TEMPORAL CONSTRAINT ON CHOICE: SENSITIVITY AND BIAS IN MULTIPLE SCHEDULES

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In multiple schedules of reinforcement, ratios of responses in successive components are relatively insensitive to ratios of obtained reinforcers. An analysis is proposed that attributes changes in absolute response rates to concurrent interactions between programmed reinforcement and extraneous reinforcement in other components. The analysis predicts that ratios of responses in successive components vary with reinforcer ratios, qualified by a term describing the reinforcement context, that is, programmed and extraneous reinforcers. Two main predictions from the analysis were confirmed in an experiment in which pigeons' responses were reinforced in the components of a multiple schedule and analog extraneous reinforcement was scheduled for an alternative response in each component. Sensitivity of response and time ratios to reinforcer ratios in the multiple schedules varied as a function of the rate of extraneous reinforcers. Bias towards responding in one component of the multiple schedule varied as an inverse function of the ratios of extraneous reinforcer rate in the two components. The data from this and previous studies of multiple-concurrent performance were accurately predicted by our analysis and supported our contention that the allocation of behavior in multiple-schedule components depends on the relative values of concurrently-available reinforcers within each component.

Key words: sensitivity to reinforcement, bias, generalized matching relation, extraneous reinforcement, concurrent interaction, multiple-concurrent schedules, key peck, pigeons

A major concern in the study of behavior has been the different performances observed in successive and simultaneous discriminations. One well-known though poorly understood difference is the greater difficulty of successive discriminations (Mackintosh, 1974). This apparent difference in difficulty is consistent with another feature distinguishing successive from simultaneous discriminative performance. That is, differences between response rates in the presence of the different discriminative stimuli vary with changes in the associated reinforcement rates to a far lesser extent in successive discriminations (Herrnstein, 1970). Our aim is to show that this relative insensitivity of response differ-

entials to variation in reinforcement differentials can be understood in terms of the temporal constraint placed on behavior by successive discrimination procedures. Specifically, the options available to the subject at any time are constrained to those associated with either of the successively presented discriminative stimuli. In simultaneous discriminations, options include those associated with both stimuli. In the present paper we identify both empirical and theoretical difficulties encountered by Herrnstein's (1970) analysis of performance in multiple schedules and present an alternative analysis based solely on concurrent interactions within multiple-schedule components. Two experiments reported here arranged conditions where the extent of concurrent interactions was systematically varied to produce changes in the free parameters of the generalized matching law, which we retain as a means of describing the phenomena in simultaneous and successive discriminations. The changes observed provided convincing empirical evidence for our "concurrent analysis" of multiple-schedule performance.

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Generalized Matching Law

Successive and simultaneous discriminations are studied in compound schedules of reinforcement where two or more components (usually variable-interval schedules), each associated with a different discriminative stimulus, are combined according to either of two main temporal configurations. In a concurrent schedule, the components are simultaneously available and the subject is free to choose between them. In a multiple schedule the components alternate successively, with the result that when one component is in effect the other is unavailable.

Performance under concurrent schedules of reinforcement is well described by a power function relating ratios of responses in the components to the ratio of reinforcers. For the example of a choice between left and right responses,

$$\frac{P_L}{P_R} = c \left(\frac{R_L}{R_R} \right)^a, \quad (1)$$

where P and R represent responses and reinforcers respectively and their subscripts (L and R) identify the components in which they occur. The power function of Equation 1 is termed the generalized matching law (Baum, 1974). Two free parameters in Equation 1, a and c , are estimated empirically by fitting a straight line to a logarithmic transformation of response and reinforcer ratios. The slope of the line provides an index of sensitivity with which response ratios change when reinforcement ratios are varied (a), and the intercept provides an index of bias towards responding in one component versus the other (c). For concurrent schedules the exponent of the power function given by Equation 1 typically ranges between about .8 and 1.0 (Baum, 1979; de Villiers, 1977; Lobb & Davison, 1975; Mullins, Agunwamba, & Donohoe, 1982; Myers & Myers, 1977) but may take smaller values if the stimuli signaling the different components are physically similar (Bourland & Miller, 1981). Thus Equation 1 provides a useful measure of the strength of discrimination in concurrent schedules of reinforcement.

A power function similar to Equation 1 also provides an accurate description of performance in multiple schedules. The ratio of responses (P_1/P_2) in temporally separated Components 1 and 2 varies with the ratio of rein-

forcers for those responses (R_1/R_2) according to:

$$\frac{P_1}{P_2} = c' \left(\frac{R_1}{R_2} \right)^{a'}. \quad (2)$$

Values of the exponent a' (sensitivity of response ratios to variation in multiple-schedule reinforcer ratios) are typically in the region of .3 to .5, markedly lower than those for concurrent schedules (Charman & Davison, 1982; Davison & Ferguson, 1978; Lander & Irwin, 1968; Lobb & Davison, 1977; McLean & White, 1981; White, Pipe, & McLean, Note 1). The low sensitivity to reinforcer ratios is termed *undermatching*, since a change in response ratios is always less extreme than the corresponding change in reinforcement ratios. Bias, given by c' in Equation 2, may also be evident in multiple schedules.

In the present paper, our intention is to show that both undermatching and bias in multiple schedules may be understood in terms of the concurrent choice between programmed reinforcement and other sources of reinforcement in each multiple-schedule component. That is, we will attempt to advance a theoretical and empirical analysis of multiple-schedule performance that relies solely on concurrent interactions occurring during each component of the multiple schedule. First, however, we will consider Herrnstein's (1970) influential account of undermatching in multiple schedules, partly in order to identify its deficiencies and partly to establish the basis for concepts, such as the notion of extraneous reinforcement, which are common to Herrnstein's and our own account.

HERRNSTEIN'S ANALYSIS

The most influential account of why undermatching is more marked in multiple than in concurrent schedules has been offered by Herrnstein (1970). In concurrent schedules, behavior is allocated to responding on the higher-valued of two components at the expense of responding on the lower-valued component. Thus when reinforcement for one response is increased, concurrent interactions increase the rate of that response and concomitantly decrease the rate of alternative responses (Catania, 1963). Herrnstein's concurrent-schedule analysis therefore describes the strength of one response in terms of the rela-

tion between reinforcement for that response and reinforcement for alternative competing responses:

$$P_1 = \frac{k \cdot R_1}{(R_1 + R_2 + R_0)} \quad (3)$$

In Equation 3, R_0 is extraneous reinforcement obtained from sources other than R_1 and R_2 and maintains an aggregate of other unmeasured behavior, P_0 . The term k refers to the total behavior occurring in the session and is measured in the same units as P_1 , that is, $k = P_1 + P_2 + P_0$. When R_2 and R_0 are zero, Equation 3 indicates that all of k is allocated to the response P_1 . Therefore, k defines the maximal or asymptotic rate at which the response P_1 can be emitted, which is invariant with respect to total reinforcement and to reinforcement for competing responses (see Herrnstein, 1974, pp. 161-163). Thus the essence of Herrnstein's concurrent-schedule analysis is that during any period of time a constant quantity of behavior (k) is allocated between alternative responses (P_1 , P_2 , and P_0) in a way that depends on the relative rates of the available sources of reinforcement (R_1 , R_2 , and R_0).

In extending this analysis to multiple schedules, Herrnstein (1970, p. 259) assumed that the effects of programmed and other reinforcement in one component on responding in the other may be attenuated by virtue of the temporal separation of the multiple-schedule components. The extent to which responses are dependent upon temporally distant reinforcers is given by the parameter m in Equation 4. During Component 1,

$$P_1 = \frac{kR_1}{R_1 + R_{01} + mR_2 + mR_{02}}, \quad (4a)$$

and during Component 2,

$$P_2 = \frac{kR_2}{R_2 + R_{02} + mR_1 + mR_{01}}. \quad (4b)$$

One main feature of Equation 4 is that it predicts the phenomenon of behavioral contrast. That is, for $m > 0$ decreases in R_2 in Component 2 generate increases in P_1 , in the temporally distant Component 1. For small values of m , predicted contrast is small, and a matching equation derived by dividing Equation 4a by Equation 4b predicts strong undermatching. Thus Herrnstein proposed that approximate matching obtains in concurrent schedules because the extent of component interactions is

maximal ($m = 1.0$), whereas undermatching obtains in multiple schedules because the temporal separation of components constrains their interaction ($m < 1.0$). Herrnstein's multiple- and concurrent-schedule analyses therefore describe response rate as a function of relative reinforcement available during the entire session.

