MATCHING AND MAXIMIZING WITH VARIABLE-TIME SCHEDULES

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Pigeons were offered choices between a variable-time schedule that arranged reinforcers throughout the session and a variable-time schedule that arranged reinforcers only when the pigeon was spending time on it. The subjects could maximize the overall rate of reinforcement in this situation by biasing their time allocation towards the latter schedule. This arrangement provides an alternative to concurrent variable-interval variable-ratio schedules for testing whether animals maximize overall rates or match relative rates, and has the advantage of being free of the asymmetrical response requirements present with those schedules. The results were contrary to those predicted by maximizing: The bias it predicts did not appear.

Key words: matching, maximizing, concurrent schedules, variable-time schedules, variable-interval schedules, variable-ratio schedules, key peck, pigeons

Concurrent variable-interval variable-interval (VI VI) schedules offer choices between two VI schedules that program reinforcers simultaneously and independently. The typical result is an approximate match between both the ratio of responses made and times spent on each schedule and the ratio of reinforcers obtained on those schedules (for a review, see de Villiers, 1977). This simple relation has been generalized as follows (Baum, 1974):

$$B_1/B_2 = b(R_1/R_2)^s, \quad (1)$$

where B_1 and B_2 represent the responses made or times spent on schedules 1 and 2, R_1 and R_2 are the numbers of reinforcers obtained on those schedules, and b and s are empirically derived constants. Equation 1 is called the generalized matching law; strict matching requires that parameters be equal to unity. Deviations of the parameters from unity are called bias ($b \neq 1$), undermatching ($s < 1$), and overmatching ($s > 1$). Taking the logarithm of both sides of this expression produces the equation of a straight line, and this is the form typically used to display and fit data.

Although matching as a phenomenon is well established, there is debate as to whether or not it is the fundamental process of choice. An alternative approach considers matching to be the by-product of a maximizing process. One type of maximizing account, called molar maximizing, views organisms as distributing their behavior across alternatives in ways that maximize the overall rate of reinforcement (Rachlin, Green, Kagel, & Battalio, 1976; Staddon & Motheral, 1978). Molecular-maximizing accounts, on the other hand, appeal to the momentary probability of reinforcement (Shimp, 1969) or the local rate of reinforcement (Herrnstein & Vaughan, 1980) as the controlling variable. The present study addresses the debate between matching and molar maximizing.

Concurrent VI VI schedules do not provide an adequate test of matching versus molar maximizing because both accounts predict nearly identical distributions of behavior. However, the predictions of these accounts diverge on concurrent variable-interval variable-ratio (VI VR) schedules. In these situations, the overall rate of reinforcement will be maximized only if the subject allocates much more time to the VR schedule than is predicted by matching. Nevertheless, Herrnstein and Heyman (1979) found that pigeons' response and time

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distributions on concurrent VI VR schedules were close to matching, despite the fact that this considerably reduced the overall rate of reinforcement. This result provides evidence against molar maximizing, but a firm conclusion cannot be made because of the complicating effects of the different response requirements across the schedules. Several authors (Herrnstein & Heyman, 1979; Rachlin, Kagel, & Battalio, 1980) have noted that bias towards the ratio schedule may not be found because of the greater response effort required by this schedule. Rachlin, Battalio, Kagel, and Green (1981) and Green, Kagel, and Battalio (1982) have shown that the results of Herrnstein and Heyman are compatible with maximizing if this effort is taken into account—that is, if the subject is viewed as maximizing both reinforcement and leisure. Concurrent VI VR schedules, therefore, may not be well suited for comparing matching with molar maximizing.

The present design eliminates the complicating effects of asymmetrical response requirements that are intrinsic to concurrent VI VR schedules, yet still allows a comparison between matching and molar maximizing. Variable-time (VT) schedules were used, which do not require any response for a reinforcer to be delivered, other than spending time on the schedule. Each schedule was correlated with a red or green keylight, and the pigeons could alternate between the colors by pecking a second key (Findley, 1958). One schedule, which will be referred to as the “independent” VT, programmed reinforcers throughout the session, but delivered them only when the keylight was green. The other schedule, which will be referred to as the “dependent” VT, simultaneously scheduled and delivered reinforcers only when the keylight was red. The maximizing pattern in this situation would be to spend time on the dependent VT, with switches being made to the independent VT only to collect reinforcers that have been scheduled for delivery. Thus, maximizing predicts a sizable bias in time allocation towards the dependent VT, whereas matching predicts approximate

equality between behavior ratios and reinforcer ratios.

METHOD

Subjects

Four male Silver King pigeons with no previous experimental history served as subjects. They were maintained at approximately 80% of their free-feeding weights.

