FUNCTIONAL CLASSES AND EQUIVALENCE RELATIONS

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Three adult subjects were taught a set of two-choice simultaneous discriminations, with three positive and three negative stimuli; all possible combinations of positive and negative stimuli yielded nine different pairs. The discriminations were repeatedly reversed and rereversed, the former positive stimuli becoming negative and the former negative stimuli becoming positive. With all subjects, a reversal of the contingencies for one pair of stimuli became sufficient to change their responses to all of the other pairs. The reversals had produced functional stimulus classes. Then, all subjects showed conditional discriminations emerging between members of a functional class; given a sample from one class and comparisons from both classes, they selected the comparison that was in the same class as the sample. Next, 2 of the subjects showed that the within-class conditional relations possessed the symmetric and transitive properties of equivalence relations; after having been taught to relate new stimuli to existing class members, the subjects then matched other class members to the new stimuli. Subsequent tests of two-choice discriminations showed that the conditional discriminations had transferred functional class membership to the new stimuli. The 3rd subject, who did not show equivalence relations among functional class members, was also found to have lost the within-class conditional relations after the equivalence tests.

Key words: equivalence relations, functional equivalence, conditional discrimination, discrimination, matching to sample, humans

After learning arbitrary conditional discriminations, human subjects demonstrate new conditional relations that they were never taught explicitly (e.g., Sidman, 1971; Spradlin, Cotter, & Baxley, 1973). These emergent conditional discriminations provide behavioral definitions of the three formal properties of equivalence relations: reflexivity, symmetry, and transitivity (e.g., Sidman, in press; Sidman, Kirk, & Willson-Morris, 1985; Sidman & Tailby, 1982).

So far, studies of equivalence relations have used conditional discriminations to test for reflexivity, symmetry, and transitivity. Vaughan (1988) has suggested another way to view and to test for equivalence classes. He first taught pigeons to peck at any of a set of positive stimuli and to refrain from pecking at any of a set of negative stimuli. Then, after repeatedly reversing the discriminations, he observed the pigeons changing their responses to all of the

stimuli after experiencing the reversed contingency with just a few.

This result indicated that the pigeons had partitioned the stimuli into two subsets. Mathematically (with some qualifications), such a partition implies equivalence. The behavioral analogue of the partition is the functional class. With respect to the behavior that stimuli in the class control in common, the stimuli are substitutable for each other (Goldiamond, 1966). In Vaughan's experiment, when a few stimuli in one set became discriminative for pecking and a few in the other set became discriminative for not pecking, responses to the rest of the stimuli in each set also changed appropriately. The discrimination reversals had generated two functional classes.

What is not yet clear, however, is whether functional classes (identified by their members' common behavioral functions) and equivalence classes (identified when relations among their members meet the three defining features of equivalence relations) are behaviorally the same. As Vaughan (1988) pointed out, "Mathematically, an equivalence relation and a partition are two ways of looking at the same mathematical structure. It remains to be seen to what extent behavioral analogues share that literal structure" (p. 42).

A reasonable first approach to this question

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Fig. 1. The upper section shows the stimuli used with Subject DJK and the Greek letter names used to identify them in the text. UC and LC indicate upper- and lower-case. Class A stimuli are in the first row and Class B stimuli in the second. The lower section shows the numerals used as stimuli with Subjects PJV and JDB; one class contained odd numbers and the other, even.

would be to inquire whether members of equivalence classes also form functional classes. This question, which can be asked only when subjects form equivalence classes, has been answered in the affirmative with humans (Lazar, 1977; Lazar & Kotlarchyk, 1986; Mackay, 1985; Mackay & Sidman, 1984; Silverman, Anderson, Marshall, & Baer, 1986; Wulfert & Hayes, 1988). The present study asks the opposite question: Do relations among members of a functional class meet the reflexivity, symmetry, and transitivity criteria that define equivalence relations? Although Vaughan's reversal technique permits this to be asked with nonhumans, the present experiments used human subjects.

METHOD

The basic methodology was the same for all subjects. Differences will be noted in conjunction with the presentation of results. The experimental plan was as follows:

1. Teach the subjects several two-choice simultaneous discriminations. Continue reversing these discriminations until the subjects make an error just at the beginning of each reversal. This will establish the two functional classes.

2. Determine whether the subjects will match members of a functional class to each other.

3. If conditional relations between functional class members do emerge, teach the subjects conditional relations between some of the

functional-class members and new stimuli. Test the relations between other functional-class members and the new stimuli for equivalence.

4. Return to the two-choice discrimination procedure, with stimulus pairs that include the new stimuli, and test the functional-class membership of the new stimuli. Did the new stimuli join the same functional classes as the stimuli to which they were shown to be equivalent?

Subjects

Subject DJK was female, 22 years old, and a student in speech pathology. Subjects PJV and JDB, males in their upper teens, were students at the New England Center for Autism. Although Subjects PJV and JDB were quite skilled socially and verbally, the concepts *odd* and *even* were unfamiliar to them—a necessary condition, because odd numbers were to constitute one class and even numbers the other class. Before the experiment, these subjects were asked several times whether numbers to be used were odd or even, and to select the odd and even members of number pairs; their replies and selections were inconsistent. After the experiment, Subject PJV was taught the concepts, but only with great difficulty; attempts to teach Subject JDB have so far been unsuccessful.

Apparatus

The equipment has been described in detail elsewhere (Bush, Sidman, & de Rose, 1989). In brief, a computer monitor displayed five keys; the key in the center had a key above, below, to the left, and to the right of it. Sample stimuli always appeared in the center, and comparisons appeared in three of the surrounding keys. The position of the blank key varied from trial to trial. Figure 1 illustrates the stimuli, which were computer-generated forms that resembled Greek letters (used with Subject DJK) or numerals (used with Subjects PJV and JDB). When subjects touched a transparent screen mounted over the face of the monitor, the computer recorded the location of the touch.