Although Herrnstein's equations for single and concurrent schedules appear to do well in accounting for data variance (de Villiers, 1977; de Villiers & Herrnstein, 1976), these equations have the flaw that they assume perfect matching without bias (a and c equal to 1.0 in Equation 1) for behavior allocation between simultaneously-available reinforcement sources. In Equation 4, the same relationship is assumed for the choice between R_1 (or R_2) and R_0 . Yet concurrent performances are now well established as a power function of relative reinforcement rates with non-unit values of the exponent and bias terms. The matching assumption must inevitably lead to inaccuracies in empirical estimates of the free parameters of Equation 3 and 4 that could not be empirically identified. With Herrnstein's single-schedule equation, for example, similar measures of variance accounted for will obtain for any extent of undermatching or bias with a different value for R_0 (cf. Wearden, 1981).

Criticisms

There are a number of empirical problems with Equation 4. First, it does not predict changes in contrast that occur when extraneous reinforcement is varied. When R_{01} and R_{02} are large relative to R_2 , Equation 4 predicts relatively small contrast when R_2 is varied. Hinson and Staddon (1978) found larger contrast effects when they arranged an alternative source of reinforcement in multiple-schedule components, compared to the contrast effects obtained without the alternative source, although changes in response ratios or proportions observed when extraneous reinforcement is varied do appear consistent with Equation 4 (Herrnstein & Loveland, 1974; Pliskoff, Shull, & Gollub, 1968).

Second, Equation 4 does not predict the changes in absolute response rates that occur when component duration is manipulated. Herrnstein (1970) suggested that component interaction (m) would be large when components were short and alternated rapidly. Al-

though the decrease in undermatching (Charman & Davison, 1982; Shimp & Wheatley, 1971; Todorov, 1972) and the increase in contrast (Ettinger & Staddon, 1982; McSweeney, 1982) as component duration becomes shorter are consistent with predictions from Equation 4, actual changes in response rates are not. Equation 4 predicts a decrease in response rate when m increases (Edmon, 1978), whereas response rates in the richer component of a multiple schedule with a constant reinforcement differential increase as component duration shortens (Charman & Davison, 1982; McSweeney, 1982).

Third, response rates predicted by Herrnstein's single-schedule equation (Equation 3 for the case where R_2 is zero and R_1 is varied) are higher than those predicted by Equation 4 for a nondifferential multiple schedule, i.e., where both components provide the same rate of reinforcement. This curious prediction was noted by Herrnstein (1970) and arises because predicted response rates in multiple-schedule components are decreased by reinforcement obtained in the other component. Speelman and Gollub (1974) and McSweeney (1980) examined performance over several transitions between nondifferential multiple schedules and equal-valued single schedules and found no reliable change in response rate.

The source of the above problems lies in an inconsistency between the notion that reinforcers in one component affect behavior in the other (successive interaction) and the notion that k remains constant for multiple schedules. In a concurrent schedule total behavior k is allocated between response alternatives in proportion to the relative reinforcements maintaining them. In a multiple schedule, k can be allocated only to those options that are available during one component at a time—response alternatives in the other component are unavailable. Herrnstein (1970) is unclear, however, whether k for multiple schedules refers to the total behavior during a component ($P_1 + P_{01}$) or to total behavior summed over components ($P_1 + P_{01} + P_2 + P_{02}$). If the former were the case, versions of Equation 4 written for P_1 and P_{01} , or alternatively for P_2 and P_{02} , will sum to k only if $m = 0$. If the latter were the case (as in concurrent schedules), versions of Equation 4 written for the four response alternatives (P_1, P_2, P_{01}, P_{02}) will sum to k only if $m = 1.0$. In either case, therefore, instances of $0 < m < 1.0$ render Equation

4 invalid. The major problem with Herrnstein's (1970) analysis of multiple-schedule performance stems from the proposal of a *successive interaction*—the possibility that reinforcement rate in one component influences responding in the other. In the remainder of this paper, therefore, we advance an alternative multiple-schedule analysis that invokes only *concurrent interactions* during each multiple-schedule component.

CONCURRENT ANALYSIS OF MULTIPLE-SCHEDULE PERFORMANCE

Our own analysis is premised on the assumption that during each component of a multiple schedule the subject is presented with a choice between emitting the response defined by the explicitly programmed reinforcement contingency and engaging in other behavior—no other choice exists for the subject. Further, we assume that over any period of time, some behavior is allocated to making the defined response, and the remainder is allocated to emitting other responses (P_0). If all responses are expressed in the same units (P), then reallocation of behavior between available response alternatives allows the rate of any particular response to vary, but leaves total behavior within a component (k) invariant (see also Baum, 1981; Rachlin, 1978; Staddon, 1982). Thus the term k equals total behavior in Component 1 ($P_1 + P_{01}$) and also equals total behavior in Component 2 ($P_2 + P_{02}$), given equal component duration.

Finally, we assume that the allocation of behavior in Component 1 between P_1 and P_{01} depends only on reinforcement obtained during the component. That is, response rate P_1 varies directly with reinforcement R_1 , and inversely with reinforcement R_{01} (concurrent interaction), but is insensitive to reinforcement obtained in the alternated component (R_2 and R_{02}). We will present an empirical examination of this assumption, which we term *successive independence*, and a consideration of its implications for the phenomenon of behavioral contrast later in this paper.

Given these assumptions, absolute response rates in each component can be predicted on the basis of a stable relationship between response allocation and relative obtained reinforcement rates within each component. Re-

writing Equation 1 in proportional terms for the concurrent choice between R_1 and R_{01} during Component 1 yields a generalized version of Herrnstein's (1970) single-schedule equation:

$$\frac{P_1}{P_1 + P_{01}} = \frac{R_1^a}{R_1^a + b \cdot R_{01}^a},$$

where a is the concurrent sensitivity in Component 1, and b is the reciprocal of the concurrent-schedule bias term, c (Equation 1). Both of these terms are assumed independent of reinforcement obtained during Component 2. Now since P_1 and P_{01} are the only activities the subject can engage in during Component 1, the expression $P_1 + P_{01}$ exhausts k , the total behavior in the component. Substituting k for $P_1 + P_{01}$ and rearranging terms yields, for Component 1,

$$P_1 = \frac{k \cdot R_1^a}{R_1^a + b \cdot R_{01}^a}, \quad (5a)$$

and for Component 2,

$$P_2 = \frac{k \cdot R_2^a}{R_2^a + b \cdot R_{02}^a}. \quad (5b)$$

Davison and Hunter (1976) and Wearden (1981) proposed similar equations for absolute response rates in concurrent-schedule components and single schedules, respectively. An expression predicting the ratios of responses P_1/P_2 in the two components of the multiple schedule is derived by dividing Equation 5a by Equation 5b and canceling k :

$$\frac{P_1}{P_2} = \left[\frac{R_1^a}{R_1^a + b \cdot R_{01}^a} \right] \cdot \left[\frac{R_2^a}{R_2^a + b \cdot R_{02}^a} \right]^{-1},$$

simplified,

$$\frac{P_1}{P_2} = \left[\frac{R_1}{R_2} \right]^a \cdot \frac{R_2^a + b \cdot R_{02}^a}{R_1^a + b \cdot R_{01}^a}. \quad (6)$$

In Equation 5, a is the sensitivity of behavior to concurrently available reinforcers within each component as in Equation 1 (here, sensitivity is assumed to be the same for each component), and b is the inverse of the bias between concurrently available alternatives. It does not matter what determines the values a and b , so long as it is not the reinforcement obtained in the alternated component. For the case where a and b are both 1.0 (matching with-

out bias within each component), Equation 6 reduces to

$$\frac{P_1}{P_2} = \frac{R_1}{R_2} \cdot \frac{R_2 + R_{02}}{R_1 + R_{01}}. \quad (7)$$

