UNDERMATCHING AND OVERMATCHING AS DEVIATIONS FROM THE MATCHING LAW

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A model of performance under concurrent variable-interval reinforcement schedules that takes as its starting point the hypothetical "burst" structure of operant responding is presented. Undermatching and overmatching are derived from two separate, and opposing, tendencies. The first is a tendency to allocate a certain proportion of response bursts randomly to a response alternative without regard for the rate of reinforcement it provides, others being allocated according to the simple matching law. This produces undermatching. The second is a tendency to prolong response bursts that have a high probability of initiation relative to those for which initiation probability is lower. This process produces overmatching. A model embodying both tendencies predicts (1) that undermatching will be more common than overmatching, (2) that overmatching, when it occurs, will tend to be of limited extent. Both predictions are consistent with available data. The model thus accounts for undermatching and overmatching deviations from the matching law in terms of additional processes added on to behavior allocation obeying the simple matching relation. Such a model thus enables processes that have been hypothesized to underlie matching, such as some type of reinforcement rate or probability optimization, to remain as explanatory mechanisms even though the simple matching law may not generally be obeyed.

Key words: concurrent schedules, undermatching, overmatching, random-response allocation, burst length

Research in the early 1960's by Herrnstein (1961) and Catania (1963) suggested that animal performance under concurrent variable-interval variable-interval (VI VI) reinforcement schedules might be described by the matching law:

$$\frac{R_a}{R_a + R_b} = \frac{r_a}{r_a + r_b} \quad (1)$$

or, equivalently,

$$\frac{R_a}{R_b} = \frac{r_a}{r_b}, \quad (2)$$

where R_a and R_b are the numbers of responses made to the concurrent VI VI schedules a and b , and r_a and r_b are the rates of reinforcement

they deliver. Baum and Rachlin (1969) later found an analogous relationship between the lengths of time spent on each of two concurrent VI schedules (T_a and T_b) and reinforcement rates:

$$\frac{T_a}{T_b} = \frac{r_a}{r_b}. \quad (3)$$

Reviews of more recent data from experiments employing concurrent VI VI schedules suggest, however, that Equations 1, 2, and 3 should be replaced by

$$\log \left(\frac{R_a}{R_b} \right) = x \log \left(\frac{r_a}{r_b} \right) + \log a \quad (4)$$

or by an analogous relationship for the lengths of time spent on the schedules (Baum, 1979; Myers & Myers, 1977; Wearden & Burgess, 1982). Equation 4 yields a straight line (of slope x and intercept $\log a$) when the log of the choice measure taken (R_a/R_b or T_a/T_b) is plotted against the log of the ratio of reinforcement rates (r_a/r_b). Equation 4 is said to describe undermatching (Baum, 1974) if $x < 1.0$ and overmatching if $x > 1.0$. If behavior undermatches, it is less sensitive to reinforcement-ratio values (r_a/r_b) than the matching

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law (Equation 1) predicts; if behavior overmatches, it is more sensitive. The intercept of Equation 4 (log a) is most frequently characterized as a bias toward responding on one response alternative rather than another, regardless of the reinforcement rate it delivers (Baum, 1974).

Reviews of studies employing concurrent VI VI schedules by Baum (1979), Myers and Myers (1977), and Wearden and Burgess (1982) all found that undermatching was predominant (i.e., $x < 1.0$) when responses were measured. There is some disagreement about whether time allocation measures undermatch to the same degree (Baum, 1979; Mullins, Agunwamba, & Donohoe, 1982), but review of the most recent data (Wearden & Burgess, 1982) suggests that they generally do. The existence of common deviations from the matching law such as bias and undermatching are important not only because they complicate descriptions of concurrent performance but also because simple matching is often said to be compatible with a number of different behavioral processes. Among these are the maximization of overall reinforcement rate (i.e., obtaining the largest number of reinforcers each session, see Houston and McNamara, 1981, and Prelec, 1982), or a decision rule such as always picking the response alternative that yields the likelier payoff for the next response (i.e., momentary maximizing, see Shimp, 1969, and Staddon, Hinson, & Kram, 1981). Bias and undermatching call into question, by implication, the utility of such notions as explanations of behavior. However, it is possible to preserve these notions if deviations from the matching law can be interpreted theoretically in terms of additional processes added to a behavior-allocation principle based on simple matching, rather than requiring interpretation in terms of principles incompatible with matching.