Procedures

Reinforcement. The monitor continuously displayed the number of points a subject had earned. A "beep" sounded along with each addition to the point counter. At the end of every session, points were exchanged for money

at the rate of 2 pennies per point for Subject DJK and 1 penny per 15 points for Subject PJV. The beeps (and perhaps the points) sufficed to keep Subject JDB working at high levels of accuracy. At the end of every session, the 2 boys were also given 50 cents to spend immediately in a candy/snack dispenser.

Instructions to subjects. When subjects first sat at the terminal all five keys were displayed, but the center key was blank and only one outer key contained a stimulus. Subjects were told only, "Touch it." Eventually, all subjects touched the key that contained a stimulus, producing the beep, the first point on the counter, and another stimulus on another key. The counter was then brought to their attention; they were told that sometimes a point would be added, the monetary value (if any) of the points, and that the money (if any) would be paid at the end of the session. Then, they were once again told, "Touch it." They were occasionally asked, "How many points do you have?" Because tests were carried out in extinction (see below), subjects were told before each test, "This time you will not get any beeps or points, but you can make up the points on some easy ones later."

Preteaching. The customary backchaining procedures (e.g., Bush et al., 1989), using stimuli that differed from those to be used later, taught the subjects the conditional-discrimination procedures. If the subject touched a correct comparison and reinforcement was scheduled (see Tests in Extinction, below), the sample and comparison stimuli disappeared, the computer beeped and added a point to the reinforcement counter, and a 0.68-s intertrial interval began. Neither beeps nor points followed errors, and the intertrial interval was the same as on correct trials. The procedure was noncorrection.

Standard learning criteria. Each combination of sample and comparison stimuli was defined as a trial type. For example, one trial type might have the numeral 1 as a sample and the numerals 2 and 3 as comparisons; another trial type might have the same sample with the comparisons 2 and 5; another might have 3 as the sample with 2 and 5 as comparisons. Trial types in simple discriminations (see below) had no sample; stimuli appeared only on two comparison keys. The definition of a trial type did not include a specification of key location.

A block of trials included one occurrence of every trial type that was required in a partic-

ular experimental phase (see Results), and consecutive blocks contained different sequences of trial types and correct keys. No trial type was repeated on consecutive trials; no key was correct on consecutive trials. (At the beginning of the experiment, when there were only two trial types, no trial type could occur on more than three consecutive trials.) Subjects completed a preteaching or teaching phase by meeting the following criteria: an overall accuracy of at least 95% and no trial type with more than one error during six consecutive trial blocks. Occasionally, if the experimenters were unsure of the stability of a subject's performance, the number of criterion trial blocks was increased.

Delayed-cue procedure. Each new discrimination was taught by a variant of the delayed-cue procedure (Touchette, 1971). During the first block of trials, all incorrect comparisons disappeared after 0.1 s, leaving the correct key obvious. After every errorless trial block, a longer interval elapsed before the incorrect comparisons disappeared. Eventually, subjects began to select the correct comparison while the incorrect stimuli were still present. Once subjects had become familiar with the delayed-cue procedure during preteaching, they learned subsequent discriminations nearly errorlessly.

Tests in extinction. Test trials that assessed emergent conditional discriminations were inserted as probes among baseline trials of conditional discriminations that the subject had been taught explicitly. In tests, however, neither beeps nor points followed any trials, baseline or probe; no differential consequences followed correct or incorrect choices; baseline and probe trials were always mixed in unpredictable sequences. After each test, subjects were given enough trials of simple auditory-visual and visual-visual number and number-name matching to make up for the points they had missed.

Before the first test with probe trials, baseline conditional discriminations that had been taught explicitly were tested without beeps or points, using the same number of unreinforced trials that subsequent tests would require. Subjects had to meet the standard accuracy criteria.

Simple discriminations. After preteaching, subjects were taught the first of three two-choice simultaneous discriminations. The trials used no samples. Stimuli appeared simultaneously on two comparison keys that varied in

position from trial to trial; blank keys were nonfunctional. One stimulus was from the group designated as Class A (for Subject DJK) or Odd (for the other subjects), and the other stimulus was from the Class B or Even group (Figure 1). Later, when the second and third simple discriminations were added, all stimuli designated as one class were positive, and all stimuli in the other class were negative. Touching the correct stimulus—one from the group designated as the positive class—produced a beep and a point. Reinforcement, the delayed-cue procedure, trial sequence restrictions, error specifications, and the standard learning criteria were the same as in conditional-discrimination trials. Subjects had no problem when presented with simple discriminations.

Discrimination reversals. When subjects met the standard learning criteria, the first discrimination was reversed and rereversed until subjects met the reversal accuracy criteria (see below). Then, a second two-choice discrimination was taught, after which the subjects were given all four possible combinations of positive and negative stimuli from the first two discriminations. A second series of discrimination reversals was then carried out with these four trial types and their reversed counterparts. Then, a third two-choice discrimination was taught, after which the subjects were given all nine possible combinations of positive and negative stimuli from the three discriminations. A third series of discrimination reversals was then carried out with these nine trial types and their reversed counterparts.

Reversal accuracy criteria. The subject's selections of stimuli from the group designated as the positive class continued to be reinforced until the subject met the standard learning criteria, described above. The contingencies were then reversed, with the other group being designated as the positive class. Contingencies continued to be reversed until the subject met the following additional criteria on at least three consecutive reversals: First, no more than one error could be made in the first block of trials; this gave the subject one opportunity to find out or to confirm that the contingencies had been reversed. Second, no more than one subsequent error could occur; this permitted one "error of inattention" after the first block.

After meeting the standard learning criteria, the subject moved to another seat while the experimenter selected parameters for the next

set of trials. During reversal phases, stimuli accompanying this change of seating position permitted subjects to anticipate reversals of the contingencies.

RESULTS

Functional Stimulus-Class Training

The first question was: Will all stimuli correlated with the same consequence (reinforcement or nonreinforcement) become members of a functional class? A replication of Vaughan's (1988) findings would pave the way for an investigation of relations between functional and equivalence classes.

The experimenters classified the stimuli (Figure 1) into subsets; stimuli within a subset always functioned together as positive or negative. For Subject DJK, the Greek letters were divided into Class A and Class B; for Subjects PJV and JDB, the numerals were classed as odd or even. Class A or odd stimuli were positive whenever new discriminations were introduced. Table 1 shows the discriminations and reversals that the subjects learned in each phase of functional stimulus-class training.