Figure 1 illustrates how Equation 7 predicts the relationship between multiple-schedule response ratios and reinforcer ratios, when the values of R_{01} and R_{02} (reinforcers per hour) are varied. Predictions are for the following series of multiple schedules (in minutes): VI 12 VI 2.4, VI 6 VI 3, VI 4 VI 4, VI 3 VI 6, VI 2.4 VI 12, where average reinforcer rates (across

Fig. 1. Response ratios predicted from Equation 7 plotted as a function of reinforcement ratios (top panel). Three sets of predictions are shown, one for each of the rates of R_0 indicated. The lower panel shows logarithms of predicted response ratios plotted against logarithms of reinforcement ratios for different R_{01}/R_{02} ratios. Solid lines are least-squares regression lines.

components) are constant. In the upper panel of Figure 1, these relationships are plotted on linear coordinates to illustrate their strong similarity to the power functions described by the generalized matching relation (Equation 2). For high R_0 (100), the predictions are consistent with a power function having an exponent of the order of .9, and for low R_0 (5), the equivalent power function has an exponent of the order of .3. Thus the extent to which response ratios match reinforcement ratios (indexed by a' in Equation 2) is a direct function of the values of R_{01} and R_{02} . That is, Equation 7 predicts that multiple-schedule sensitivity is determined by the value of reinforcement for other behavior relative to that of reinforcement R_1 and R_2 .

The lower panel of Figure 1 shows the effect of unequal rates of extraneous reinforcement across components on predicted response ratios. The predictions are for the same series of multiple schedules as in the upper panel and are plotted on log-log coordinates. The middle function for $R_{01} = R_{02} = 25$ has a zero intercept reflecting no bias. The top function for $R_{01} = 10$ and $R_{02} = 40$ has a positive intercept, reflecting a bias towards responding in Component 1, and the bottom function for $R_{01} = 40$ and $R_{02} = 10$ has a negative intercept reflecting bias towards responding in Component 2. Thus the extent of bias is an inverse function of the ratio of obtained rates of extraneous reinforcement in the two components.

AN EXPERIMENTAL ANALOG OF R_0

In the following experiments we investigated the contribution of extraneous reinforcement to multiple-schedule sensitivity and bias by arranging food reinforcers for responses on a second key during each component of a series of multiple schedules. Actual extraneous reinforcement cannot be measured, but by explicitly programming reinforcers concurrently with R_1 and R_2 in the multiple-schedule components, we are able to define an experimental analog of R_0 . That is, variations in the analog R_0 should affect multiple-schedule performance in the same manner that identical variations in actual R_0 would be expected to do, notwithstanding the fact that in this procedure there will still remain actual reinforcers for other behavior. Further, analog R_0 in the present experiments was programmed accord-

ing to independent variable-interval (VI) schedules. It is a matter of debate whether actual extraneous reinforcement is time-based or response-based. We chose VI schedules for the analog R_0 primarily for the pragmatic reason of maintaining control over their values in components. However, it is likely that the conditions under which behavior such as resting and grooming will be reinforced (fatigue, "itchiness") are met only occasionally and at irregular intervals throughout a session (see also Staddon, 1982).

EXPERIMENT 1 SENSITIVITY

The contribution of R_0 to multiple-schedule sensitivity was studied in Experiment 1. In a two-key experimental chamber, ratios of reinforcers (R_{L1}/R_{L2}) for responses on the left key in the two components of a multiple schedule were varied in the context of high or low rates of right-key reinforcers (R_R), which served as the analog of R_0 . Equation 7 predicts that the sensitivity (a' in Equation 2) of left-key response ratios (P_{L1}/P_{L2}) to variations in left-key reinforcer ratios will be greater when R_R is high than when R_R is low.

METHOD

Subjects

Four homing pigeons, each with previous training in multiple schedules, were maintained at $80 \pm 5\%$ of their free-feeding weights. Water and grit were always available in home cages, and supplementary feeding maintained prescribed body weights.

Apparatus

A light-proof, sound-attenuating pigeon chamber, 35 by 33 by 34 cm, contained an interface panel. Two 2.5-cm diameter response keys were 25 cm from the floor and 9 cm either side of the center of the interface panel. A hopper was central in the interface panel and 8 cm from the floor. Each key was illuminated red (S1) or green (S2), and pecks exceeding about .2 N produced .05-sec keylight offset. Reinforcement was 2.5-sec access to wheat during which the hopper was illuminated white and both keys were dark. A ventilation fan at the rear of the chamber helped mask extraneous sounds. Standard electromechanical programming apparatus and electronic timers,

located in an adjacent room, scheduled and recorded all experimental events.

Procedure

Throughout all conditions, 90-sec presentations of S1 (simultaneously on both keys) alternated directly with 90-sec presentations of S2, for a total of 32 components per session. Pecks on each key in each component were reinforced according to separate, independent variable-interval (VI) schedules. Each VI schedule comprised a multiple of five randomized intervals derived from Fleshler and Hoffman's (1962) equation. Within each component, a changeover delay (COD) prevented reinforcement of responses on a key that followed responses on the other key, and all subsequent responses for a fixed period thereafter. The COD was introduced and gradually increased up to a final value of 1.5 sec before birds were placed in their first conditions. The schedule combinations used in each condition are given in Table 1. There were two series of conditions. In one series (Conditions 1 to 5), the multiple-schedule reinforcer ratio was varied on the left key while the schedules on the right

key were held at equal, low values. In the other series (Conditions 6 to 10), left-key reinforcer ratios were again varied and right-key reinforcer rates were equal and high. The scheduled total reinforcers for left-key responding over components was held constant across conditions and series. Birds A1 and A2 were exposed first to Conditions 1 to 5 and Birds A3 and A4 were exposed first to Conditions 6 to 10. For each bird, daily 48-min sessions were continued within a condition for at least 25 sessions and until a stability criterion had been met. The criterion was that for five consecutive sessions, left-key responses in S1 as a proportion of total left-key responses did not deviate from the mean proportion of left-key responses over the five sessions by more than .025. For Birds A1 and A3 in Condition 9, this criterion was not met after 55 sessions, and the condition was abandoned and repeated later. The orders of conditions within each series were different for each bird and are given in Table 1 along with the number of sessions in each condition required to meet the stability criterion.

Responses and reinforcers on each key in each component were recorded separately. In addition, time allocated to left-key responding in S1 and S2 was measured by accumulating the times between a left-key response following a right-key response and a right-key response following a left-key response separately in each component. Time allocated to the right was obtained by subtracting left-key time allocation from total session time. Our procedure thus afforded a measure of multiple-schedule time allocation that may be comparable to those obtained by measuring time spent in the region of spatially separated manipulanda (White, 1978) or by observation (Rand, 1977) in standard multiple procedures.

RESULTS

The summary data used for all analyses were means from the five days when performance was stable according to the criterion. Since the criterion specified a maximum allowable variability over sessions, these means accurately represent stable multiple-schedule performance. Appendix A gives each bird's mean time, response rates, and total reinforcers on each key in each component for each condition. The time base for calculation of absolute response rates in each stimulus was the time

Table 1

Schedule combinations in each condition (in minutes), order of exposure for each bird, and number of sessions required to reach stability (in parentheses) in Experiment 1.