The present article shows that undermatching and overmatching deviations from the simple matching law can be encompassed by the addition of two processes to behavior allocation based on matching. The first of these is a small amount of random behavior allocation; the second, a rule for governing the lengths of response bursts (Nevin & Baum, 1980) to the concurrent alternatives. A model embodying both these processes is thus able to account for undermatching and overmatching

while preserving, as an underlying process, behavior allocation compatible with the simple matching law.

Undermatching and Random Behavior Allocation

Consider a subject allocating bursts of responses (each of length t and within which responding proceeds at k responses per minute) to each of two concurrent VI alternatives, a and b , according to the simple matching law. Then

$$\frac{P_a}{P_b} = \frac{r_a}{r_b}, \quad (5)$$

where P_a and P_b are the probabilities of burst allocation to a and b . Then

$$\frac{T_a}{T_b} = \frac{tr_a}{tr_b} = \frac{r_a}{r_b} \quad (6)$$

and

$$\frac{R_a}{R_b} = \frac{ktr_a}{ktr_b} = \frac{r_a}{r_b}, \quad (7)$$

where T_a and T_b are the lengths of time, and R_a and R_b the numbers of responses, allocated to a and b , with r_a and r_b the rates of delivered reinforcement. Suppose, however, the behavior allocation does not always perfectly obey the matching law. Let p be the probability that a response burst will be allocated according to simple matching (Equation 5) and q the probability that the burst is allocated to a or b at random, without regard to reinforcement rate. Then

$$\frac{P'_a}{P'_b} = \frac{p P_a + q/2}{p P_b + q/2}, \quad (8)$$

where P'_a and P'_b are the probabilities of burst allocation to a and b given some probability of random burst allocation (q), and P_a and P_b are calculated according to simple matching (Equation 5). If we assume that $p = 1 - q$, then Equation 8 can be written as an expression in q alone. Equations for choice in terms of responses (R_a/R_b) or time (T_a/T_b) can be found by multiplying both the numerator and denominator of Equation 8 by kt or t , respectively. Both expressions obviously reduce to Equation 8, which therefore describes the effect on choice performance of a certain degree of random behavior allocation, q .

To obtain predictions of choice performance using Equation 8, r_a/r_b was varied over

values of 20, 10, 5, 2, 1, .5, .2, .1, and .05, and $\log (P'_a/P'_b)$ was plotted against $\log (r_a/r_b)$. In all cases the plot resulted in a function very close to a straight line, with all r^2 values for goodness of linear fit being greater than .98. The effect of varying q on such plots is shown in Table 1.

Table 1 shows that choice ratios generated by the random-responding model (Equation 8) undermatch more with increases in the value of q . Slopes of data produced by experiments employing concurrent VI schedules, which only rarely have values of less than .7 (Baum, 1979; Wearden & Burgess, 1982), would require the probability of a random response to be one-fifth or less, and the typical slope values reported by Baum (1979) and Wearden and Burgess (1982) would be generated by the random responding model if q was about one-tenth.

