THE CASE AGAINST ALLEN'S GENERALIZATION OF THE MATCHING LAW

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Allen (1981) claims to have established the formal validity of a power-function generalization of the matching law. This paper argues that Allen's proof is not correct when schedules are presented in pairs and that his initial assumptions are too restrictive when all schedules are simultaneously available.

Key words: generalized matching, power law, functional equation

The matching law was introduced by Herrnstein (1961) to describe choice behavior on concurrent variable-interval (VI) schedules. When there are n schedules, the law can be written as

$$\frac{B_1}{\sum_{i=1}^n B_i} = \frac{R_1}{\sum_{i=1}^n R_i} \quad (1a)$$

or equivalently

$$\frac{B_1}{R_1} = \frac{B_2}{R_2} = \dots = \frac{B_n}{R_n} \quad (1b)$$

where B_i is the number of pecks made per session on Schedule i (e.g., Herrnstein, 1961) or the amount of time per session spent on Schedule i (e.g., Baum & Rachlin, 1969) ($i = 1, 2, \dots, n$) and R_i is the number of reinforcements obtained from Schedule i per session ($i = 1, 2, \dots, n$).

The following equation has been widely used as a generalization of the matching law (Baum, 1974, 1979; de Villiers, 1977; Staddon, 1968):

$$\frac{B_1}{B_2} = b \left(\frac{R_1}{R_2} \right)^s \quad (2)$$

The parameter b is referred to as bias by Baum (1974). It is taken to be an indication of uncontrolled variables that influence choice.

Allen (1981) claims that the exponent s is a mathematical necessity of consistent deviations from matching. This paper presents a case against Allen's argument for a generalized matching equation.

A generalization of the matching law is an equation relating the various B_i and R_i , which reduces to Equation (1) under certain circumstances. Thus, Equation (2) is a generalization, in that it reduces to matching when $a = b = 1$. Allen (1981) arrives at a particular generalization of the matching law by the following argument:

Allen assumes that behavioral allocation does not perfectly conform to matching. He represents this deviation by

$$B_i/B_j = f(R_i/R_j) \quad \text{for all } i, j \quad (3)$$

"The function f is any arbitrary function describing how the ratio of responses might vary from the ratio of reinforcement, but is the same for all possible pairs, i, j " (Allen, 1981, p. 126).

Allen's next step is to write

$$\frac{B_i}{B_j} = \frac{B_i}{B_k} \cdot \frac{B_k}{B_j} \quad \text{for all } i, j, k \quad (4)$$

He then uses Equation (3) to replace the behavior ratios, giving the equation

$$f\left(\frac{R_i}{R_j}\right) = f\left(\frac{R_i}{R_k}\right) \cdot f\left(\frac{R_k}{R_j}\right) \quad (5)$$

Introducing positive constants

$$c = R_i/R_k$$

and

$$d = R_k/R_j$$

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results in the functional equation

$$f(c \cdot d) = f(c) \cdot f(d) \quad (6)$$

Allen gives the most general continuous solution to Equation (6) as

$$f(x) = x^s \quad (7)$$

From this, Allen concludes:

Thus if behavior *tends* to follow the matching law but varies from this relation in some consistent way, then it follows that deviations from this law must be captured by a power function; and thus it is shown that the power function equation is formally a generalization of the matching law. (p. 126)

The first point to note is that, as Allen himself says, his argument gives the exponent s but no bias term b . In fact, his argument is incompatible with any value of b other than one in equation (2). The assumption that the same function f holds for all i, j [see Equation (3) above] rules out asymmetries. In effect, deviations from matching must be invariant to relabeling of the choice alternatives. This is a strong condition, leading to a functional equation in its own right. It also has other drawbacks. When the animal is faced with n alternatives, Equation (3) assumes that deviations from matching for a pair of alternatives depends on just their reinforcement rates. This need not be so. A less restrictive equation would be

$$\frac{B_1}{\sum B_i} = f \frac{R_1}{\sum R_i} \quad (8)$$

Note that as soon as failure to match is considered, formulations based on Equations (1a) and (1b) need not be equivalent. Thus, using Equation (8),

$$\frac{B_1}{B_2} = f \left(\frac{R_1}{\sum R_i} \right) / f \left(\frac{R_2}{\sum R_i} \right),$$

which may not reduce to a simple function of R_1/R_2 .

When there are only two alternatives, Equation (3) is a bit more reasonable, although still fairly restrictive. But the two-choice case has problems of its own. Equation (4) is only a tautology when all the alternatives are present simultaneously. When, as is usually the case, the alternatives are presented in pairs, the equation may not hold. To see why, let $B_i(i, j)$ denote the behavior on Schedule i when

the choices are Schedules i and j . Equation (4) can now be written as

$$\frac{B_i(i, j)}{B_j(i, j)} = \frac{B_i(i, k)}{B_k(i, k)} \cdot \frac{B_k(j, k)}{B_j(j, k)} \quad (9)$$

Or, in words, the ratio of behavioral allocation when the alternatives are i and j is given by the ratio when they are i and k times the ratio when they are j and k .

[Equation (9) is formally-identical to a consequence of Luce's choice axiom—see Luce (1959), Theorem 2, pp. 16-17].

It cannot be assumed that Equation (9) will always hold; it is an empirically testable assumption, not an identity.

Allen claims to have formally validated the equation

$$\frac{B_1}{B_2} = \left(\frac{R_1}{R_2} \right)^s$$

as the generalization of the matching law. I am unconvinced by this claim. It is true that when $n > 3$ schedules are simultaneously present, Allen's argument is formally valid. I would argue, however, that in such circumstances Equation (3) is not acceptable, in that it does not include all the n sources of reinforcement. When $n = 2$, Equation (2) may be more acceptable (although it still excludes bias and the possibility of deviations based on terms just in R_1 or just in R_2) but now Equation (4) is no longer an identity and the argument breaks down.

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