With the delayed-cue procedure, the subjects learned each new discrimination quickly and usually errorlessly. After learning the first discrimination in Phase I, they all met the reversal accuracy criteria with few errors, and those all occurred on the first trial after a reversal. Subject DJK made no errors in the final two reversals.

After subjects learned the second discrimination, positive and negative stimuli from the two discriminations were combined; Phase II in Table 1 shows all four stimulus pairs in which Class A letters or odd numbers were positive and the four pairs in which the Class B letters or even numbers were positive. Subjects DJK and PJV again met the reversal criteria almost errorlessly, often without an error even after a reversal. Subject JDB, however, after only two errors on the first reversal (one on the first trial), began to make many errors, sometimes as many as 15, before meeting the learning criteria in the next series of Phase II reversals. After 17 reversals, however, his performance became more accurate, and three of the final four reversals in Phase II were errorless.

After the subjects learned the third discrimination, the three discriminations were combined; each block of trials contained all nine

Table 1

The discriminations taught to Subject DJK (Greek letters) and to Subjects PJV and JDB (numerals). In original discriminations, Class A letters or odd numbers were positive; in reversals, Class B letters or even numbers were positive. Greek letters are designated by their names (some abbreviated—see Figure 1); LC and UC indicate lower- and upper-case. Plus and minus signs indicate the positive and negative stimulus in each pair. Stimuli within a class functioned together as positive or negative. When discriminations were combined (Phases II and III), stimulus pairs were presented in mixed orders.

Phase	Subject DJK		Subjects PJV and JDB	
	Class A/Class B	Class B/Class A	Odd/Even	Even/Odd
I	*LCdel +/LCgam –	LCgam +/LCdel –	*1 +/2 –	2 +/1 –
II	*UCphi +/UCpi –	UCpi +/UCphi –	*7 +/4 –	4 +/7 –
	UCphi +/LCgam –	UCpi +/LCdel –	7 +/2 –	4 +/1 –
	LCdel +/UCpi –	LCgam +/UCphi –	1 +/4 –	2 +/7 –
	LCdel +/LCgam –	LCgam +/LCdel –	1 +/2 –	2 +/1 –
III	*UCsig +/UCxi –	UCxi +/UCsig –	*3 +/8 –	8 +/3 –
	UCsig +/UCpi –	UCxi +/UCphi –	3 +/4 –	8 +/7 –
	UCsig +/LCgam –	UCxi +/LCdel –	3 +/2 –	8 +/1 –
	UCphi +/UCxi –	UCpi +/UCsig –	7 +/8 –	4 +/3 –
	UCphi +/UCpi –	UCpi +/UCphi –	7 +/4 –	4 +/7 –
	UCphi +/LCgam –	UCpi +/LCdel –	7 +/2 –	4 +/1 –
	LCdel +/UCxi –	LCgam +/UCsig –	1 +/8 –	2 +/3 –
	LCdel +/UCpi –	LCgam +/UCphi –	1 +/4 –	2 +/7 –
	LCdel +/LCgam –	LCgam +/LCdel –	1 +/2 –	2 +/1 –

* New discrimination.

stimulus pairs in which Class A or odd stimuli were positive (Table 1, Phase III). Subject DJK made five errors in the first block of combined trials, and one subsequent error, before meeting the learning criteria. Then, she made errors only on Trials 1 and 8 in the first reversal; the next three were errorless. At the start of the next session, the same set of contingencies prevailed (no reversal), and she made errors on Trials 1 and 2. Given one final reversal, she made no errors.

Subject PJV made no errors when the three discriminations were combined. He then made an error on Trial 1 of the first reversal and few errors on the final three reversals, none in the first block of nine trials. Subject JDB made three errors when the three discriminations were combined into nine trial types, but he then went on to meet the reversal criteria in Phase III nearly errorlessly.

With each reversal, the subjects shifted their selections to stimuli from the other class, sometimes after an error on the first trial, and often with no errors. They continued to select members of the class that Trial 1 had shown to be positive (or to reject members of the class that Trial 1 had shown to be negative). Because a reversal of the contingencies for one pair of stimuli became sufficient to change their re-

sponse to all the other pairs, the subjects could be said to have partitioned the stimuli, Greek letters or numerals, into functional classes.

After Phase I, the subjects could not have met the reversal accuracy criteria unless the functional classes had formed. They therefore demonstrated functional classes as early as Phase II, which involved only two positive and two negative stimuli—four trial types.

By making no errors even on Trial 1 in the final reversals of Phase II and III, the subjects also showed that they had learned to anticipate the contingency changes; the reversals themselves had come under the conditional control of “between-run” stimuli (changing of seats, etc.). This conditional control over the contingency reversals does not differ in principle from conditional control exerted by the first unreinforced trial; both gave the subjects the same “instruction.” If functional classes based on the reinforcement contingency had not formed, no conditional control by between-run stimuli could have developed.

Conditional Relations Within Functional Classes

The next tests determined whether the subjects would match functional class members to

Table 2

Subject DJK. Section I shows the functional classes that were established during discrimination reversals, and, in parentheses, the stimuli that were to be added to each class later. In Sections II and III, stimuli are identified by class name and stimulus number, as designated in Section I (for example, LCdel is A1, UCxi is B3, etc.). Section II shows the trial types in the tests for conditional relations within functional classes. Sections III and IV show the trial types in equivalence tests after new stimuli were related conditionally to functional class members (Test 1) and to each other (Test 2).