The foregoing account of undermatching adds precision to previous accounts of the phenomenon such as the subject's failure to discriminate between the available concurrent schedules (Baum, 1974). To the extent that such discrimination fails, random behavior allocation might result, with consequent undermatching. However, one serious deficiency of the account is that it cannot deal with the occurrence of overmatching which, though much rarer than undermatching (Wearden & Burgess, 1982), does occur. When no random behavior allocation occurs (i.e., $p = 1.0$ and $q = 0.0$), the model embodied in Equation 8 will perfectly match, but it can never overmatch. Another problem with the model above is that it makes assumptions about bursts of responding that are probably unrealistic. Although it does appear that responding on many reinforcement schedules consists of a sequence of alternating pauses and bursts during which responding proceeds at a constant tempo (Pear & Rector, 1979), it may be unreal-

istic in the case of VI schedules to assume, as was done above, that bursts are of constant length. A more appropriate assumption for VI performance might be that bursts with a high probability of initiation tend to last longer than those with a lower probability of initiation. Such an assumption was used by Nevin and Baum (1980) to derive a feedback function (see Baum, 1973, for a rationale of this approach) for VI performance. The consequences of incorporating such a burst-length rule into the model outlined above are explored in the next section.

Overmatching and Burst Length

Response bursts to the preferred alternative of a concurrent pair (where preference is measured in terms of rate or probability of behavior initiation) will tend to last longer than those emitted to the nonpreferred alternative, if we assume that rates of initiating and terminating response bursts are inversely related (Nevin & Baum, 1980). This assumption might be stated as

$$\text{Rate of initiating burst} + \text{Rate of terminating burst} = u, \text{ where } u \text{ is some constant.}$$

We will assume below that bursts allocated to each of two response alternatives a and b also obey the additive relation between rate of initiation and termination, which we will call, below, the burst-length rule.

When applied to concurrent VI schedules, the burst-length rule tends to produce overmatching. Suppose that burst allocation occurs according to simple matching (Equation 5). If the burst-length rule operates, then the higher the probability of burst allocation to some response alternative the longer the burst will tend to last. Conversely, bursts to the nonpreferred alternative will tend to be shortened. The overall effect of this process is overmatching.

The rate of behavior initiation to one of the concurrent alternatives in any time period is simply proportional to the probability of behavior initiation (P_a or P_b):

$$\text{rate}(a) = s P_a$$

and

$$\text{rate}(b) = s P_b,$$

where rate (a) and rate (b) are the rates of behavior initiation to a and b in some time pe-

Table 1

Slope of regression line of $\log (P'_a/P'_b)$ against $\log (r_a/r_b)$ for different values of the probability of making a random response (q).

q	slope	q	slope	q	slope	q	slope
.9	.068	.45	.403	.2	.656	.05	.887
.8	.137	.4	.447	.15	.722	.04	.906
.7	.210	.35	.493	.1	.797	.03	.928
.6	.280	.3	.543	.08	.831	.02	.950
.5	.361	.25	.598	.06	.867	.01	.975

riod, and s is a proportionality constant. For the concurrent schedule condition, the burst-length rule therefore becomes

$$sP_a + y = u$$

$$sP_b + z = u,$$

where P_a and P_b are the probability of initiation and y and z the rate of termination of bursts to a and b , respectively. Thus,

$$y = s(c - P_a) \quad (9a)$$

$$z = s(c - P_b), \quad (9b)$$

where

$$c = u/s.$$

The simplest model of concurrent performance consistent with the above assumptions is one in which there are only two defined activities, responding to a and responding to b . In this case, the rate of terminating one activity and the rate of initiating the other must be equal. Therefore, $y = sP_b$ and $z = sP_a$. Substituting these values and rearranging, we have

$$c = P_a + P_b = 1.0.$$

Appendix 1 shows that application of the burst-length rule to the concurrent schedule situation involving random behavioral allocation yields a basic choice equation which is

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} = \frac{P'_a(1 - P'_b)}{P'_b(1 - P'_a)}, \quad (10)$$

where P'_a and P'_b are generated by Equation 8. If P_a/P_b equals r_a/r_b , Equation 10 represents a model of choice that takes into account the possibility of random-burst allocation and the rule for determination of burst length. Since P'_a and P'_b are related to r_a/r_b by Equation 8, behavior ratios generated by Equation 10 may be plotted as functions of r_a/r_b .

