*THE APPLICATION OF HERRNSTEIN'S LAW OF EFFECT
TO DISRUPTIVE AND ON-TASK BEHAVIOR OF A
RETARDED ADOLESCENT GIRL*

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The purpose of the present study was to evaluate Herrnstein's law of effect as a description of socially significant behavior in an applied setting. The subject was an 18-year-old retarded girl with a history of autistic-like and aggressive behavior. Using a baseline design for two response classes and stimulus conditions, eight categories of subject and staff behavior were monitored over a 3-week period. A computerized observation system, developed for use in the present investigation, was used to obtain real-time durations of the behavior categories alone and in combination. Overlapping durations of teacher and subject behavior were then correlated to yield approximations to a functional definition of reinforcement. Plots of behavior by contingent reinforcement revealed a hyperbolic relationship for each response class, the shape of which varied as a function of extraneous reinforcement r_0 . In addition, estimated parameters in Herrnstein's equation did not differ significantly from those obtained through independent observation. Finally, Herrnstein's equation accounted for an average 63% of variance in response allocation. Results are discussed in terms of the relevance of matching-law theory to behavior in applied settings.

Key words: Herrnstein's equation, matching law, concurrent schedules, time allocation, classroom behavior, retarded adolescents

Herrnstein (1961) described a mathematical relationship, termed the matching law, that relates frequencies of responding to frequencies of reinforcement in a two-choice concurrent-schedule paradigm. According to the matching law, responding will be distributed in the same proportion as obtained reinforcers across response alternatives. In 1970, a mathematically equivalent form of the matching law was proposed to describe the effects of reinforcement on behavior in single-alternative environments (Herrnstein, 1970; McDowell, 1986). As a theory of choice, the single-alternative form of the matching equation states that contingent reinforcement indeed strengthens behavior but does so relative to all reinforcers contacting the subject.

Matching theory has been supported repeatedly in animal research as a description of operant choice within a concurrent-schedule paradigm (e.g., Catania, 1962, 1963, 1968; de Villiers, 1977; Findley, 1958). Perhaps as a

result of this support, there has been increased interest in attempts to apply matching theory to the description of human behavior (McDowell, 1982; Myerson & Hale, 1984). Existing efforts to apply matching theory to human behavior can be classified into three general categories: those that evaluate the theory on mathematical or conceptual grounds (Killeen, 1972; Martens & Witt, 1988; McDowell, 1980, 1982, 1986; Myerson & Hale, 1984); those that demonstrate matching on analogue laboratory tasks (Baum, 1975; Bradshaw, Szabadi, & Bevan, 1976, 1977, 1978; Schroeder & Holland, 1969); and those that demonstrate processes such as behavioral contrast and responding under differential-reinforcement-of-other-behavior (DRO) schedules, which approximate matching in applied settings (Deitz, Repp, & Deitz, 1976; Gross & Drabman, 1981; Parrish, Cataldo, Kolko, Neef, & Egel, 1986; Poling & Ryan, 1982; Repp, Barton, & Brulle, 1983; Simon, Ayllon, & Milan, 1982).

Efforts to evaluate Herrnstein's law of effect in relation to human behavior are attractive to applied behavior analysts for several reasons. First, it is generally acknowledged that reinforcers contingent upon a variety of responses are available concurrently in applied settings

The authors gratefully acknowledge the help of Joseph C. Witt in discussing the conceptual bases of the data collection procedures and Frank M. Gresham for reviewing an earlier draft of the article. Reprints may be obtained from Brian K. Martens, Department of Psychology, Syracuse University, 430 Huntington Hall, Syracuse, New York 13244-2340.

(Myerson & Hale, 1984). Second, by viewing behavior as choice, alternatives arise for maximizing the attractiveness of desired responding over time and across settings, thereby enhancing the generalization of treatment effects (Martens & Witt, 1988; McDowell, 1982). Third, systems-level models for the functional analysis of behavior (e.g., matching theory) have been suggested in the explanation of unanticipated treatment effects (Kazdin, 1982; Parrish et al., 1986; Sajwaj, Twardosz, & Burke, 1972; Voeltz & Evans, 1982). Although empirical support for these models is lacking, they have served as useful heuristics in guiding research (Wahler, 1975; Willems, 1974).