I. Functional classes			II. Conditional discriminations within functional classes				
Stimulus	Class A	Class B	Sample	Baseline comparisons		Probe comparisons	
				Correct	Incorrect	Correct	Incorrect
1	LCdel	LCgam	A1	A1	B1	A2	B2
2	UCphi	UCpi	B1	B1	A1	B2	A2
3	UCsig	UCxi	A2	A2	B2	A1	B1
(4)	(LCsig)	(UCdel)	B2	B2	A2	B1	A1
(5)	(UCome)	(LClam)					

III. Equivalence Test 1						IV. Equivalence Test 2					
New baseline			Probe			New baseline			Probe		
Comparisons			Comparisons			Comparisons			Comparisons		
Sample	Correct	Incorrect	Sample	Correct	Incorrect	Sample	Correct	Incorrect	Sample	Correct	Incorrect
A4	A1	B1	A3	A4	B4	A5	A4	B4	A2	A5	B5
B4	B1	A1	B3	B4	A4	B5	B4	A4	B2	B5	A5
						A4	A1	B1			
						B4	B1	A1			

each other. In the previous simple discriminations and reversals, the subjects had been selecting members of the same class on virtually every trial. Now, if they were suddenly tested for conditional discriminations without reinforcement to guide their choices, they could be expected to continue selecting comparison stimuli from just one class—ignoring the samples. To prevent the subjects from treating the probes as simple rather than conditional discriminations, a baseline of reinforced identity matching was established first, with the stimuli being the same functional class members that were to be used in the tests. All subjects showed themselves immediately capable of identity matching, thereby demonstrating reflexivity.

Section I of Table 2 shows the three stimuli in each functional class that the discrimination reversals had established for Subject DJK (and two stimuli that were to be added to each class later). In Sections II, III, and IV of Table 2, stimuli are denoted by their class and their number, as specified in Section I of the table (e.g., LCdel is A1, LCgam is B1, UCsig is A3, etc.).

Section II of Table 2 shows the trial types

in the test for conditional discriminations within the functional classes. Only two stimuli from each class were used, leaving the others available for later equivalence tests. The left column lists the four sample stimuli, and the next two columns show the comparisons in identity-matching baseline trials. The two right columns show the probe comparisons. Probe trials tested symmetric relations between Stimulus 1 and Stimulus 2 within each class; the samples in one pair of probe trial types functioned as comparisons in the other pair.

Subject DJK made no “errors” in the 72-trial test (nine presentations of each baseline and probe trial type), always selecting a comparison that was in the same functional class as the sample. Such selections were arbitrarily called “correct.” When tested, therefore, conditional relations emerged between functional class members.

Section I of Table 3 shows the three numerals in each functional class that the discrimination reversals had established (and two numerals that were to be added to each class later) for Subjects PJV and JDB. Section II shows the identity baseline trial types and the

Table 3

Subjects PJV and JDB. Section I shows the functional classes (odd and even numerals) that were established during discrimination reversals, and, in parentheses, the numerals that were to be added to each class later. Section II shows the trial types in the tests for conditional relations within functional classes. Sections III and IV show the trial types in equivalence tests after new stimuli were related conditionally to functional class members (Test 1) and to each other (Test 2). Subject JDB did not go beyond Test 1.

I. Functional classes		II. Conditional discriminations within functional classes				
Odd	Even	Sample	Baseline comparisons		Probe comparisons	
			Correct	Incorrect	Correct	Incorrect
1	2	1	1	2	7	4
3	4	2	2	1	4	7
7	8	7	7	4	1	2
(5)	(0)	4	4	7	2	1
(9)	(6)					

III. Equivalence Test 1						IV. Equivalence Test 2					
New baseline			Probe			New baseline			Probe		
Comparisons			Comparisons			Comparisons			Comparisons		
Sample	Correct	Incorrect	Sample	Correct	Incorrect	Sample	Correct	Incorrect	Sample	Correct	Incorrect
5	1	2	3	5	0	9	5	0	7	9	6
0	2	1	8	0	5	6	0	5	4	6	9
						5	1	2			
						0	2	1			

probe trial types that tested for emergent conditional discriminations within the functional classes. Again, only two stimuli from each class were used.

Subject PJV made no baseline errors and only two errors in the 36 probe trials. Subject JDB made no baseline errors in his first test, but only 24 of the 36 probe trials were correct. In subsequent tests, however, probe selections became highly accurate and stable. In six consecutive tests, each with 36 probes, Subject JDB scored 24, 30, 30, 34, 35, and 33 correct. By choosing an odd comparison when the sample was odd and an even comparison when the sample was even, Subjects PJV and JDB showed conditional relations emerging between functional class members.

The First Equivalence Test

The emergence of conditional discriminations within functional classes, although a necessary first step, did not suffice to demonstrate equivalence relations among the class members. Given that subjects will match any stimulus in a functional class to any other member of that class, the emergence of symmetric re-

lations was inevitable; equivalence was not needed to account for any of the symmetries. The same may be said of transitivity.

Symmetry and transitivity could be tested, however, by first teaching the subjects conditional relations between class members and new stimuli. These new relations could then be directly tested for equivalence. Therefore, each subject was taught a conditional discrimination in which the samples were new stimuli and the comparisons were original members of the functional classes. Subjects were then tested for the emergence of a conditional discrimination in which the samples were other class members and the comparisons were the new stimuli.

Subject D/JK. The new samples were Stimuli A4 and B4 (LCsig and UCdel, listed in Section I of Table 2). The subject was taught to relate one new sample, A4 (LCsig), to an original Class A member, A1 (LCdel), and the other new sample, B4 (UCdel), to an original Class B member, B1 (LCgam). The left side of Section III in Table 2 shows that the baseline for Equivalence Test 1 consisted of the trial types in the new conditional discrimination.

Fig. 2. Subject DJK. Conditional relations involved in the first equivalence tests. Sample stimuli are enclosed in individual boxes to indicate that they were presented separately; pairs of comparisons are enclosed in the same box to indicate that they were presented together. (The names were not shown to the subjects.) Each arrow points from samples to comparisons. The solid arrow indicates the directly taught conditional discriminations; dashed arrows indicate emergent relations (see text).

In probe trials (Table 2, right side of Section III), the samples were Stimuli A3 (*UCsig*) and B3 (*UCxi*), original class members that had not yet taken part in any conditional discriminations. Comparisons were the new stimuli, A4 and B4, that served as samples in the baseline. During the 36-trial test, Subject DJK made no errors. On the 18 probes, she always selected Stimulus A4 (*LCsig*) when A3 (*UCsig*) was the sample, and B4 (*UCdel*) when B3 (*UCxi*) was the sample.

