978) have suggested that food and water are not substitutable (Rachlin & Burkhard, 1978; Rachlin, Kagel, & Battalio, 1980). However, Herrnstein's equation requires that all sources of reinforcement be completely substitutable. The changes in r_o found in the present study indicate that food and water are at least somewhat substitutable.

In conclusion, the r_o parameter of Herrnstein's equation covaried with an independent measure of alternative reinforcement. Functional changes in water consumed across conditions, however, suggest that alternative reinforcement is not a constant as required by the equation. The parameter r_o may be the average rate of alternative reinforcement across conditions. If it is, Herrnstein's equation can provide a mathematical description of behavior but cannot provide an explanation of behavior. Finally, the low percentage of variance accounted for suggests that the equation is very limited even as a purely descriptive device.

REFERENCES

- Anderson, N. H. (1978). Measurement of motivation and incentive. *Behavior Research Methods & Instrumentation*, 10, 360-375.
- Baum, W. M. (1981). Optimization and the matching law as accounts of instrumental behavior. *Journal of the Experimental Analysis of Behavior*, 36, 387-403.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. (1976). Behavior of humans in variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 26, 135-141.
- Catania, A. C. (1963). Concurrent performances: Reinforcement interaction and response independence. *Journal of the Experimental Analysis of Behavior*, 6, 253-263.
- Catania, A. C., & Reynolds, G. S. (1968). A quantitative analysis of the responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 11, 327-383.
- de Villiers, P. A. (1977). Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.) *Handbook of operant behavior* (pp. 233-287). Englewood Cliffs, NJ: Prentice-Hall.
- de Villiers, P. A., & Herrnstein, R. J. (1976). Toward a law of response strength. *Psychological Bulletin*, 83, 1131-1153.
- Dougan, J. D., & Eacker, J. N. (1982). Interference between licking and bar-pressing responses during a multiple schedule of food reinforcement. *Behaviour Analysis Letters*, 2, 79-86.
- Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior*, 13, 243-266.
- Herrnstein, R. J. (1974). Formal properties of the matching law. *Journal of the Experimental Analysis of Behavior*, 21, 159-164.
- Herrnstein, R. J., & Loveland, D. H. (1974). Hunger and contrast in a multiple schedule. *Journal of the Experimental Analysis of Behavior*, 21, 511-517.
- Hinson, J. M., & Staddon, J. E. R. (1978). Behavioral competition: A mechanism for schedule interactions. *Science*, 202, 432-434.

- Hursh, S. R. (1978). The economics of daily consumption controlling food- and water-reinforced responding. *Journal of the Experimental Analysis of Behavior*, **29**, 475-491.
- Innis, N. K., Simmelhag-Grant, V. L., & Staddon, J. E. R. (1983). Behavior induced by periodic food delivery: The effects of interfood interval. *Journal of the Experimental Analysis of Behavior*, **39**, 309-322.
- McDowell, J. J. (1981). Wilkenson's method of estimating the parameters of Herrnstein's hyperbola. *Journal of the Experimental Analysis of Behavior*, **35**, 413-414.
- McDowell, J. J. (1982). The importance of Herrnstein's mathematical statement of the law of effect for behavior therapy. *American Psychologist*, **37**, 771-779.
- McDowell, J. J., & Wood, H. M. (1984). Confirmation of linear system theory prediction: Changes in Herrnstein's k as a function of changes in reinforcer magnitude. *Journal of the Experimental Analysis of Behavior*, **41**, 183-192.
- McSweeney, F. K. (1975). Concurrent schedule responding as a function of body weight. *Animal Learning & Behavior*, **3**, 264-270.
- McSweeney, F. K. (1982). Prediction of concurrent treadle-pressing from simple schedule performance. *Behaviour Analysis Letters*, **2**, 11-20.
- McSweeney, F. K., Melville, C. L., & Whipple, J. E. (1983). Herrnstein's equation for the rates of responding during concurrent schedules. *Animal Learning & Behavior*, **11**, 275-289.
- Rachlin, H., & Baum, W. M. (1972). Effects of alternative reinforcement: Does the source matter? *Journal of the Experimental Analysis of Behavior*, **18**, 231-241.
- Rachlin, H., & Burkhard, B. (1978). The temporal triangle: Response substitution in instrumental conditioning. *Psychological Review*, **85**, 22-47.
- Rachlin, H., Kagel, J. H., & Battalio, R. C. (1980). Substitutability in time allocation. *Psychological Review*, **87**, 355-374.
- Staddon, J. E. R. (1977). Schedule-induced behavior. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 125-152). Englewood Cliffs, NJ: Prentice-Hall.
- Staddon, J. E. R. (1979). Operant behavior as adaptation to constraint. *Journal of Experimental Psychology: General*, **108**, 48-67.
- Staddon, J. E. R., & Simmelhag, V. L. (1971). The "superstition" experiment: A reexamination of its implications for the principles of adaptive behavior. *Psychological Review*, **78**, 3-43.
- Timberlake, W. (1982). The emperor's clothes: Assumptions of the matching theory. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2: Matching and maximizing accounts* (pp. 549-568). Cambridge, MA: Ballinger.
- Warren-Boulton, F. R., Silberberg, A., Gray, M., & Ollom, R. (1985). Reanalysis of the equation for simple action. *Journal of the Experimental Analysis of Behavior*, **43**, 265-277.
- Wood, K. A., Martinez, E. S., & Willis, R. D. (1975). Ratio requirement and reinforcer effects in concurrent fixed-interval fixed-ratio schedules. *Journal of the Experimental Analysis of Behavior*, **23**, 87-94.

Received December 6, 1983

Final acceptance January 1, 1985