Despite the apparent value of matching theory as a description of behavior, investigation of its parameters in applied settings has been difficult to accomplish. As noted by Fuqua (1984), "Published empirical validations of the matching law with topographically dissimilar responses are noticeably lacking and any attempts at such validation in applied settings will need to grapple with several quantification problems" (p. 382). Chief among these problems are the selection of measures along which topographically different kinds of behavior can be equated (e.g., time allocation) and the development of procedures for identifying various reinforcers contingent upon each of several behavior categories (Herrnstein, 1979).

The purpose of the present study was to evaluate Herrnstein's law of effect as a description of classroom behavior. Toward this goal, a computer-based system for taking real-time durational measures of behavior was developed to obtain independent estimates of the equation parameters. Because response and time allocation are isomorphic in concurrent schedules (Herrnstein, 1979; Myerson & Miezin, 1980), the observation system used in the present study was useful in several respects. First, by using duration as a measure, topographically different response classes were equated along the single dimension of time. Second, the observation system permitted measurement of the degree of overlap among multiple behavior categories (e.g., teacher praise during student seat work). This feature enabled the identification of teacher-behavior categories that occurred during, and were positively correlated with, various student-behavior

categories. The present study therefore used a correlational paradigm to approximate a functional definition of reinforcement in a classroom setting.

METHOD

Subject

The subject was an 18-year, 6-month-old girl enrolled in a class for the moderately mentally retarded in a public high school. Prior to her current placement, she attended a class for profoundly retarded and autistic adolescents. The change in placement was prompted by disruptive and aggressive behavior toward students and staff for which physical restraint was the modal response. The subject was referred to the authors by her present teacher to assist in controlling these aggressive outbursts. The most recent psychoeducational evaluation of the subject indicated a Full Scale IQ of 58 on the Weschler Intelligence Scale for Children (Revised). Academic assessment showed a lack of independent reading skills, but the subject could make change for purchases with use of a calculator.

During the course of the investigation, no aggressive behavior was observed, although one incident requiring physical restraint was reported by staff. In general, the subject's classroom behavior was characterized by attending to instructions and participating in small-group activities (approximately 70% of the time), scanning the room while grimacing, and making repetitive hand movements (e.g., picking sores, waving fingers in front of face). The subject rarely initiated interactions with other students or staff.

Setting

Observations were conducted for 18 to 30 min during the morning in a classroom staffed by one teacher and three teaching assistants. There were 13 other students also present in the room ranging in age from 16 to 20 years, all of whom were classified as moderately mentally retarded with no additional handicapping conditions. The curriculum focused on community-based living and communication skills with on-site training in such jobs as stocking shelves, cleaning, packaging, and providing customer assistance. All students participated in supervised work experiences in the community.

The daily routine began with a morning meeting involving all students and staff. During the meeting, activities for the day were discussed and deductions were made for inappropriate behavior from the previous day's allotment of points. Remaining points could be exchanged for privileges. For the subject, these privileges included taking another student to lunch, participating in field trips or special in-school activities (e.g., cooking, attending wood shop), listening to music through headphones, or eating snacks. Independent of the point system, a response-cost procedure was in effect for the subject in which she lost access to the next available activity reinforcer contingent upon "losing control or becoming aggressive."

Following the morning meeting, the class was divided into small instructional groups of two to three students. The subject participated in group instruction in the mornings, and was typically out of the room on field trips, in special classes, or at on-site work experience after lunch.

While in the classroom, the subject had a staff member assigned to her at all times. During class meetings or instructional group activities, the assigned staff member sat next to the subject with other students separated from them by a table or desk. Occasionally the class sat around a table as a group with a staff member on either side of the subject. During activity transitions, the subject was required to wait until the other students had finished their movement about the classroom before changing location.

Target Behavior

Eight categories were used to represent subject and staff/peer behavior. Two categories were mutually exclusive and exhaustive of subject behavior, five categories were mutually exclusive and exhaustive of assigned staff behavior, and the remaining category was used to represent contact with the subject by other individuals in the classroom.

On-task. The subject category *on-task* included a variety of behavior appropriate for participation in classroom activities. Examples of on-task behavior included instructional interactions with the teacher, independent seat work on assigned tasks, and compliance with teacher requests.

Disruption. The second category of subject

behavior was coded whenever the subject engaged in off-task or disruptive behavior. Specific examples of this category included picking at skin, gesturing or grimacing, off-task use of materials, waving fingers in front of face, and vocal outbursts unrelated to an ongoing activity.