Undermatching and Overmatching—Simulation and Evaluation of the Model

The model of choice under concurrent schedules embodied in Equation 10 encompasses two opposing types of deviation from simple matching. The first is a tendency to randomly allocate bursts (which, as shown above, tends to produce undermatching); the second is the burst-length rule (which tends to produce overmatching).

Fig. 1. Performance of the model embodied in Equation 10 when the relative rate of reinforcement was varied about 200-fold (6 natural log units). Results are shown for probabilities of random behavior allocation (q) of .0, .2, and .5. Also shown is performance conforming to simple matching (diagonal dotted line). The left-hand corner square illustrates performance of the model when relative reinforcement rates were varied about 20-fold.

To illustrate the operation of the model choice, measures (either $\log [R_a/R_b]$ or $\log [T_a/T_b]$) were plotted against reinforcement ratios ($\log r_a/r_b$) for a wide range of r_a/r_b values (more than 200-fold, or greater than six natural log units). The probability of random behavior allocation was varied across values of 0, .2, and .5. P_a/P_b was set at r_a/r_b (i.e., the basis of the model was matching). P'_a and P'_b were calculated according to Equation 8, and the results were substituted into Equation 10. Figure 1 shows the resulting plots and also illustrates the choice measures observed when performance conformed to matching (diagonal dotted line).

The model makes several predictions. First, when all behavior is allocated at random, both P'_a/P'_b (from Equation 8) and overall choice measures are equal to 1.0 (therefore $\log [R_a/R_b] = .0$). That is, under this condition, performance is insensitive to reinforcement rate and half of all responses (or half of total time) is allocated to each response alternative. On the other hand, when $q = 0$ and no random behavior allocation occurs, $P'_a/P'_b = P_a/P_b$ (using Equation 8). Substituting this value into Equation 10 gives

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} = \frac{P_a(1 - P_b)}{P_b(1 - P_a)} = \left(\frac{P_a}{P_b}\right)^2 = \left(\frac{r_a}{r_b}\right)^2.$$

Thus, the most extreme overmatching value that can occur is for choice performance to equal the square of the reinforcement ratio. This corresponds to overmatching with a slope of 2 when log choice measures are plotted against log reinforcement ratios and is shown in Figure 1.

Second, between these two extreme values of random behavior allocation probability, the model suggests that the choice measure value is influenced by two factors. One is the proportion of behavior bursts that are allocated at random. For all r_a/r_b values, the larger the value of q (i.e., the more random behavior allocation), the smaller the value of the choice measures. A tendency towards undermatching is also promoted by large r_a/r_b values, that is, by conditions in which there is a large discrepancy between the rates of reinforcement delivered by the two concurrently presented schedules, for any value of q (apart from 0). However, the values of r_a/r_b needed to test this prediction may be unrealizable in practice and do not occur in any published experiments. Although some studies employing concurrent VI schedules have used VI schedules varying widely in value (e.g., the range from VI 8-sec to VI 720-sec employed by Bradshaw, Szabadi, Bevan, & Ruddle, 1979), they have not usually employed the most extreme schedule values as concurrent alternatives.

These two tendencies taken together mean that for some q values (e.g., $q = .5$), undermatching will occur at all r_a/r_b values. For others (e.g., $q = .2$), undermatching will occur when r_a/r_b is very large, but matching or overmatching will occur at lower r_a/r_b values.

Third, within a range of r_a/r_b values similar to those used in actual experiments (a range from 20 to 1, shown in the smaller corner square), the slope of the log choice measure appears close to a straight line function of the log reinforcement ratio. This is consistent with the data from real experiments, which are initially well fitted by straight lines (e.g., see Baum, 1979).