Apparatus

Two three-key Behavior Systems International pigeon chambers (Model #417), 32.2 cm high, 34.8 cm long, and 32.5 cm wide, were used. A sound-attenuating box served as the top, right, and rear wall of the chamber. The door to the chamber, which opened on the left side, was transparent plastic, and the front slide-in wall was aluminum. Only the center and right keys were used; the left key was dark and inoperative. The right key (Gerbrands) was 2 cm in diameter, 23.5 cm from the floor, and 8.2 cm to the right of the 2.5-cm-diameter plastic center key, which was inoperative. The operative key required a minimum force of about 0.15 N to operate and a feedback click was provided. The opening of the grain hopper was 8.5 cm above the wire-mesh floor of the chamber and was located directly beneath the center key. The houselight was a 28-V dc lamp located at the top and center of the rear wall. White noise and a ventilation fan masked extraneous sounds. Solid-state and electro-mechanical equipment controlled the experiment from an adjoining room.

Procedure

The center key, designated the main key, could be transilluminated with red or green light; the right key, designated the changeover key, with white light. A single peck on the changeover key alternated the color of the main key and initiated a 2-s changeover delay (COD), which prevented reinforcers from being delivered for 2 s after the last changeover. The dependent VT simultaneously scheduled and delivered reinforcers

only when the keylight was red. The independent VT, on the other hand, scheduled reinforcers throughout the session, but delivered them only when the keylight was green. A reinforcer scheduled while the pigeon was at the red keylight was held, and the timer was stopped until the pigeon returned to the green keylight. The intervals of both schedules were approximately exponentially distributed (Fleshler & Hoffman, 1962).

These final conditions were preceded by a 10-day training period. The subjects received 1 day of shaping to peck the changeover key, 2 days of concurrent dependent VT independent VT with VT values of 15 s, and 7 days with VT values of 45 s. The COD was introduced as 1.5 s midway through this 7-day period, and was increased to its final value of 2 s on the 10th day.

Reinforcement consisted of 3-s access to mixed grain, during which only the hopper light was illuminated and all recording (except for the independent VT programmer) was stopped. Sessions were conducted 7 days a week, and ended when 60 reinforcers had been delivered. Conditions were maintained for at least 30 days, and were changed when no trend was observed in the data over 10 consecutive days. Table 1 presents the sequence of schedules used, their nominal mean values, and the number of sessions in each condition.

RESULTS

The time spent and reinforcers received on each schedule, and the number of changeovers during a session, are shown in Table 1. Each data entry represents the mean of the results obtained during the last 10 days in each condition.

Figure 1 presents, in separate logarithmic coordinates (base 10) for each subject, the ratio of time on the dependent VT to time on the independent VT as a function of the ratio of reinforcers obtained on the dependent VT to reinforcers obtained on the independent VT. The method of least squares was used to fit a straight line to the logarithms of the

Table 1

The sequence of schedules, time spent and reinforcers obtained on each schedule, number of changeovers per session, and number of sessions in each condition

Schedule values (sec)		Time (sec)		Reinforcers		Change-overs	Sessions
Subject 980							
45	45	845	1054	18	42	504	67
15	45	623	267	41	19	124	47
60	45	745	1393	12	48	386	69
45	90	1508	821	34	26	337	41
45	60	1104	1054	24	36	361	49
45	270	2268	400	51	9	228	42
Subject 6734							
45	45	164	2382	4	56	64	67
15	45	575	409	38	22	166	47
60	45	366	2002	6	54	114	69
45	90	1399	1130	32	28	268	41
45	60	858	1553	20	40	234	49
45	270	270	2129	48	12	132	42
Subject 6773							
45	45	38	2592	1	59	12	67
15	45	686	207	46	14	64	47
60	45	327	2059	6	54	119	69
45	90	1595	695	36	24	193	41
45	60	1138	989	24	36	352	49
45	270	2210	487	50	10	213	42
Subject 136							
45	45	120	2402	3	57	42	67
15	45	851	60	58	2	4	47
60	45	202	2266	3	57	35	69
45	90	1543	917	35	25	147	41
45	60	1125	1066	25	35	218	49
45	270	2319	255	52	8	85	42

Note. Order of presentation is dependent VT, independent VT.

ratios of the data, and the resulting equations (in arithmetic form) and coefficients of determination (r^2) are shown.

Bias (b) towards the dependent VT was obtained for three of the subjects; the other subject showed bias towards the independent VT. The degree of bias obtained is well within the range typically found with concurrent VI VI (see Baum, 1974).

Baum (1979) has suggested a range of sensitivity (s) of 0.90 to 1.11 as approximating matching. The sensitivities obtained for two of the subjects are within this range; the sensitivities of the other two subjects show undermatching, but within the range typically found with concurrent VI VI (see de Villiers, 1977). Undermatching is the most commonly observed deviation from matching with concurrent VI VI (Baum, 1974; Myers & Myers, 1977).

Fig. 1. Ratio of time on the dependent VT to time on the independent VT as a function of the ratio of reinforcers obtained on the dependent VT to reinforcers obtained on the independent VT for each individual subject. The best-fitting lines, their equations, and coefficients of determination are also shown. Note the shifted ordinate and abscissa scales for Bird 136, relative to those for the other three birds.

Figure 2 presents the pooled data. The parameters of the best-fitting line show that the central tendency of the data is towards matching, but with a slight bias towards the dependent VT.