Instruction. This category of staff behavior included verbalizations by the assigned staff person that were of an instructional nature. Providing information to initiate or change a task, questioning or prompting related to an ongoing activity, and questioning to obtain information about a social event were examples of this category.

Praise. Praise was coded whenever the assigned staff person made a positive statement evaluating the subject's behavior.

Reprimand. This category was coded when the assigned staff person made negative statements evaluating the subject's behavior. Providing warnings and stating negative consequences for inappropriate behavior were also coded as reprimands.

Proximity. When the assigned staff person was within 3 ft of the subject but not interacting verbally with the subject or a peer, the category *proximity* was coded. This category was frequently coded between instructional exchanges when the staff person was observing but not interacting with the subject.

Attend others. This category was coded whenever a verbal or nonverbal interaction occurred between the assigned staff person and an individual other than the subject. This behavior was notable as the exception to one-on-one interactions that typically occurred between the subject and assigned staff members.

Nonassigned contact. This category included verbal or nonverbal interaction with the subject by another individual in the classroom other than the assigned staff person. Because it occurred infrequently and while the assigned staff person was attending to individuals other than the subject, this category took precedence over other staff-behavior categories for coding purposes.

Apparatus

Two Zenith® personal computers (model Z-170) with supporting software (Martens, Meller, & Springer, 1987) were used to record real-time durations of student and staff behavior. The computers were self-contained,

battery-packed units standing approximately 24 cm high, 33 cm wide, and 17 cm deep. A keyboard folded away from each unit into the conventional typing position exposing the screen. The observation program displayed a simulated keyboard on the screen in which each key was labeled with the corresponding behavior category. Subject-behavior categories appeared in the top row of keys, and staff-behavior categories appeared in the second row of keys. Pressing a key corresponding to a behavior category activated a feedback prompt on the display and a timer until the key was again pressed to terminate timing. Timing was also terminated when any other key in the same row was activated. Thus, behavior categories within each row were mutually exclusive, although all possible combinations could occur between rows. The observation program recorded durations of each key press and all combinations of simultaneous key presses to the nearest second for each minute of observation.

Observational Procedures

The subject was observed for a total of 276 min in 13 sessions over a 3-week period. Observations were conducted under two stimulus conditions: assignment to the teacher (145 min) and assignment to the aide (131 min). A range of classroom activities was sampled, and observations were conducted only in the classroom.

Before entering the classroom for observation, a starting time and ending time were entered into the computer setting the session length. In the case of reliability observations, starting and ending times were synchronized for the two observers. Upon entering the classroom, observers sat at a table approximately 4 m from the subject. Because the classroom received frequent visitors, staff and students rarely responded to the observers' presence.

Reliability Assessment

Estimates of interobserver reliability were obtained randomly during 36% of the observation sessions for the two subject-behavior categories as well as for all combinations of subject and teacher behavior. The duration of behavior in seconds during consecutive 1-min intervals was used as the unit of measure in computing reliability values. Reliability was assessed using both correlational (Pearson r) and percentage-agreement indices. The former

method has been suggested for use with continuous-trial data, whereas the latter is attractive for its interpretive simplicity (Baer, 1977; Hartmann, 1977). Percentage agreement was calculated by dividing the larger total obtained by either observer by the smaller total and multiplying by 100 (Kelly, 1977). A summary of the reliability estimates obtained for each behavior category is presented in Table 1. As indicated in Table 1, only those behavior categories meeting the .60 criterion for correlational estimates (Gelfand & Hartmann, 1975) or the .80 criterion of percentage agreement were retained for further analyses. These criteria were adopted for all target behavior categories even though teacher categories actually required the joint coding of both teacher and subject behavior. One behavior combination, teacher praise/subject off-task, did not occur during the reliability sessions.

Design

Observations were conducted using a baseline design for two response classes and stimulus conditions (Hayes, 1981). The two stimulus conditions sampled included classroom activities in which either the teacher or the primary aide was assigned to the subject.