The range of r_a/r_b values shown in Figure 1 is, however, unrealistically large, and Figure 2 shows the performance of the model over a range of r_a/r_b values much more similar to

those employed in actual experiments on concurrent VI schedules. r_a/r_b was varied across values of 20, 10, 5, 2, and 1 and log response (or time) ratios were calculated by Equation 10 as previously. In all cases the r^2 value for goodness of linear fit was greater than .95 (although it did increase as q decreased), supporting the impression gained from Figure 1 that over the r_a/r_b range of 20 to 1, the model predicts that log choice measures will be close to a linear function of log (r_a/r_b) .

In Figure 2 the slopes of lines produced by plotting log choice measures against log (r_a/r_b) were drawn as a function of q (the probability of random burst allocation). It can be seen that q values less than about .3 produce overmatching in choice measures, values greater than .3, undermatching.

Thus, under the model embodied in Equation 10, the typical slope value found in concurrent VI VI experiments (about .9, see Wearden & Burgess, 1982) would require about one-third of all behavior bursts to be

Fig. 2. Slope of regression line relating log of choice measure (R_a/R_b or T_a/T_b) to log relative reinforcement rate, when relative reinforcement rate was varied across values of 20, 10, 5, 2, and 1. Slope values were calculated using Equation 10, with q (the probability of random behavior allocation) varied.

randomly allocated. This value is higher than that needed to produce this degree of undermatching in the simpler random responding model (Equation 8), because of the addition of the burst-length rule, which tends to promote overmatching. The curve relating regression line slope to probability of random behavior allocation shown in Figure 2 is steeper at small q values than at larger ones. This suggests that, although the maximum possible value of overmatching in the model is a slope of 2, any small amount of random behavior allocation will reduce this value sharply. Consistent with this, slopes of choice functions derived from real organisms tend to be smaller than 2, even when overmatching does occur (see Wearden & Burgess, 1982, and Baum, 1979). The model is therefore consistent with two general trends in data from concurrent VI experiments. The first is the restriction of values of overmatching that appears to occur, although deviations from matching in the direction of undermatching can be large (see Matthews & Temple, 1979). The second is the prevalence of undermatching, which results in the present model from undermatching being consistent with a wider range of random behavior allocation probabilities (.3 and above) than is overmatching.

The model presented above might also, more speculatively, be related to certain specific details of experiments that have investigated factors influencing degree of conformity with the matching law. Although undermatching is the most common result of studies of concurrent VI performance, there is evidence that manipulation of some procedural variables within subjects can affect conformity to the matching law in systematic ways. For example, increasing changeover delay values in pigeons (Brownstein & Pliskoff, 1968) and hens (Scown, Foster, & Temple, 1981), the imposition of changeover ratios (Pliskoff & Fetterman, 1981), or punishment of changeover responses (Todorov, 1971) all seem to promote overmatching. Similarly, Baum (1982) found that increasing the time required to travel between two response keys tended to shift performance from undermatching to overmatching. Manipulations such as these are usually interpreted (e.g., Baum, 1981) in terms of their effects on changeover behavior and, similarly, within the context of the present model, might be seen as acting to discourage random change-

overs and thus generally decreasing the tendency to emit random bursts. Although a completely rigorous account of the effects of changeover contingencies on choice behavior remains to be fully developed, it is suggestive that procedures that tend to make random burst allocations between response manipulanda difficult (such as the imposition of a barrier between the keys, e.g., Baum, 1981, 1982) also tend to shift choice measures away from undermatching.