The diagram in Figure 2 helps to clarify how these results must have come about. As indicated by the upper (solid) arrow in the center, the subject had been explicitly taught to relate the new samples, *LCsig* (A4) and *UCdel* (B4), to functional-class members *LCdel* (A1) and *LCgam* (B1), respectively. The lower (dashed) arrow in the center denotes conditional relations between functional-

class members, *LCdel* and *UCsig* in Class A and *LCgam* and *UCxi* in Class B. Although these particular relations between functional-class members had neither been taught directly nor tested, they can reasonably be assumed on the basis of the positive within-class tests that had used other class members (Table 2, Section II).

If the conditional relations represented by the center arrows were symmetric, as demanded by equivalence, samples could serve as comparisons and comparisons as samples; this would yield the derived relations denoted by the innermost dashed arrows at the left. Transitivity of these derived relations would then bring about the emergent conditional discrimination in the first equivalence test, as indicated by the leftmost arrow. This is termed a test for equivalence, however, rather than transitivity, because it requires the derived symmetry relations, whereas simple transitivity (not tested here, but indicated by the arrow at the right) might be found even if the relations depicted in the center were not symmetric. The conditional relations demonstrating equivalence could have come about only if the relations between class members were symmetric and transitive.

Subjects PJV and JDB. The new samples were the numerals 5 and 0. Subjects PJV and JDB learned to relate one new sample, 5, to an original odd class member 1, and the other new sample, 0, to even class member 2. The new conditional discrimination formed the baseline for Equivalence Test 1 (Table 3, left side of Section III). In the probe trials (Table 3, right side of Section III), class members 3 and 8, not yet used in any conditional discriminations, were samples, and the comparisons were the new stimuli, 5 and 0, that served as samples in the baseline.

During the first 36-trial test, Subject PJV made two errors, both on probes. In a second test, he again made two probe errors. On 89% of the 72 probes, he selected 5 when 3 was the sample and 0 when 8 was the sample.

Figure 3 indicates how equivalence could have brought about the emergence of the untaught conditional discrimination. The uppermost (solid) arrow in the center shows the conditional discrimination that had been explicitly taught, and the lower (dashed) arrow shows the conditional discrimination that was to be expected on the basis of the within-class tests (Table 3, Section II). If these conditional

relations were also equivalence relations, the derived symmetric relations (denoted by the two innermost arrows on the left) would, via transitivity, yield the conditional relations that emerged in the equivalence test (outermost arrow).

After showing the emergent conditional discrimination within the functional classes, Subject JDB differed from the other subjects in his remaining tests. In the first test for equivalence (Table 3, Section III), Subject JDB made 10 errors on the 18 baseline trials and 12 on the 18 probes. A second test produced only a slight improvement in the baseline. The baseline was therefore retaught and was maintained in the next two tests. In the probes, however, the subject showed a complete comparison stimulus preference, selecting 5 whether the sample was 3 or 8.

Subject JDB's remaining data need only be summarized: On subsequent tests, the baseline again deteriorated. The subject started to respond as though the instructions, "No beeps and no points," meant, "Do everything wrong"; his scores approached zero. With beeps and points, however, he rarely made errors. Even when given his familiar number matching without reinforcement, he consistently selected the wrong choice. Correct performances were finally produced by asking him before each choice, "What is the right answer," and then, when he named the correct comparison, telling him, "Touch it." These instructions were gradually faded out, the baseline for the equivalence test was reintroduced, the number of extinction trials per test was gradually increased, and Subject JDB was once again given equivalence tests.

During these tests, he maintained the baseline, and in the first test, made only one error on the probes. In three more tests, however, his probe score deteriorated to 50% and remained at that level for four more tests. Unlike the other subjects, Subject JDB's test for equivalence relations was clearly negative.

At the start of almost every test session, Subject JDB had been given the simple discriminations and a reversal. With the exception of occasional errors on Trial 1, his performance was perfect in 25 repetitions of these reversal tests. A breakdown of the functional classes, therefore, was not responsible for his failure to show equivalence.

If, however, conditional relations no longer existed within the functional classes, then con-

Fig. 3. Subject PJV. Conditional relations involved in the first equivalence test. Sample stimuli are enclosed in individual boxes to indicate that they were presented separately; pairs of comparisons are enclosed in the same box to indicate that they were presented together. Each arrow points from samples to comparisons. The solid arrow indicates the directly taught conditional discriminations; dashed arrows indicate emergent relations (see text).

ditional discriminations that would have shown equivalence could not have been expected to emerge; the relations denoted by the lower dashed arrow in the center of Figure 3 would not have existed. The subject was therefore retested for conditional relations within the classes, with baseline and probe trials as indicated in Table 3, Section II.

All subsequent baseline identity-matching performances were perfect. On 24 probe trials in each of two tests, however, Subject JDB was correct at first on only 15, and then on 12 trials. The two probe trial types that had 7 or 4 as samples were then eliminated, but in 11 more tests with this reduced set of probes, the subject settled down to scores of 50%.

To check on the possibility that some peculiarity of the stimuli was responsible for the negative results, functional-class members 3 and 8 were substituted for 7 and 4 in the probe trials. In the first test for the new conditional

discriminations, the subject scored only 11 correct out of 19 trials (an experimenter error produced a smaller number of trials than was planned), and in the second test, scored only 10 correct out of 24 trials.

Subject JDB, therefore, formed and maintained the functional classes throughout testing, and at first showed conditional relations between class members. After he learned to match new stimuli to class members, however, the new stimuli failed to enter the classes; equivalence relations between class members could not be demonstrated. These negative equivalence tests were consistent with negative retests for conditional relations between class members.

The Second Equivalence Test

For 2 subjects, the first equivalence tests showed that the conditional relation between each new stimulus and one functional-class member was also an equivalence relation. Having become equivalent to one class member, the new stimulus also entered into conditional relations with other class members. To demonstrate this interaction between functional classes and equivalence classes required the establishment of two three-member equivalence classes. A more stringent test was attempted next.