RESULTS

Identification of Reinforcers

A reinforcer was defined as any staff or peer behavior that (a) was directed toward the subject, (b) occurred while the subject was engaged in an activity, and (c) was significantly and positively correlated with total duration of that activity. Because it was correlationally based, this definition fell short of identifying the necessary and sufficient conditions of a functional relationship. The sufficient condition for identifying a functional relationship is the experimental manipulation of an independent variable (Johnston & Pennypacker, 1980). Because it lacked this feature, this definition of reinforcement merely approximated a functional definition and probably included relationships between subject and staff behavior beyond those of a reinforcing nature (e.g., staff behavior that served as discriminative stimuli or subject and staff behavior that covaried together with other events). However, by limiting durations of staff behavior only to those that overlapped the response class of interest, gross misrepresentations of potential reinforc-

ers (e.g., staff behavior occurring before the onset of subject behavior) were excluded.

Despite its limitations, the present definition of reinforcement would be satisfied by any true functional relationship when viewed within a correlational framework. The loss in precision therefore was weighed against the definition's ability to capture all possible reinforcing interactions in the setting. In addition, because the loss in precision could be likened to random error, its influence on the data analyses would be detrimental only vis-à-vis matching theory. Because the definition would produce data less likely to fit Herrnstein's hyperbola, it was viewed as adequate for use in the present study. Finally, the definition was adopted because it enabled the most accurate interpretation of behavior-environment interactions with the use of real-time correlational measures (Sanson-Fisher, Poole, Small, & Fleming, 1979).

Because the goal at this stage of analysis was to identify all possible reinforcers rather than to construct a predictive model, zero-order correlation coefficients were computed between staff, student, and peer behavior categories under each stimulus condition. In keeping with the above definition, correlations were obtained between the overall durations of each student behavior and *only* the durations of staff or peer behavior that occurred simultaneously. For example, assume the subject is on-task for 30 consecutive seconds followed by a period of repetitive motor movements. Halfway through the 30 s of on-task behavior, the teacher begins interacting with the subject and stops interacting while the subject is self-stimulating (perhaps to ignore the self-stimulation). The correlation between teacher interaction and subject *on-task* behavior would then be computed using 30 s of total time on-task by the subject and 15 s of teacher interaction *while* the subject was on-task.

It bears mention here that zero-order correlations are based on a linear model. As a result, the circular logic of using the matching theory (which proposes a hyperbolic relationship between behavior and reinforcement) to build parameters for evaluating the theory was avoided in favor of using more conservative linear estimates of association.

Results of the correlational analyses indicated that the same two types of teacher behavior (*instruct* and *proximity*) were identified as reinforcers for both disruptive and on-task student behavior using the present definition.

Table 1

Estimates of reliability for subject and teacher behavior.

Behavior combination		Pearson <i>r</i>	% Total agree- ment
Subject	Teacher		
Off-task	—	.61	96
On-task	—	.58	99
Off-task	Instruct	.67	89
On-task	Instruct	.85	86
Off-task	Praise ^a	—	—
Off-task	Reprimand ^b	-.01	31
Off-task	Proximity	.73	53
Off-task	Attend others ^b	.10	66
Off-task	Nonassigned contact	.82	81
On-task	Praise	.70	51
On-task	Reprimand ^b	.29	47
On-task	Proximity	.78	80
On-task	Attend others	.77	88
On-task	Nonassigned contact ^b	.20	42

^a Nonoccurrence precluded estimates of reliability.

^b Excluded from further analyses.

Thus, on occasion the teacher was observed to interact with the subject while she was being disruptive, thereby potentially reinforcing disruptive behavior. In contrast, the teacher made infrequent use of verbal praise, and praise that was delivered failed to correlate significantly with total duration of student on-task behavior.

Three types of aide behavior (*instruct*, *praise*, and *proximity*) were identified as reinforcers for disruptive student behavior, whereas only one aide behavior (*instruct*) was identified as a reinforcer for being on-task. Although the aide made more frequent use of verbal praise than the teacher, positive statements were often made to the subject while she was engaged in repetitive motor movements. A summary of results from the correlational analyses is presented in Table 2.

Durations of staff behavior identified as reinforcers for each subject-behavior category were summed to yield a single index of contingent reinforcement for that response class (r_1). Because extraneous reinforcement (r_0) is composed of concurrently available as well as noncontingent reinforcement (McDowell, 1982), r_1 values for one response class were used as the independent estimates of r_0 for the other response class. Thus, r_1 for disruptive subject behavior was used as the independent estimate of r_0 for being on-task (and vice versa) for each stimulus condition.