Figure 3 presents for each subject the changeover rates as a function of preference. In every case, the rate of changeover between the schedules was highest around indifference, and decreased as preference for one of the schedules increased. This produces an inverted U-shaped pattern in the distribution of points. This type of pattern in

changeover rates is also typically found with concurrent VI VI schedules (Baum, 1973, 1975, 1976; Brownstein & Pliskoff, 1968; Herrnstein, 1961; Shull & Pliskoff, 1967).

DISCUSSION

The generalized matching law adequately described the distribution of time between the schedules for both the individual subjects and for their pooled data. The coefficients of determination were above .96 in every case. The deviations from strict matching that occurred (as bias and undermatching) were

Fig. 2. Pooled data for four birds. The solid line is the best-fitting line; its equation and coefficient of determination are also shown. The broken line shows the locus of points that would correspond to maximizing; the thin line shows a comparable locus relative to matching.

within the range frequently found with concurrent VI VI schedules (Baum, 1974, 1979); also, the pattern of changeover rates was identical to that typically observed.

The maximizing strategy can be derived using the approach of Herrnstein & Heyman (1979). The following feedback function for the expected rate of reinforcement from both schedules was used (see Heyman & Luce, 1979):

$$R = P/VT_1 + (1 - P)/(VT_1 + I/P) + (1 - P)/VT_2, \quad (2)$$

where P is the proportion of time spent on the independent VT, VT_1 and VT_2 are the mean interreinforcement intervals for the independent VT and dependent VT, respectively, and I is one-half the harmonic mean of the interchangeover times. For each subject, I was determined for each pair of schedules from the equation

$$I = [2P(1 - P)T]/CO,$$

where T is the session time and CO is the number of changeovers per session. The accuracy of Equation 2 was then assessed by substituting the scheduled values of VT_1 and VT_2 and the calculated values of I , and comparing the predicted reinforcement rates to

Fig. 3. Rate of changeover as a function of the ratio of times spent in each component for each individual subject.

the obtained reinforcement rates. The rates predicted by this equation account for .998 of the variance in the obtained reinforcement rates.

The distribution of time that would have maximized the reinforcement rates for the obtained values of I and the programmed values of VT_1 and VT_2 was then found by varying P in Equation 2 and extracting the values that gave the highest reinforcement rates for each condition. The parameters of the best-fitting function for these time allocations and reinforcement rates for the pooled data are $s = 0.795$ and $b = 3.648$. Thus, maximization of overall reinforcement rate predicts undermatching and considerable bias towards the dependent VT. This equation is plotted as the broken line in Figure 2. It is apparent that this line does not describe the distribution of points for the pooled data or for any individual pigeon. In fact, the pigeons typically lost about 20 reinforcers an hour by not spending more time on the dependent VT. This loss was not merely hypothetical, but was actually experienced by the subjects. For example, during some conditions (e.g., dependent VT 15 s independent VT 45 s), the pigeons decreased the overall rate of reinforcement by up to 20 reinforcers an hour across several sessions, and sometimes from one session to the next, by decreasing the time they spent on the dependent VT. During other conditions, large shifts in time allocation had little effect on the overall rate of reinforcement, but behavior still shifted towards matching. Taking both these points into consideration makes it difficult, if not impossible, to point to the overall rate of reinforcement as the controlling variable. This study, therefore, joins others (Herrnstein & Vaughan, 1980; Mazur, 1981; Vaughan, 1981) that have shown that pigeons fail to maximize the overall rate of reinforcement.

Green, Rachlin, and Hanson (1983) have presented data that may be interpreted as providing support for molar maximizing. Those authors used a procedure analogous to the present design, except the pigeons in the experiment of Green and his colleagues

were offered a choice between two VR schedules (in lieu of VT schedules) correlated with different keys. One VR advanced only when the subject responded on one particular key; the other VR advanced when the subject responded on either key. Bias towards the first schedule appeared, as predicted by maximizing, but it was not as large as expected. This bias could reflect control by the overall reinforcement rates, but the results of the experiments mentioned above suggest otherwise. Another possibility exists: The response allocation that maximizes the overall reinforcement rate in the study of Green et al. also minimizes the response requirement. The bias obtained could reflect sensitivity to this second factor; that is, the pigeons may have allocated their responses so as to reduce their total response output.

Ziriaux and Silberberg (1984) have shown that the matching obtained in experiments using concurrent VI VR schedules can provide only weak evidence for a matching process, because matching is substantially dictated by procedure on these schedules. This is also true to some extent of the present design: It does not provide direct evidence for a matching process. Indeed, the results are also consistent with melioration (Herrnstein, 1982; Herrnstein & Vaughan, 1980; Vaughan, 1981), where behavior shifts towards alternatives providing higher local rates of reinforcement. An analysis of choice procedures presented by Staddon, Hinson, and Kram (1981) suggests that the results are also in agreement with a momentary-maximizing account. Additional experiments are necessary in order to determine the process that leads to matching. In any case, the results of the present experiment show that it is not likely to be molar maximizing.

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