Subject DJK. This subject was taught conditional discriminations in which two more new stimuli, A5 (UCome) and B5 (LClam), were related to the former new stimuli, A4 (LCsig) and B4 (UCdel). These trial types, along with the baseline trial types from the first equivalence test, comprised the baseline for the second equivalence test (Table 2, left side of Section IV).

In probe trials (Table 2, right side of Section IV), the samples were existing class members A2 (UCphi) and B2 (UCpi), which had been used in testing for conditional discriminations within the functional classes (Table 2, Section II). The most recently introduced stimuli, A5 and B5, served as comparisons in probe trials.

In 36 baseline trials, Subject DJK made no errors. With only one exception during the 18 probe trials, Subject DJK always selected A5 (UCome) when A2 (UCphi) was the sample and B5 (LClam) when B2 (UCpi) was the sample.

Figure 4 illustrates the directly taught and

derived relations that made this outcome possible. The upper solid arrow in the center shows the conditional relations that Subject DJK had been most recently taught (Table II, Section IV), with the newest stimuli (A5 and B5) as samples and the former new stimuli (A4 and B4) as comparisons. The solid arrow below shows the relations she had learned in preparation for the first equivalence test (Table 2, Section III), with the first pair of new stimuli (A4 and B4) as samples and existing class members (A1 and B1) as comparisons. The lowest (dashed) arrow in the center denotes conditional relations between existing class members that were never explicitly taught but were demonstrated in the earlier within-class tests (Table 2, Section II). The dashed arrows at the left of Figure 4 denote the three symmetric counterparts of the relations depicted in the center. On the right side, the two innermost dashed arrows denote the derived equivalence relations that each pair of symmetric relations would make possible. If the conditional discriminations had indeed generated these equivalence relations, their combination would yield the results observed in the second equivalence test, indicated by the dashed arrow at the far right. This more stringent test required three-member classes of equivalent stimuli to combine into four-member classes, each with two nodes (Fields & Verhave, 1987; Fields, Verhave, & Fath, 1984).

In the second equivalence test, the newest stimuli, A5 (UCome) and B5 (LClam), entered the functional classes because their explicitly taught conditional relations with the previous new stimuli, A4 (LCsig) and B4 (UCdel), were also equivalence relations. In turn, Stimuli A4 and B4 had to have joined the classes via equivalence relations with original class members during the first test.

Subject PJV. In preparation for Subject PJV's second equivalence test, he was taught conditional discriminations relating two more new stimuli, 9 and 6, to the first new stimuli, 5 and 0. The newest trial types, along with those that had been introduced as the baseline for the first equivalence test, comprised the baseline for the second equivalence test (Table 3, left side of Section IV). Samples in the probe trials (Table 3, right side of Section IV) were original class members 7 and 4, which had been used in testing for conditional discriminations within the functional classes (Table 3,

Fig. 4. Subject DJK. Conditional relations involved in the second equivalence test. Sample stimuli are enclosed in individual boxes to indicate that they were presented separately; pairs of comparisons are enclosed in the same box to indicate that they were presented together. (The names were not shown to the subjects.) Each arrow points from samples to comparisons. The solid arrows indicate directly taught conditional discriminations; dashed arrows indicate emergent relations (see text).

Section II). Comparisons in the probes were the most recently introduced stimuli, 9 and 6.

In 54 trials, Subject PJV made one error (on a baseline trial). On all 18 probes, he selected 9 when 7 was the sample and 6 when 4 was the sample. Figure 5 illustrates the directly taught and derived relations that made this outcome possible. The center arrows show the conditional relations the subject had either been explicitly taught (solid arrows) or (dashed arrow at the bottom center) that he had demonstrated in the within-class tests (Table 3, Section II). Given that the conditional relations were also equivalence relations, their symmetric counterparts (indicated by the dashed arrows at the left) would, via transitivity, give rise to the equivalence relations indicated by the innermost arrows at the right.

Fig. 5. Subject PJV. Conditional relations involved in the second equivalence test. Sample stimuli are enclosed in individual boxes to indicate that they were presented separately; pairs of comparisons are enclosed in the same box to indicate that they were presented together. Each arrow points from samples to comparisons. The solid arrows indicate directly taught conditional discriminations; dashed arrows indicate emergent relations (see text).

These derived relations, each indicating a three-member equivalence class, would, in turn, yield the four-member classes documented in the second equivalence test, as indicated by the dashed arrow at the far right. Again, the new stimuli had apparently entered the functional classes through their conditional relations with existing class members, even when the existing members had gained that status only through conditional discriminations.

Final Functional-Class Test

Not yet directly confirmed was the actual inclusion of the new stimuli in the existing functional classes that had been established on the basis of the differential reinforcement contingencies. Simple discrimination tests would show whether the new stimuli that had been related to functional-class members via conditional discriminations had actually joined the

Table 4

Functional class test. The 25 stimulus pairs presented as simple discriminations and reversals (+ and - indicate positive and negative stimuli) to determine whether the new stimuli had entered the classes. NEW denotes stimulus pairs that contained at least one of the new stimuli.