Duration was used as the parameter of rel-

Table 2

Zero-order correlations between staff and student behavior.

Staff behavior	Student behavior	
	Disruption	On-task
Instruct	.17* (.38*)	.29* (.19*)
Praise	-.02 (.36*)	.08 (.04)
Reprimand	.13 (0)	.03 (0)
Proximity	.39* (.33*)	.33* (.14)
Attend others	.83* (.90*)	.31* (.28*)
Nonassigned contact	.05 (.10)	.09 (.06)

Note. Values to the left represent correlations between student behavior and teacher behavior. Values in parentheses were obtained under assignment to the aide.

* $p < .05$.

ative reinforcement for two reasons. First, each category of reinforcer involved teacher-student interaction. It is generally held that extended interactions (e.g., lengthy explanations, arguments) are more potent than brief contacts in maintaining appropriate as well as inappropriate student behavior (e.g., Harris, 1985). Second, although reinforcement relativity may be affected by numerous factors, meaningful comparisons can only be made across values of reinforcement that reflect a common parameter (McDowell, 1986). All categories of reinforcement in the present study could be represented meaningfully in terms of duration.

Prior to fitting Herrnstein's equation, durations of subject behavior and reinforcement in seconds per minute were averaged within 5-s bands of contingent reinforcement for each response class and stimulus condition. Thus, all minutes of data that fell within a 5-s band of contingent reinforcement were combined to yield average values of subject behavior, contingent reinforcement, and extraneous reinforcement. This procedure reduced considerably the scatter present in the momentary data, thereby permitting conventional hyperbolic fits. The mean duration values of reinforcement and behavior used in subsequent analyses for each response class and stimulus condition are presented in the Appendix.

Inspection of Scatterplots

The strict matching equation poses a hyperbolic relationship between behavior and contingent reinforcement (Herrnstein, 1979). The shape of this relationship, or more specifically, the rapidity with which the hyperbola approaches its asymptote (i.e., response rate

Fig. 1. Durations of disruptive and on-task subject behavior plotted against durations of contingent reinforcement when assigned to the teacher and aide. Data points represent averages within 5-s bands of contingent reinforcement. The r_0 values indicate mean durations in seconds per minute of concurrently available reinforcement (obtained through direct observation). The estimated values of r_0 appear in parentheses. The hyperbolic function for each plot was fitted using the estimated values of r_0 .

equals the maximum possible response rate) is a function of r_0 (extraneous reinforcement) (McDowell, 1982). As r_0 increases, the downward concave shape of the hyperbolic relationship becomes less pronounced.

Because values of r_0 in the present investigation were obtained independently of the other equation parameters, it was possible to compare the mean values of extraneous reinforcement with their corresponding scatterplots. Presented in Figure 1 are the plots of average subject behavior by average contingent reinforcement in seconds per minute for each 5-s band of contingent reinforcement. Also presented in Figure 1 are the observed and estimated values of extraneous reinforcement in seconds per minute. As shown in the figure, hyperbolas fitted to the data sets using Wilkinson's method (McDowell, 1981) varied in shape as a function of extraneous reinforcement in the manner predicted by Herrnstein's law of effect.

For example, the plot of disruptive subject

behavior while assigned to the aide had a mean r_0 value of 20.15 s per minute, showing a flat curve relating behavior to contingent reinforcement that did not exceed 40 s per minute of behavior at maximum levels of contingent reinforcement. In contrast, the plot of on-task subject behavior while assigned to the aide had a mean r_0 value of only 1.33 s per minute, showing near maximum response durations (i.e., 60 s per minute) at virtually all values of contingent reinforcement. Thus, as suggested by matching theory, increases in contingent reinforcement were associated with slower rates of increase in subject behavior when values of r_0 were high.