Schedule Combinations				
Condition	S1		S2	
	Left	Right	Left	Right
1	VI 6	VI 6	VI 1.2	VI 6
2	VI 3	VI 6	VI 1.5	VI 6
3	VI 2	VI 6	VI 2	VI 6
4	VI 1.5	VI 6	VI 3	VI 6
5	VI 1.2	VI 6	VI 6	VI 6
6	VI 6	VI .85	VI 1.2	VI .85
7	VI 3	VI .85	VI 1.5	VI .85
8	VI 2	VI .85	VI 2	VI .85
9	VI 1.5	VI .85	VI 3	VI .85
10	VI 1.2	VI .85	VI 6	VI .85

Orders of Exposure and Sessions to Stability

Bird					
A1	1(37), 7(38),	4(47), 9(46),	2(25), 6(42),	5(26), 10(34),	3(35), 8(24).
A2	2(25), 9(31),	1(25), 10(43),	5(25), 6(27),	4(29), 7(25),	3(26), 8(25).
A3	9(26), 2(35),	6(29), 1(25),	7(35), 5(26),	8(31), 4(33),	10(31), 3(30).
A4	10(25), 1(31),	7(45), 5(45),	9(33), 2(32),	6(41), 4(33),	8(50), 3(30).

Fig. 2. Filled circles represent log response ratios (P_{L1}/P_{L2}) and are plotted against log reinforcement ratios (R_{L1}/R_{L2}). Upper panels show data from Conditions 1 to 5, where right-key reinforcement (R_r) was 10/hr. Lower panels show data from conditions where R_r was 70/hr. Equations of least-squares regression lines and standard errors of estimate are given in each panel. The small diagonal crosses and VACs in each panel concern predictions from Equation 9 (see text).

during which a response could occur; that is, time in a component less reinforcement time in that component. This follows Herrnstein's (1961) method of calculating rates and corrects them for the different reinforcement times in the two components. The measure of time allocation in Appendix A is total time on the left key (less left-key reinforcement time) in a component, divided by the time base for that component. These measures therefore represent the time allocated to the left key as a proportion of available time in a component, with the complementary proportion allocated to the right key.

Figure 2 shows logarithms of ratios of left-key response rates (filled circles) plotted as a function of logarithms of ratios of left-key reinforcers. Ratios in this and all subsequent figures are rates in S1 divided by rates in S2. Upper panels show data from Conditions 1 to 5 where scheduled R_r was 10/hr in both components. Lower panels show data from Conditions 6 to 10 where R_r was 70/hr. Equations of least-squares regression lines are given in each panel. These equations correspond to the logarithmic form of the power function given by Equation 2. Regression line slopes (which give an index of multiple-schedule sensitivity) in Conditions 1 to 5 were within the range

typical of multiple-schedule performance, the slopes ranging from .24 to .54. Consistent with Equation 7, the performance of each bird showed greater sensitivity to reinforcement ratios in Conditions 6 to 10 where R_r was high (lower panels) than in Conditions 1 to 5 where R_r was low (upper panels). That is, undermatching was stronger when right-key reinforcement rates were lower. Figure 3 shows the same analysis for time ratios on the left key. Sensitivities for time allocation were always lower than those for response allocation. Again, multiple-schedule sensitivities for time ratios increased for all birds with increasing R_r .

Concurrent performance within each component of the multiple schedule was examined by calculating slopes of straight lines, which best fitted logarithms of concurrent response ratios from both components (P_{L1}/P_{R1} and P_{L2}/P_{R2}) plotted as a function of the corresponding reinforcement ratios. These slopes index sensitivity of concurrent response ratios to reinforcer ratios as described by Equation 1. For Birds A1, A2, A3, and A4, respectively, concurrent sensitivities were .77, .52, .37, and .69 for $R_r = 10$, and .71, .81, .33, and .88 for $R_r = 70$. Corresponding concurrent time-allocation sensitivities were .85, .48, .47, and .93

Fig. 3. Filled circles are log time ratios (T_{L1}/T_{L2}) plotted against log reinforcement ratios for Conditions 1 to 5 (upper panels) and Conditions 6 to 10 (lower panels). Equations of least-squares regression lines and standard errors of estimate are given in each panel. Diagonal crosses and VACs are from Equation 9 (see text).

for $R_R = 10$, and .60, .79, .37, and 1.05 for $R_R = 70$. There was no reliable difference between concurrent sensitivities for the different R_R conditions. The reliable increase in multiple-schedule sensitivity cannot therefore be attributed to a change in the sensitivity of concurrent response ratios to concurrent reinforcement ratios as a result of changing R_R .

Quantitative Predictions

Ratios of response rates in the successive components of the multiple schedule can be predicted by substituting the obtained reinforcement rates from the different conditions of the experiment for the reinforcement terms in Equation 6, having first estimated the obtained values for a (concurrent sensitivity) and b (concurrent bias) using Equation 1. It should be noted, however, that the analog values for R_0 (that is, right-key reinforcement) do not replace actual extraneous reinforcement that still occurs. That is, during each component there was extraneous reinforcement in addition to the obtained analog values. Thus, in effect our procedure presented the subject with a 3-ply concurrent schedule in each component (VI_L, VI_R, R_0). If R_0 is non-zero, it will main-

tain P_0 in each component, so the response totals ($P_{L1} + P_{R1}$ and $P_{L2} + P_{R2}$) in the two components do not equal k . Moreover, since the reinforcement totals ($R_{L1} + R_{R1}$ and $R_{L2} + R_{R2}$) differed in most conditions, concurrent interactions involving real R_{01} and R_{02} would differentially affect response totals in the two components, so they were not equal and constant across conditions (Appendix A). However, Davison and Hunter (1976) and Davison (1982) have shown that the rate of reinforcement on a third alternative does not affect behavior allocation between two other components of a 3-ply concurrent schedule. Therefore the following relations are unaffected by the presence of actual extraneous reinforcement. In Component 1, behavior allocation between left and right keys is given by

$$\frac{P_{L1}}{P_{L1} + P_{R1}} = \frac{R_{L1}^a}{R_{L1}^a + bR_{R1}^a}, \quad (8a)$$

and in Component 2,

$$\frac{P_{L2}}{P_{L2} + P_{R2}} = \frac{R_{L2}^a}{R_{L2}^a + bR_{R2}^a}. \quad (8b)$$

Dividing Equation 8a by Equation 8b and rearranging terms gives an expression predicting

the multiple-schedule (left-key) response ratio that is analogous to Equation 6:

$$\frac{P_{L1}}{P_{L2}} = \left[\frac{R_{L1}}{R_{L2}} \right]^a \cdot \frac{R_{L2}^a + bR_{R2}^a}{R_{L1}^a + bR_{R1}^a} \cdot \frac{P_{L1} + P_{R1}}{P_{L2} + P_{R2}}. \quad (9)$$

Predictions for all data points were made from a logarithmic form of Equation 9 and are shown as crosses in Figures 2 and 3. The VACs given in these figures refer to the proportion of variance in the data accounted for by the predictions. In all cases, the predictions from Equation 9 matched obtained response ratios very closely indeed, and regression lines through the crosses in Figures 2 and 3 have slopes and intercepts that closely correspond to those given for the obtained data.

DISCUSSION

Predictions from Equation 7 were closely supported by our data. Sensitivity of response and time ratios to reinforcement ratios in the left-key multiple schedule increased as a function of right-key reinforcement rates, with higher sensitivity when the right-key reinforcement rate was higher. Thus the result is consistent with that of Pliskoff, Shull, and Gollub (1968) who exposed three pigeons to two multiple-schedule conditions (multiple VI 3-min VI 3-min and multiple VI 1.5-min VI 6-min) with concurrently-available reinforcement for responses on a second key. Three rates of reinforcement were used on the second key: 0, 20/hr, and 60/hr. They found more extreme response ratios in their differential multiple-schedule condition when concurrently-available reinforcement was high. Pliskoff, Shull, and Gollub derived an expression similar to Equation 7, based on the notion that the components of a multiple schedule could be treated as separate and independent concurrent schedules. The general tenor of our argument, and the data of Experiment 1, are therefore consistent with the conclusions of Pliskoff et al.