Subject DJK		Subject PJV	
Class A positive	Class B positive	Odd positive	Even positive
LCdel +/LCgam -	LCgam +/LCdel -	1 +/2 -	2 +/1 -
LCdel +/UCpi -	UCpi +/LCdel -	1 +/4 -	4 +/1 -
LCdel +/UCxi -	UCxi +/LCdel -	1 +/6 -	6 +/1 -
LCdel +/UCdel - (NEW)	UCdel +/LCdel -	1 +/8 -	8 +/1 -
LCdel +/UClam - (NEW)	UClam +/LCdel -	1 +/0 -	0 +/1 -
UCphi +/LCgam -	LCgam +/UCphi -	3 +/2 -	2 +/3 -
UCphi +/UCpi -	UCpi +/UCphi -	3 +/4 -	4 +/3 -
UCphi +/UCxi -	UCxi +/UCphi -	3 +/6 -	6 +/3 -
UCphi +/UCdel - (NEW)	UCdel +/UCphi -	3 +/8 -	8 +/3 -
UCphi +/UClam - (NEW)	UClam +/UCphi -	3 +/0 -	0 +/3 -
UCsig +/LCgam -	LCgam +/UCsig -	5 +/2 -	2 +/5 -
UCsig +/UCpi -	UCpi +/UCsig -	5 +/4 -	4 +/5 -
UCsig +/UCxi -	UCxi +/UCsig -	5 +/6 -	6 +/5 -
UCsig +/UCdel - (NEW)	UCdel +/UCsig -	5 +/8 -	8 +/5 -
UCsig +/UClam - (NEW)	UClam +/UCsig -	5 +/0 -	0 +/5 -
LCsig +/LCgam - (NEW)	LCgam +/LCsig -	7 +/2 -	2 +/7 -
LCsig +/UCpi - (NEW)	UCpi +/LCsig -	7 +/4 -	4 +/7 -
LCsig +/UCxi - (NEW)	UCxi +/LCsig -	7 +/6 -	6 +/7 -
LCsig +/UCdel - (NEW)	UCdel +/LCsig -	7 +/8 -	8 +/7 -
LCsig +/UClam - (NEW)	UClam +/LCsig -	7 +/0 -	0 +/7 -
UCome +/LCgam - (NEW)	LCgam +/UCome -	9 +/2 -	2 +/9 -
UCome +/UCpi - (NEW)	UCpi +/UCome -	9 +/4 -	4 +/9 -
UCome +/UCxi - (NEW)	UCxi +/UCome -	9 +/6 -	6 +/9 -
UCome +/UCdel - (NEW)	UCdel +/UCome -	9 +/8 -	8 +/9 -
UCome +/UClam - (NEW)	UClam +/UCome -	9 +/0 -	0 +/9 -

functional classes. New trial types presented each new stimulus along with each original class member, and with each other, adding 16 stimulus pairs to the nine that had comprised the final phase of functional stimulus-class training (Table 1, Phase III).

The two left columns of Table 4 list all of the Greek-letter stimulus pairs, with Class A or Class B positive, for Subject DJK; the two right columns list the stimulus pairs, with odd or even numbers positive, for Subject PJV. Tests contained 75 trials; three 25-trial blocks presented the stimulus pairs in mixed orders—three presentations of each pair. All responses consistent with the experimentally defined contingencies (Class A or Class B positive; odd or even positive) were reinforced.

On her first 75-trial test, with Class B stimulus positive, Subject DJK made two errors, one on Trial 1 before she had any indication of which class was positive, and one on Trial 3. Neither of these errors involved new stimuli. When the contingencies were reversed, she made one error, but not until Trial 24.

On Subject PJV's first test, with odd numbers positive, he made just one error, on Trial 40. On the reversal, he made two errors. One, on Trial 4, did not involve a new stimulus; the other, on Trial 20, did.

The first pair of new stimuli had been related conditionally to two existing class members, and the second pair of new stimuli had then been related conditionally to the first pair. The simple discrimination tests demonstrated directly that the new stimuli had indeed joined the functional classes. Both subjects, therefore, showed that the conditional discriminations had transferred functional-class membership appropriately to the new stimuli.

GENERAL DISCUSSION

When changes in the contingencies controlled by one pair of stimuli are sufficient to change the subject's behavior with respect to other pairs, classes of functionally equivalent stimuli are demonstrated. When explicitly taught conditional relations give rise to un-

taught conditional discriminations that show the original relations to be reflexive, symmetric, and transitive, classes of stimuli related by equivalence are demonstrated. Do these behavioral analogues of two mathematical ways of looking at equivalence represent the same behavioral process?

With 3 human subjects, successive reversals of a set of two-choice simultaneous discriminations established two functional classes. This systematically replicated Vaughan's (1988) study, which was done with pigeons as subjects. The subjects were then given conditional-discrimination tests for equivalence classes.

For 2 subjects, members of the functional classes did prove to be related by equivalence. The tests given to these subjects, and the results, illustrate the kinds of investigation that are needed to ascertain whether relations between members of a functional class meet the defining criteria for equivalence relations. Related studies are those of de Rose, McIlvane, Dube, Galpin, and Stoddard (1988) and de Rose, McIlvane, Dube, and Stoddard (1988), who showed that conditional relations between a positive discriminative stimulus and a neutral stimulus transferred the discriminative function to the formerly neutral stimulus. In those studies, however, the existence of functional classes had to be inferred from the transfer test.

Studies similar to the present one, but with nonhuman subjects, will be required to help answer the theoretical question of where equivalence relations come from (Sidman, in press). It has been suggested that linguistic competence may be required for subjects to demonstrate the formal properties of equivalence (Devany, Hayes, & Nelson, 1986).² On the other hand, it has also been suggested that equivalence relations may underlie some aspects of language (Sidman, 1986, in press). This problem may never receive a satisfactory resolution. For example, would the absence of language explain a subject's failure to show equivalence, or would that failure help explain the absence of language? A demonstration that nonhumans can form equivalence relations would, however, settle the issue; it would be

clear, then, that equivalence relations do not require language.

Nonhuman subjects have not yet shown the emergent conditional discriminations that would demonstrate equivalence relations (e.g., D'Amato, Salmon, Loukas, & Tomie, 1985; Lipkens, Kop, & Matthijs, 1988; Sidman et al., 1982). (In the one seeming exception, reported by McIntire, Cleary, & Thompson, 1987, the "emergent" performances had actually been taught directly to the subjects; this provocative study, however, deserves to be followed up.) A continued search with nonhuman subjects may yet provide the key to the problem of which is primary, equivalence or language. Studies like those of de Rose et al. (1988) and the ones reported here remain to be done with nonhumans. Only then will it be known whether functional classes formed by nonhumans share properties in common with equivalence relations. Positive outcomes of such tests would prove language to be unnecessary for the formation of equivalence relations.

The 3rd subject in the present experiment formed functional classes without being able to demonstrate equivalence relations between class members. Why this subject differed from the others is not known, but the lesson he taught is clear: A set of stimuli partitioned into subsets of functionally equivalent members does not represent the same behavioral process as conditional-discrimination tests for equivalence relations, even with human subjects.