Under the present model, the principal determinant of conformity to the matching law is the proportion of random burst allocations, and such random allocations in a sense represent failures of the basic matching relation. As noted above, the matching relation can be derived by assuming perfect adherence to a number of different underlying mechanisms (Houston & McNamara, 1981; Myerson & Miezin, 1980; Shimp, 1969; Staddon & Motheral, 1978; Staddon et al., 1981), but such perfect performance, or perfect knowledge of the reinforcement contingencies, seems implausible in the case of animal subjects. Humans may, however, be able to verbalize the contingencies of a concurrent VI schedule perfectly, or may be able to perfectly execute some optimal response strategy. Thus, within the context of the present model, random burst allocation might be expected to have a low probability in experiments with human subjects, with consequent lack of undermatching, or even overmatching. Bradshaw, Ruddle, and Szabadi (1981) conclude, on the basis of a review of human concurrent VI performance, that humans do not systematically deviate from conformity to the matching law. An examination of some recent experiments by Bradshaw and his colleagues suggests, in fact, that overmatching might be much more common in concurrent schedules studies with humans than it is with animals (Wearden & Burgess, 1982). The preponderance of overmatching obtained from humans is consistent, under the present model, with close conformity to the basic matching relation in allocating response bursts to the two concurrent response alternatives.

Under the present model, the normal performance produced by perfect allocation of bursts according to the matching law is overmatching. Undermatching results from failures of the basic choice contingency. The model therefore captures some similar previ-

ous speculations by Baum (1981) that overmatching might be the performance typical of an organism's foodsearching in the wild. Forays in patches with a high probability of food might tend to be prolonged, those in lean patches shortened, in a way that the burst-length rule of the present model exaggerates preferences for concurrent alternatives. In the natural environment, the probability of random behavior allocation might be very low, since patches are likely to be spatially or in some other way separated, so the burst-length rule (or some analogous principle applicable to behavior outside the laboratory) might operate to produce performance which in a concurrent VI VI experiment would be described as overmatching.

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APPENDIX 1

The average burst length to each of the concurrent alternatives (t_a and t_b) is the reciprocal of its rate of termination. Thus, from Equations 9a and 9b

$$t_a = \frac{1}{s(c - P_a)} \text{ and } t_b = \frac{1}{s(c - P_b)}.$$

If the subjects respond at a constant tempo within bursts (e.g., k responses per minute), then the average number of responses per burst is

$$\frac{k}{s(c - P_a)} \text{ on alternative } a, \text{ and}$$

$$\frac{k}{s(c - P_b)} \text{ on alternative } b.$$

The ratio of the number of responses to a and b is found by multiplying the average number of responses per burst by probability of burst initiation, that is,

$$\frac{R_a}{R_b} = \frac{P_a \frac{k}{s(c - P_a)}}{P_b \frac{k}{s(c - P_b)}}$$

or

$$\frac{R_a}{R_b} = \frac{P_a(c - P_b)}{P_b(c - P_a)}.$$

Similar reasoning leads to an identical relation for time allocation measures, that is,

$$\frac{T_a}{T_b} = \frac{P_a(c - P_b)}{P_b(c - P_a)}.$$

but for both types of choice, when there are only two behavior states, $c = 1$ (see text for discussion) giving

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} = \frac{P_a(1 - P_b)}{P_b(1 - P_a)}. \quad (1i)$$

If preferences are made according to simple matching (i.e., $\frac{P_a}{P_b} = \frac{r_a}{r_b}$) then Equation 1i can be shown to produce more exaggerated choice than the matching law requires. Let

$$E = \frac{c - P_b}{c - P_a} = \frac{P_a}{P_b}$$

and suppose that $P_a = kP_b$. Then

$$E = \frac{kP_b}{P_b} = k.$$

If $k > 1$ (i.e., $P_a > P_b$) then $E > 1$ and

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} > \frac{r_a}{r_b}. \quad (2i)$$

If $k < 1$ (i.e., $P_a < P_b$) then $E < 1$ and

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} < \frac{r_a}{r_b}.$$

Equations 1i and 2i can be derived however P_a and P_b are produced. Therefore, the values from the previous section (P'_a and P'_b) may be substituted. This yields

$$\frac{R_a}{R_b} \text{ or } \frac{T_a}{T_b} = \frac{P'_a(1 - P'_b)}{P'_b(1 - P'_a)}.$$