Lobb and Davison (1977) compared performances in standard multiple schedules and multiple-concurrent schedules where a common VI 60-sec schedule was accessible via a changeover-key response (Findley, 1958). Lobb and Davison found no systematic differences between sensitivities in multiple schedules with and without concurrently-available rein-

forcement, a result that is apparently inconsistent with our reliable changes. The inconsistency is resolved when two aspects of their procedure are considered. First, their common VI schedule allowed obtained common-key reinforcer ratios to vary, and second, total reinforcers from the multiple schedule varied relative to total common-key reinforcers across multiple-schedule conditions. Specifically, the group-average obtained reinforcer rate from the multiple-schedule components increased steadily from about 125/hr (with about 50/hr from the common schedule) in their non-differential Condition 1, to about 600/hr (with only 24/hr from the common schedule) in their extreme multiple-schedule conditions. The schedules used in the latter conditions (VI 5-sec and VI 55-sec) were not used as components of their multiple schedules. Equation 6, supported by our finding that changes in multiple schedule ratios on one key depend on total reinforcement on that key relative to total reinforcers on the other, predicts a negatively-accelerating increase in log response ratios as a function of log reinforcement ratios under these conditions, and indeed there is quite clear evidence of such nonlinearity in Lobb and Davison's Figure 6. In fact, the deviations from linearity in these data indicate an important advantage of the present analysis over Equation 2, which must predict a linear function on log-log coordinates. Figure 4 shows a plot of Lobb and Davison's data against response ratios predicted by Equation 9, plotted on log-log coordinates. Only data from conditions with noninfinite concurrent reinforcement ratios were used for reanalysis. Despite changes in obtained reinforcer rates from their common schedule between components and across conditions, obtained ratios match predictions closely, and VACs by predictions from Equation 9 are high in all cases. Lobb and Davison's data are therefore entirely consistent with our analysis.

The present analysis may also afford a satisfactory interpretation of other manipulations sometimes used in multiple schedules. For example, Bouzas (1978) studied performance under a multiple VI 1-min VI 4-min schedule where a VI 30-sec schedule of shock was superimposed on both multiple-schedule components. Food and shock were both contingent on key pecking, and as the shock intensity was increased, absolute response rates decreased in

Fig. 4. Log response ratios plotted as a function of log predicted response ratios. The diagonal line represents perfect prediction, and the VACs given in each panel are proportions of variance in obtained log response ratios accounted for by predictions (data from Lobb & Davison, 1977).

both components. Superimposed shock may be seen as increasing the value of R_0 relative to that of food. Consistent with this interpretation, the effect of increasing shock intensity was a differential reduction in responding in the lower-valued component, and performance approached matching in the multiple schedule as intensity increased.

EXPERIMENT 2 BIAS

In terms of the power function relating response ratios to reinforcer ratios in the successive components of a multiple schedule (Equation 2), bias towards responding in one component versus the other is quantified by the constant c' . According to our concurrent analysis summarized by Equation 7, bias will result when different values of R_0 are obtained in the two components (for example, a color preference). In particular, if extraneous reinforcement maintains more behavior in one component than in the other, concurrent interactions will differentially reduce responding in that component producing a bias in response ratios that favors the component with relatively low R_0 .

In Experiment 2, the effects on bias of different rates of "extraneous" reinforcement in the two components of a multiple schedule were studied in the two-key procedure where right-key reinforcement schedules were designated as analogs of R_0 , as in Experiment 1. Ratios of left-key reinforcers were varied as in Experiment 1 but with different rates of right-key reinforcers in the two successive components, specifically R_{R1} higher than R_{R2} in one series of conditions and R_{R1} lower than R_{R2} in another. Equation 7 predicts that bias is an inverse function of the right-key reinforcer ratio, R_{R1}/R_{R2} .

METHOD

Two adult homing pigeons were maintained at $80 \pm 5\%$ of their free-feeding weights. The two-key experimental chamber and all aspects of the procedure except for schedule combinations were identical to those in Experiment 1. In each of two series of conditions, the reinforcer ratios on the left key were varied over the same range as in Experiment 1, as shown in Table 2. In Conditions 1 to 5, right-key reinforcement was low in S1 (10/hr) and high in S2

Table 2

Schedule combinations in each condition (in minutes), order of exposure for each bird, and number of sessions required to reach stability (in parentheses) in Experiment 2.

Schedule Combinations				
Condition	S1		S2	
	Left	Right	Left	Right
1	VI 6	VI 6	VI 1.2	VI .85
2	VI 3	VI 6	VI 1.5	VI .85
3	VI 2	VI 6	VI 2	VI .85
4	VI 1.5	VI 6	VI 3	VI .85
5	VI 1.2	VI 6	VI 6	VI .85
6	VI 6	VI .85	VI 1.2	VI 6
7	VI 3	VI .85	VI 1.5	VI 6
8	VI 2	VI .85	VI 2	VI 6
9	VI 1.5	VI .85	VI 3	VI 6
10	VI 1.2	VI .85	VI 6	VI 6

Orders of Exposure and Sessions to Stability

Bird

A5	1(53), 9(34), 9*(28).	2(39), 7(26),	5(48), 10(34),	4(55), 6(33),	3(25), 8(25),
A6	10(51), 5(28), 5*(30).	7(41), 2(25),	9(49), 4(25),	6(45), 3(38),	8(44), 1(25),

*repetitions of a condition

(70/hr). In Conditions 6 to 10, right-key reinforcement was high in S1 (70/hr) and low in S2 (10/hr). The different orders in which the conditions were conducted for the two birds, and the number of sessions required in each condition to achieve stable performance, are given in Table 2. In several conditions, Bird A5 failed to meet the stability criterion with 55 sessions, and these conditions were repeated later. The first condition of each bird's second series was replicated at the end of the experiment.

RESULTS

Appendix A gives each bird's mean response rates, times, and total reinforcers on each key for the last five days in each condition. All analyses were conducted in the same manner as in Experiment 1. Figure 5 (upper panel) shows logarithms of left-key response ratios (P_{L1}/P_{L2}) plotted as a function of logarithms of left-key reinforcer ratios for the values of R_{R1} and R_{R2} given. Lower panels show the same analyses for time ratios on the left key. For both time and response ratios, regression line intercepts indicate that both birds showed bias towards responding in Component 1 in conditions where

Fig. 5. Log response ratios (filled circles) are plotted as a function of log reinforcement ratios in upper panels for Experiment 2, Conditions 1 to 5 ($R_{R1} = 10, R_{R2} = 70$) and Conditions 6 to 10. Lower panels show the same analysis for time ratios. Equations of least-squares regression lines and standard errors of estimate are given in each panel. Diagonal crosses and VACs are from Equation 9 (see text).

R_{R1} was smaller than R_{R2} and bias towards responding in Component 2 in conditions where R_{R1} was greater than R_{R2} . Thus, bias in the left-key multiple schedule was an inverse function of the reinforcement ratio R_{R1}/R_{R2} .

Predictions from Equation 9 are shown as crosses in Figure 5. The VACs in Figure 5 provide an estimate of the variance in obtained data points accounted for by the predictions. These estimates demonstrate the success that Equation 9 has in providing quantitative predictions of the data.

DISCUSSION

The clear inverse relation between multiple-schedule bias and the right-key reinforcer ratios in Experiment 2 is consistent with our concurrent analysis. A further demonstration of the way in which multiple-schedule response ratios are biased by concurrent interactions involving different rates of extraneous reinforcement in the successive components is afforded by the data of Experiment 1. By regarding the left-key reinforcers in Experiment 1 as analog R_0 , the effect of changing the left-key reinforcer ratio on the right-key response ratio represents a change in bias (for right-key responding) towards one or other component,

since R_R was held constant in both components at either 10/hr or 70/hr.