Estimated Versus Obtained Equation Parameters

Herrnstein's equation was fitted to the data sets for each response class and stimulus condition using Wilkinson's method (Davison & McCarthy, 1988; McDowell, 1981). Values of k and r_0 estimated from fitting the equation were then compared to values of the same parameters obtained through direct observation using a t comparison statistic. Presented in Table 3 are obtained and estimated values of the equation parameters, standard errors of estimate and the corresponding t statistics, and percentage of variance accounted for (%VAF) for each response class and stimulus condition. As indicated in this table, the estimated values of k closely approximated the obtained value of 60 s per minute for each response class and stimulus condition (failure to reject the null hypothesis at $\alpha = .05$ in all cases). Using the same criteria, obtained values of r_0 were also approximated by the estimated values for disruptive behavior under both stimulus conditions. In contrast, the estimated value of r_0 for on-task behavior under assignment to the teacher significantly underestimated the obtained value ($\alpha = .05$). Obtained and estimated values of r_0 for on-task behavior under assignment to the aide did not differ significantly at $\alpha = .05$.

Using %VAF as a criterion (McDowell, 1981; Wetherington & Lucas, 1980), Herrnstein's law of effect provided a good fit to disruptive behavior under assignment to the aide (%VAF = 87) and to the teacher (%VAF = 79). To examine the data for significant bias and undermatching, the base 10 logarithms of disruption/on-task behavior ratios were re-

Table 3

Equation parameters and percentage of variance accounted for by response class and stimulus condition.

Parameter	Dis- ruption aide	Dis- ruption teacher	On-task teacher	On-task aide
k obtained	60	60	60	60
k estimated	53.14	69.85	54.36	56.24
Standard error	19.67	10.98	5.29	5.28
t	0.35	0.90	1.07	0.71
df	5	8	11	11
r_0 obtained	20.15	17.61	5.88	1.33
r_0 estimated	19.64	20.97	1.61	-0.07
Standard error	13.03	6.35	1.46	0.87
t	0.04	0.53	2.92*	1.61
df	5	8	11	11
% VAF	86.53	79.46	33.53 ^a	53.63 ^a

^a Following correction for nonzero y intercepts.

* $p < .05$.

gressed linearly on logarithms of disruption/on-task reinforcer ratios for each stimulus condition (Baum, 1974; Myers & Myers, 1977). The resulting slope of .80 ($SD = .09$) and intercept of .07 ($SD = .05$) under assignment to the teacher did not differ significantly from their expected values of one, $t(7) = 2.22$, $p > .05$, or zero, $t(7) = 1.44$, $p > .05$, respectively. Under assignment to the aide, the obtained slope of .68 ($SD = .21$) and intercept of $-.18$ ($SD = .10$) also did not differ significantly from the expected values ($\alpha = .05$) with $t(4) = 1.52$ and $t(4) = 1.80$, respectively.

The elevated values of on-task behavior at what were assessed to be zero durations of contingent reinforcement under both stimulus conditions resulted in poor hyperbolic fits through the origin (%VAF could not be calculated). Correcting for nonzero y intercepts by subtracting a constant (30 for the teacher data and 45 for the aide data) from the values of k and behavior, however, resulted in reasonably good hyperbolic fits to on-task behavior under assignment to the teacher (%VAF = 34) and under assignment to the aide (%VAF = 54). Linear regression analyses using the base 10 logarithms of on-task/disruptive behavior and reinforcer ratios showed significant bias toward time on-task, $t(10) = 4.3$, $p < .05$, and undermatching, $t(10) = 3.0$, $p < .05$, under assignment to the teacher with intercept = .33 ($SD = 0.8$) and slope = .76 ($SD = 0.8$). Under assignment to the aide, the obtained slope of .77 ($SD = .12$) was not significantly different from one, $t(9) = 1.92$, $p > .05$, and

the intercept of .23 ($SD = .18$) was not significantly different from zero, $t(9) = 1.32$, $p > .05$.

DISCUSSION

Results of the present investigation suggest that Herrnstein's (1970) law of effect can be used to describe the relationship between reinforcing teacher attention and classroom behavior. Specifically, Herrnstein's equation accounted for an average 83% of the variance in disruptive behavior and 44% of the variance in on-task behavior following correction for nonzero y intercepts. The correction procedure for on-task behavior prior to computing %VAF values was necessary because of the high levels of time on-task at what were defined to be zero durations of contingent reinforcement. This bias also emerged in the linear regression analyses of log ratios, contributing to the poorer fits of Herrnstein's equation to on-task versus disruptive behavior.