If the two kinds of equivalence need not coexist, it follows that even when they do, a conclusion that they represent the same behavioral process is not justified. Given that the behavioral definitions and the behavioral tests for functional equivalence and for equivalence relations differ drastically, this should not come as a great surprise. It is perhaps more useful to examine the relations between two distinct processes than to attempt to gloss over such obvious differences. It may turn out, for example, that the most important function of equivalence relations is to transfer new stimuli—for example, words—into already existing functional classes.

REFERENCES

- ² See also Lowe, C. F. (1986, May). *The role of verbal behavior in the emergence of equivalence relations*. Paper presented at the meeting of the Association for Behavior Analysis, Milwaukee, WI.
- Bush, K. M., Sidman, M., & de Rose, T. (1989). Contextual control of emergent equivalence relations. *Journal of the Experimental Analysis of Behavior*, 51, 29–45.

- D'Amato, M. R., Salmon, D. P., Loukas, E., & Tomie, A. (1985). Symmetry and transitivity of conditional relations in monkeys (*Cebus apella*) and pigeons (*Columba livia*). *Journal of the Experimental Analysis of Behavior*, **44**, 35-47.
- de Rose, J. C., McIlvane, W. J., Dube, W. V., Galpin, V. C., & Stoddard, L. T. (1988). Emergent simple discrimination established by indirect relation to differential consequences. *Journal of the Experimental Analysis of Behavior*, **50**, 1-20.
- de Rose, J. C., McIlvane, W. J., Dube, W. V., & Stoddard, L. T. (1988). Stimulus class formation and functional equivalence in moderately retarded individuals' conditional discrimination. *Behavioural Processes*, **17**, 167-175.
- Devany, J. M., Hayes, S. C., & Nelson, R. O. (1986). Equivalence class formation in language-able and language-disabled children. *Journal of the Experimental Analysis of Behavior*, **46**, 243-257.
- Fields, L., & Verhave, T. (1987). The structure of equivalence classes. *Journal of the Experimental Analysis of Behavior*, **48**, 317-332.
- Fields, L., Verhave, T., & Fath, S. (1984). Stimulus equivalence and transitive associations: A methodological analysis. *Journal of the Experimental Analysis of Behavior*, **42**, 143-157.
- Goldiamond, I. (1966). Perception, language, and conceptualization rules. In B. Kleinmuntz (Ed.), *Problem solving* (pp. 183-224). New York: Wiley.
- Lazar, R. (1977). Extending sequence-class membership with matching to sample. *Journal of the Experimental Analysis of Behavior*, **27**, 381-392.
- Lazar, R. M., & Kotlarchyk, B. J. (1986). Second-order control of sequence-class equivalences in children. *Behavioural Processes*, **13**, 205-215.
- Lipkens, R., Kop, P. F. M., & Matthijs, W. (1988). A test of symmetry and transitivity in the conditional discrimination performances of pigeons. *Journal of the Experimental Analysis of Behavior*, **49**, 395-409.
- Mackay, H. A. (1985). Stimulus equivalence in rudimentary reading and spelling. *Analysis and Intervention in Developmental Disabilities*, **5**, 373-387.
- Mackay, H. A., & Sidman, M. (1984). Teaching new behavior via equivalence relations. In P. H. Brooks, R. Sperber, & C. McCauley (Eds.), *Learning and cognition in the mentally retarded* (pp. 493-513). Hillsdale, NJ: Erlbaum.
- McIntire, K. D., Cleary, J., & Thompson, T. (1987). Conditional relations by monkeys: Reflexivity, symmetry, and transitivity. *Journal of the Experimental Analysis of Behavior*, **47**, 279-285.
- Sidman, M. (1971). Reading and auditory-visual equivalences. *Journal of Speech and Hearing Research*, **14**, 5-13.
- Sidman, M. (1986). Functional analysis of emergent verbal classes. In T. Thompson & M. D. Zeiler (Eds.), *Analysis and integration of behavioral units* (pp. 213-245). Hillsdale, NJ: Erlbaum.
- Sidman, M. (in press). Equivalence relations: Where do they come from? In D. Blackman & H. Lejeune (Eds.), *Behavior analysis in theory and practice: Contributions and controversies*. Hillsdale, NJ: Erlbaum.
- Sidman, M., Kirk, B., & Willson-Morris, M. (1985). Six-member stimulus classes generated by conditional-discrimination procedures. *Journal of the Experimental Analysis of Behavior*, **43**, 21-42.
- Sidman, M., Rauzin, R., Lazar, R., Cunningham, S., Tailby, W., & Carrigan, P. (1982). A search for symmetry in the conditional discriminations of rhesus monkeys, baboons, and children. *Journal of the Experimental Analysis of Behavior*, **37**, 23-44.
- Sidman, M., & Tailby, W. (1982). Conditional discrimination *vs.* matching to sample: An expansion of the testing paradigm. *Journal of the Experimental Analysis of Behavior*, **37**, 5-22.
- Silverman, K., Anderson, S. R., Marshall, A. M., & Baer, D. M. (1986). Establishing and generalizing audience control of new language repertoires. *Analysis and Intervention in Developmental Disabilities*, **6**, 21-40.
- Spradlin, J. E., Cotter, V. W., & Baxley, N. (1973). Establishing a conditional discrimination without direct training: A study of transfer with retarded adolescents. *American Journal of Mental Deficiency*, **77**, 556-566.
- Touchette, P. E. (1971). Transfer of stimulus control: Measuring the moment of transfer. *Journal of the Experimental Analysis of Behavior*, **15**, 347-354.
- Vaughan, W., Jr. (1988). Formation of equivalence sets in pigeons. *Journal of Experimental Psychology: Animal Behavior Processes*, **14**, 36-42.
- Wulfert, E., & Hayes, S. C. (1988). Transfer of a conditional ordering response through conditional equivalence classes. *Journal of the Experimental Analysis of Behavior*, **50**, 125-144.

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