Figure 6 shows logarithms of right-key response ratios (P_{R1}/P_{R2}) and time ratios (T_{R1}/T_{R2}) plotted as a function of left-key reinforcer ratios, which were varied over a range of values in Experiment 1. For both R_R conditions, right-key response ratios were an inverse function of the left-key reinforcer ratios. That is, the right-key response differential, itself maintained by constant nondifferential right-key reinforcement, was biased by left-key reinforcer ratios (the analog of R_{01}/R_{02}). Also consistent with our analysis, the slopes of these "inverse bias" functions were greater when R_R was low (upper panels). This, and the main result of Experiment 1, is consistent with the generalization that ratios of relatively infrequently-reinforced responses are most sensitive to extraneous reinforcement. The data points for the inverse bias functions closely correspond to points predicted by Equation 9, shown as crosses in Figure 6. Variance in obtained data points accounted for by the predictions is given by VAC in Figure 6. The data from Experiment 1 in Figure 6 therefore extend the result of Experiment 2 by demonstrating that the magnitude of bias increases as the differen-

Fig. 6. Filled circles in the upper half of the figure show log response ratios (P_{B1}/P_{B2}) on the right key, plotted against log reinforcement ratios (R_{L1}/R_{L2}) on the left key, for Conditions 1 to 5 and 6 to 10. The lower half of the figure shows the same analysis for time ratios. Equations of least-squares regression lines and standard errors of estimate are given in each panel. Diagonal crosses and VACs are from Equation 9 (see text).

tial between analog R_{01} and R_{02} in the successive components is increased.

An inverse relation between response ratios on one key and reinforcer ratios on the other is consistent with the data of Pliskoff et al. (1968), whose procedure was similar to ours. Our analysis of their data revealed a con-

sistent inverse relation between constant-key response ratios and multiple-schedule reinforcer ratios, although there were only two multiple-schedule reinforcer ratios at each common-key reinforcer rate. As was the case for our data, the functions were steeper where the reinforcement rate on the constant key was

low relative to the average on the multiple schedule.

Changes in constant-key responding in our data and in that of Pliskoff et al. appear to be inconsistent with two studies that have reported constant rates of responding on one key despite changes in reinforcement rate on a second key, namely those of Nevin (1974) and Catania (1962).

Nevin (1974) arranged concurrently-available FI 50-sec schedules in both components of a series of multiple schedules. When we plotted Nevin's data as log ratios, we found consistent inverse relations between constant-key (FI) response ratios and reinforcer ratios on the other (VI) key as in our data (Figure 6), but slopes of best-fitting lines to Nevin's data were smaller ($-.13$, $-.08$, $-.13$ for three birds). There are two likely reasons that the effects in Nevin's data were small. First, the FI typically provided a higher rate of reinforcement than the average rate in the multiple schedule. Our Figure 6 showed strongest changes in right-key response ratios when the rate of right-key reinforcement was low relative to the average in the multiple schedule. Second, sensitivity to reinforcer ratios in concurrent VI FI schedules is typically lower than in concurrent VI VI schedules (Lobb & Davison, 1975). Where concurrent sensitivity is low, concurrent interactions will likely be weak, and Equation 6 predicts only small changes in response ratios. We analyzed Nevin's concurrent performances separately in each stimulus because of large, consistent bias differences between stimuli. Concurrent performances were rather variable, and sensitivity estimates were low: .25 and .26 for Bird 58, .4 and .53 for Bird 59, and .39 and .23 for Bird 60. In summary, Nevin's (1974) data are generally consistent with our own.

Catania (1962) arranged a multiple VI 3-min EXT schedule on one key, with a concurrently-available VI 3-min schedule arranging reinforcers for pecks on a second key in both components. Response ratios on the constant key showed an inverse relation to multiple-schedule reinforcer ratios, but only in conditions where no changeover delay was used. In conditions with a .5-sec COD, constant-key response rates were the same regardless of the multiple-schedule component. However, Catania did not run conditions with different reinforcer ratios in the multiple schedule, and a consistent bias towards the concurrent VI VI component

(versus the concurrent VI EXT component) may have obtained that would obscure changes in constant-key ratios unless other reinforcer ratios were used in the multiple schedule. Strong response bias is indicated by the extraordinarily extreme ratio of total responses (summed across keys) over the two successive components compared with ratios of total reinforcers. In a generalization test, Catania, Silverman, and Stubbs (1974) also showed that rate on one key varied when the stimuli associated (during training) with a concurrently-available 3-component multiple schedule were presented successively on a second key. No COD was used during training, and these data are therefore consistent with Catania's (1962) conditions without a COD. Our inverse bias functions (Figure 6) demonstrate, however, that the use of a COD does not preclude differential concurrent interactions during components of multiple concurrent schedules.

GENERAL DISCUSSION

The concurrent analysis of multiple-schedule performance offered in the present paper departs from Herrnstein's (1970, 1974) analysis in two main ways. The first concerns the necessity for a generalized description of the relationship between responses and reinforcers within a component in order to maintain consistency with literature on concurrent choice and with the central assumption that the total behavior (k) allocated between possible alternatives within a component remains constant (for a given component duration). The second concerns Herrnstein's assumption that reinforcers in successive components interact in their effect on behavior, whereas we have assumed that behavior in one component is independent of reinforcer ratios in another. That is, we have assumed that choice in a multiple schedule is constrained to the specific time at which particular alternative sources of reinforcement are available and is independent of choices offered at other, temporally distant, times.

Generalized Matching

The problems stemming from Herrnstein's (1970) assumption of perfect matching in concurrent schedules seem readily rectified by including terms for concurrent sensitivity and bias (Equation 5). Although the addition of

two more free parameters may appear clumsy, its necessity is indicated by the frequent result of undermatching in concurrent choices. Thus, the ungeneralized forms (Equations 3 and 4) necessarily confound k and R_0 with bias and sensitivity, with the result that Herrnstein's interpretation that k is constant has occasionally appeared empirically unjustified. Specifically, fitting the ungeneralized form to data from single schedules when $a < 1.0$ will return underestimates of k and values for R_0 that actually equal $b \cdot R_0^a$. Baum (1981) observed that response rates on single ratio schedules often considerably exceed estimates of k from single VI performances and asserted that k must therefore be different for ratio and interval schedules. This could be seen as inconsistent with the interpretation of k as a constant, although Baum suggested that the topographies of ratio- and interval-maintained responses may differ. However, Davison (1982) has shown that the exponent in Equation 1 is higher for ratio schedules than for interval schedules, so these different response rates may reflect differences in a , not k . Similarly, a tendency for concurrent bias to favor ratio schedules more than interval schedules in some conditions (Davison, 1982) yields $b < 1.0$ in Equation 5 and therefore also increases predicted rates for ratio schedules relative to those for interval schedules, regardless of the schedule base for R_0 . The power function generalization in Equation 5 may therefore be critical to the constancy of k .

Successive Independence

A second problem with Herrnstein's analysis appears to stem from the extension of the choice analysis to multiple schedules—that is, from the assumption of successive interaction. Unlike concurrent schedules, where equations of the form of Equation 3 appear to do well, the multiple-schedule procedure imposes a temporal constraint on choice between components that precludes the allocation of behavior occurring in one component to the response defined in the other component. For k to remain constant, any change in the absolute response rate P_1 must reflect a change in the allocation of total Component 1 behavior between the alternatives P_1 and P_{01} . That is, P_{01} must undergo complementary changes, whereas a version of Equation 4 for P_{01} sometimes predicts the same change in P_{01} as in P_1 .

An increase in P_1 when R_1 remains constant and R_2 is reduced is the positive behavioral contrast that appears to constitute the main evidence for successive interaction. Given the complementarity of P_1 and P_{01} , that is $(P_1 + P_{01}) = k$, and the prediction of P_1/P_{01} following Equation 1,

$$\frac{P_1}{P_{01}} = c \left(\frac{R_1}{R_{01}} \right)^a, \quad (10)$$

the change in P_1 (contrast) can occur in only two ways. Option 1 is that there is a change in the reinforcer ratio (R_1/R_{01}) via a change in R_{01} , to which the allocation of Component 1 behavior is sensitive. Option 2 is that there is a change in the allocation of behavior between two constant sources of reinforcement. In the latter case, attributing the change in the ratio P_1/P_{01} to reinforcer rate changes in Component 2 invokes a biasing effect of Component 2 reinforcer ratios (which change when R_2 is reduced) on Component 1 response ratios. For behavioral contrast, the biasing relation is inverse and for induction it is positive. Equation 10 can be written to partial out the Component 2 reinforcer ratio as a source of bias, which would otherwise confound values of a and c :

$$\frac{P_1}{P_{01}} = c \left(\frac{R_1}{R_{01}} \right)^a \cdot \left(\frac{R_2}{R_{02}} \right)^n, \quad (11)$$

where n quantifies the extent to which the reinforcer ratio in Component 2 biases response allocation in Component 1, and a and c are defined as before. In this way, reinforcers in one component could directly affect response rate in the other and yet allow the total behavior during a component to remain constant.

The biasing effect quantified by n in Equation 11 can be studied using the analog of R_0 arranged in the present experiment. If n is negative (behavioral contrast), then fitting Equation 1 to data from the concurrent performances during Components 1 and 2 will return spurious estimates of a and c for those concurrent choices. That is, a and c in Component 1 would not be independent of the successive reinforcement context, R_2 and R_{02} . Independence of a and c was assessed by a multiple linear-regression fit of a logarithmic transformation of Equation 11:

$$\log (P_{L1}/P_{R1}) = a \log (R_{L1}/R_{R1}) + n \log (R_{L2}/R_{R2}) + \log c.$$

Multiple regression estimates of a and n for the present data are given in Table 2, along with estimates from data obtained in two other free-operant multiple-concurrent schedule studies (Lobb & Davison, 1977; Pliskoff et al., 1968). The data used in reanalyses of previous studies were from all conditions with non-infinite reinforcer ratios in components. Estimates of n in Table 3 show no systematic evidence of contrast ($n < 0$) or induction ($n > 0$) within any of the experiments. Data from a fourth study (Nevin, 1974) gave negative n s for three birds in a procedure where a fixed-interval schedule was available throughout 10-sec components of a series of multiple VI VI schedules. However, the concurrent performances in these conditions were variable and VACs by predictions from Equation 11 were below .5 in two of these cases. Moreover, the source of the negative n s in Nevin's data was a large difference in response bias between the two com-

ponents, which may well have arisen for other reasons, particularly given they were not well described by Equation 11. With this possible qualification, it would appear that our assumption of successive independence is justified.

The assertion of successive independence of a and c leaves us with Option 1 as an account of behavioral contrast—that is that the ratio R_1/R_{01} changes during the constant component owing to a change in R_{01} . Unlike programmed food reinforcement, R_0 that becomes available during one component can be obtained during the second component. Depending on the rate of, for example, P_{02} and the temporal pattern of alternation between bursts of responding and engaging in other behavior during Component 2, R_{02} may accumulate and be obtained as R_{01} after alternation to Component 1. Now following a reduction in reinforcer rate R_2 , a concurrent interaction will decrease P_2 and increase P_{02} —increasing obtained R_{02} and reducing the accumulated R_{01} available in Component 1. In this way, a change in reinforcer rate in one component may affect reinforcer ratios in both components. Such a mechanism would be consistent with our assertion of successive independence and yet could account for the sustained (e.g., Reynolds, 1961) and transient (e.g., Nevin & Shettleworth, 1966) contrast effects often found in multiple schedules. Similarly, the value of some of the events constituting R_0 may increase rapidly (relative to that of food) over periods when concurrent interactions inhibit P_0 and thereby reduce the rate at which R_0 is obtained. Much human “adjunctive” behavior is like this; for example, scratching, or visiting the bathroom can increase in urgency quite rapidly when these matters are left unattended because of concurrent alternatives that were initially more attractive (see Stadon, 1982, for a fuller account of the “reallocation” of R_0).

Sensitivity and Bias

Our finding of successive independence of component performances in multiple schedules indicates that absolute response rates depend only on reinforcers obtained during their respective components. In components where the rate of reinforcers from the multiple schedule is low relative to the rate of extraneous reinforcers, response rate will be low, and in components where it is high relative to extraneous

Table 3

Estimates of a (concurrent sensitivity) and n (successive interaction) for three free-operant multiple-concurrent schedule studies. VACs are proportions of variance in obtained log response ratios that were accounted for by predictions from Equation 11.

	Bird	a	n	VAC
Lobb & Davison	21	.80	.03	.90
(all conditions	22	.81	.04	.93
except those with	23	.91	.08	.94
indeterminate ratios)	24	.89	.07	.95
	25	.79	.07	.91
	26	.75	-.07	.89
Pliskoff et al.	B1	.73	-.04	.98
	B2	.60	.01	.88
	B3	.84	-.07	.98
<i>Responses</i>				
Present data, both series in Experiments 1 and 2	A1	.79	.12	.96
	A2	.78	.10	.96
	A3	.30	-.06	.86
	A4	.67	-.16	.93
	A5	1.02	.25	.87
	A6	.55	.09	.85
<i>Times</i>				
	A1	.92	.21	.97
	A2	.82	.21	.96
	A3	.44	.01	.91
	A4	.85	-.13	.97
	A5	1.14	.36	.85
	A6	.54	.05	.93

reinforcer rate, response rate will be high. The extent to which component response rates differ, therefore, depends on the extent to which relative reinforcer rates within components differ. When multiple-schedule reinforcer ratios are varied, the value of multiple-schedule reinforcers relative to R_0 changes in each component, and the difference between component response rates changes. Thus ratios of response rates are jointly determined by the reinforcers maintaining them (R_1 and R_2) and by the concurrently-available context for those reinforcers (R_{01} and R_{02}). The sensitivity of multiple-schedule response ratios to reinforcer ratios R_1/R_2 is well known (Equation 2 and Figure 2). Our data provide a clear demonstration that they are also sensitive to their reinforcement context (Figure 6). In Equation 7, the reinforcement context is displayed in the multiplier $(R_2 + R_{02})/(R_1 + R_{01})$, which varies as an inverse function of the multiple-schedule reinforcer ratio, R_1/R_2 . The sensitivity to the reinforcement context therefore produces behavior changes that are the reverse of the stronger changes produced by varying the reinforcer ratio, and therefore serves to attenuate changes in response ratios when reinforcer ratios are varied. This attenuation of changes in response ratios is the undermatching which characterizes multiple-schedule performance.

The extent to which sensitivity to reinforcer ratios is attenuated depends on the values of R_{01} and R_{02} . If both are zero, the multiplier reduces to the reciprocal of the reinforcer ratio, and predicted ratios cannot vary (i.e., both P_1 and P_2 reach equal, asymptotically high rates because of the absence of within-component interactions). If R_{01} and R_{02} remain equal and are increased, Equation 7 predicts increasing multiple-schedule sensitivity as the multiplier approaches constancy at all reinforcement ratios.

The specific mechanism proposed for changes in response ratios as reinforcer ratios are varied thus relies on concurrent interactions by which extraneous reinforcers inhibit responding in both components. Since R_0 is relatively large in lower-valued components, response rates are affected more in lower-valued components than in higher-valued components when R_0 is varied (Equation 5). According to our analysis, multiple-schedule response bias (c' in Equation 2) also depends on concurrent interactions during components.

That is, if R_{01} is always larger relative to R_1 than is R_{02} relative to R_2 , then response rate P_1 will be consistently low and response bias will result that favors Component 2. One possible source of different rates of R_0 in components is contact with a "preferred" key color during one component versus a less-preferred color during the other. In Equation 7 this is described by the reinforcer-context term, which is always larger for various reinforcer ratios if R_{02} is greater than R_{01} than it is when R_{02} is smaller than R_{01} . This response bias is an inverse function of the ratio of obtained rates of extraneous reinforcers in the two components.

Conclusion

Some empirical problems with Herrnstein's multiple-schedule equation arise from an inconsistency between his proposed mechanism of interaction and the interpretation that k is constant. Moreover, our own data and those from other research provide no supporting evidence for a similar direct-interaction mechanism (our Equation 11) that does not impose this inconsistency. Accordingly, we have accepted Herrnstein's interpretation that k is constant and offered an alternative analysis that assumes no successive dependence of component performances. Specifically, the allocation of k during one component is sensitive to the relative reinforcer rate (R_1 versus R_{01}) obtained during that component but is insensitive to the relative obtained reinforcer rate in an alternated component. Thus the fundamental mechanism determining multiple-schedule performance concerns independent, concurrent interactions during each component, which differentially affect component response rates when the relative rates of reinforcers differ in the two components. The data presented here have demonstrated these interactions, and Figure 6 quantified their contribution to multiple-schedule response and time ratios.

Two major predictions concerning undermatching and bias in multiple schedules are generated by our analysis, and both were confirmed in an experiment that arranged analog R_0 in each multiple-schedule component. Specifically, sensitivity to reinforcer ratios (a') increased as analog R_0 increased, and bias (c') was inversely related to the ratio of obtained R_{01} to R_{02} across components. Both of these

results were closely matched by quantitative predictions of response and time ratios in each of a series of multiple-schedule conditions. In summary, our analysis appears to provide a plausible alternative to the view that multiple-schedule performance is determined by direct successive interactions.

Although much of our description of multiple-schedule performance has been cast in the terms of the generalized matching law (Equation 2), our analysis and data present a serious qualification of that relationship. Specifically, approximations to the linear function (on log-log coordinates) predicted by Equation 2 can be expected only when the relationship between average R_1 and R_2 and average R_{01} and R_{02} remains constant, as in our procedure. The changes in sensitivity (Figure 2) between series of conditions, where changes in R_{01} and R_{02} explicitly varied this relation, indicate that nonlinear functions will obtain if either R_1 and R_2 or R_{01} and R_{02} vary within a series of conditions. This was confirmed by the data from Lobb and Davison (1977) described earlier and cannot be described by either of the free parameters in Equation 2.

Finally, our analysis has depended on the relatively small number of published studies of free-operant multiple-concurrent performance that did not have indeterminate reinforcer ratios in components. These showed that the terms a and b in Equation 6, describing concurrent performances in components, are independent of their successive reinforcer context, the ratio of reinforcers in alternated components. Accordingly, these are not "free" parameters that are estimated to maximize correspondence between predicted and obtained multiple-schedule performance, notwithstanding their post hoc estimation from concurrent performances within components. The matter of successive independence deserves further study, since there are conditions under which it may break down, such as when component stimuli are difficult to discriminate. When the conditions for successive dependence are identified, the interaction mechanism described in Equation 11 may serve as a useful extension of Equation 6 to partial out their contributions to multiple-schedule performance. Within the range of experimental conditions studied to date, however, the assumption of successive independence is justified.

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APPENDIX A

Mean absolute response rates (responses per minute), mean time proportions on the left key, and total reinforcements over the five stable sessions in each condition. *P*, *T*, and *R* represent responses, times, and reinforcements, respectively, and the subscript terms refer to key position (L or R) and stimuli (1 or 2), respectively. Replications are asterisked.

<i>Experiment 1</i>											
<i>Bird</i>	<i>Condition</i>	<i>P_{L1}</i>	<i>P_{R1}</i>	<i>P_{L2}</i>	<i>P_{R2}</i>	<i>T_{L1}</i>	<i>T_{L2}</i>	<i>R_{L1}</i>	<i>R_{R1}</i>	<i>R_{L2}</i>	<i>R_{R2}</i>
A1	1	43.8	31.8	57.6	16.8	.61	.84	29	29	103	31
	2	67.5	23.2	76.4	25.3	.80	.84	47	20	82	24
	3	63.6	16.6	59	17.2	.85	.82	65	20	59	20
	4	77.4	34.8	62.4	31.2	.78	.73	95	24	49	24
	5	85.2	18	41.4	40.2	.85	.60	110	23	19	20
	6	21.7	88.6	46.9	67.1	.22	.39	24	158	102	122
	7	26.4	73.8	36	69.7	.26	.33	44	142	85	151
	8	40.1	61.3	33.2	52.9	.36	.31	60	150	60	102
	9	45.6	53.4	31.2	73.4	.39	.26	88	145	43	146
	10	44.7	64.1	20.1	72	.42	.20	98	151	25	123
A2	1	21	43.2	33.6	88.2	.68	.83	25	26	105	19
	2	55.2	25.2	70.8	21	.75	.81	48	24	82	21
	3	55.6	27.5	54.2	23.6	.78	.78	64	19	54	17
	4	64.7	27.1	54.4	28.3	.80	.74	84	27	46	21
	5	75	21	53.4	27	.81	.69	110	22	27	23
	6	14.4	55.2	39	51.6	.21	.43	21	136	97	138
	7	24.2	57	37.9	53.2	.29	.38	40	151	83	145
	8	25	53.4	25.2	56.9	.35	.30	63	145	58	150
	9	24.8	54.8	18.1	59.8	.35	.22	79	140	45	150
	10	33	51	9.6	76.2	.47	.14	95	131	18	147
A3	1	41.4	79.2	56.4	66.6	.44	.62	23	23	92	25
	2	49.8	66	58.8	73.2	.56	.59	46	23	84	24
	3	57.5	82.9	59.7	83.2	.57	.57	58	24	62	21
	4	53.7	80.6	59.2	80.2	.55	.55	86	22	43	23
	5	62.6	57.9	38.2	85.4	.63	.41	106	24	21	25
	6	29.4	75.7	50.8	78.2	.28	.41	20	138	97	145
	7	24.9	70.9	36.5	78.5	.38	.44	38	132	68	156
	8	28.8	70.2	31.8	75.6	.30	.28	53	125	57	148
	9	33	67.2	25.2	82.2	.38	.28	75	162	40	165
	10	51	83.4	33.6	90	.38	.27	94	135	20	140
A4	1	22.2	36.6	57	29.7	.33	.63	23	23	107	22
	2	44.8	29.3	61.2	32.5	.54	.61	43	24	78	24
	3	34.5	22.8	29.6	26.3	.63	.59	63	21	61	16
	4	47.3	32.7	39.2	36.6	.64	.56	79	23	45	14
	5	63	22.8	30	46.8	.7	.29	106	22	24	23
	6	10.6	49	36.2	41.8	.12	.36	24	150	93	143
	7	18	48.6	29.4	38.4	.24	.27	41	150	72	142
	8	14.4	35.4	21.6	39	.24	.27	53	136	58	139
	9	27.6	48	17.2	51.6	.26	.15	75	160	35	169
	10	45.5	59.6	7.9	42.2	.39	.07	97	144	19	157

Experiment 2											
Bird	Condition	P_{L1}	P_{R1}	P_{L2}	P_{R2}	T_{L1}	T_{L2}	R_{L1}	R_{R1}	R_{L2}	R_{R2}
A5	1	24.6	28.8	33.6	67.2	.42	.37	19	23	95	147
	2	20.4	24.6	32.4	57.4	.48	.40	34	15	83	142
	3	51.6	15	38.2	87.8	.82	.34	66	21	57	119
	4	68.4	12	31.2	99.7	.89	.25	94	19	45	153
	5	54	18	12.6	75	.74	.13	103	20	22	156
	6	14.6	110.2	49.1	38.6	.15	.60	18	148	98	21
	7	28.7	83.2	54.6	36.5	.31	.67	37	148	87	23
	8	28.1	95.8	55.6	37.8	.28	.68	55	141	58	20
	9	41.9	74	52.6	39.8	.39	.58	84	146	42	19
	9*	31	73	48.5	23.8	.37	.73	81	140	37	23
A6	10	38.2	81.1	46.9	42.6	.36	.59	99	144	21	21
	1	36.5	46.4	40.3	64.1	.50	.42	22	22	96	148
	2	63.1	52.4	53.1	69.2	.56	.45	40	24	84	107
	3	54.6	45.1	42.7	59.8	.59	.42	61	25	61	126
	4	69.2	41.8	46.9	73.6	.64	.37	77	18	45	121
	5	73.8	63	49.8	85	.58	.34	102	22	27	154
	5*	90.7	37.3	39.2	73.4	.68	.33	104	25	25	120
	6	34.9	82.7	88.4	29.8	.28	.75	23	133	104	22
	7	36.6	61.8	66	24	.34	.71	44	136	83	19
	8	49.4	69.2	72.2	42.2	.44	.66	64	145	64	22
	9	47.4	58.8	57	23.4	.45	.68	80	144	43	14
	10	51	51	39.6	39.6	.47	.47	101	137	18	22