In addition to bias, other characteristics of the data may have contributed to the poorer hyperbolic fits to on-task behavior. First, as shown in Figure 1, the plots of on-task behavior present relatively little systematic variance to be accounted for in the analyses. Moreover, the paucity of variance and resulting lack of curvature in the data produced poorer estimates of r_0 . Second, procedural limits of the study may have contributed to the elevated levels of on-task behavior at what were defined to be zero durations of contingent reinforcement. Specifically, the role of self-reinforcement in the maintenance of behavior could not be evaluated by the present observational system. Given the subject's history of repetitive motor movements, reinforcement in the absence of contingent teacher attention was probably underestimated, thereby reducing the adequacy of hyperbolic fits through the origin.

Although Herrnstein's (1970) equation provided reasonably good predictions of response allocation, two other findings in the present study suggest its appropriateness as a description of classroom behavior. First, plots of behavior by contingent reinforcement revealed a hyperbolic relationship for each response class under both stimulus conditions, the shape of which varied with values of r_0 in the predicted manner. Second, the values for k and r_0 obtained through direct observation were closely

approximated by the estimated values. These findings provide independent verification in an applied setting for the interpretation of k as maximum response level and r_0 as extraneous reinforcement (Herrnstein, 1970; McDowell, 1982).

Results of the present investigation should be interpreted cautiously for several reasons. First, we used a correlational paradigm to identify functional reinforcers. As a result, variability in identified reinforcer durations was left to natural variation in contingent teacher attention (see Houston, 1986, for another use of this approach). In the absence of control over the independent variable, the source of this variation remains unspecified. Although functional reinforcers are captured by the definition used in the present study, teacher attention that was simply contingent on subject behavior may also have been represented. In the unlikely event that *only* contingent, as opposed to reinforcing, attention was included in the analyses, the present results might then describe a feedback function between student and teacher behavior. Student behavior would then be the source of variation in identified reinforcer durations. Because a feedback function of this type would be completely reciprocal, however, the present findings suggest that matching applies to student behavior viewed as a function of teacher behavior, and vice versa.

Second, despite the replication of results across response classes and stimulus conditions, the present investigation used only a single subject. Thus, although support for reliability of the observed relationship was obtained through replication, generality across subjects can only be established with future replications involving other individuals in similar classroom settings (Johnston & Pennypacker, 1980). In addition, data in the present investigation contained zero values of behavior and reinforcement. In reviewing the literature, we found that only nonzero rates of reinforcement had been used in previous attempts to fit Herrnstein's equation to human performance (e.g., Bradshaw et al., 1978). This difference probably contributed to the somewhat lower proportions of variance accounted for in the present investigation (mean %VAF = 63) compared to previous research (e.g., mean %VAF = 98; Bradshaw et al., 1976). Future research is needed that either samples a wider range of

reinforcers or samples reinforcers across longer time intervals to reduce the number of zero values included in the computations.

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Received April 18, 1988

Final acceptance September 6, 1988

APPENDIX

Mean duration values of behavior and reinforcement within consecutive bands of contingent reinforcement for each stimulus condition and response class.

Stimulus condition and response class	Band	Behavior		Reinforcement	
		Disruption	On-task	Disruption	On-task
Teacher/disruption	1	12.93	47.07	0	15.05
	2	15.82	44.18	3.47	23.12
	3	23.55	36.45	7.82	21.27
	4	19.33	40.67	12.83	21.17
	5	20.50	39.50	16.0	36.50
	6	37.50	22.50	24.0	17.50
	7	39.50	20.50	28.0	9.0
	8	43.50	16.50	41.50	14.0
	9	47.50	12.50	47.0	13.0
	10	54.50	5.50	52.0	5.50
Teacher/on-task	1	26.14	33.86	0.86	0
	2	16.27	43.73	7.18	3.27
	3	18.94	41.06	10.94	8.25
	4	17.60	42.40	10.90	12.50
	5	22.75	37.25	14.75	17.75
	6	16.0	44.0	7.17	23.17
	7	9.0	51.0	5.0	27.80
	8	7.0	53.0	6.50	34.0
	9	1.25	58.75	0.75	37.75
	10	7.75	52.25	6.0	42.25
	11	4.50	55.50	4.25	47.75
	12	1.44	58.56	1.33	54.10
	13	0.62	59.38	0.75	58.63
Aide/disruption	1	2.30	57.70	0	21.46
	2	6.81	53.19	3.25	33.75
	3	16.20	43.80	7.20	16.20
	4	16.33	43.67	12.67	13.67
	5	33.0	27.0	19.0	24.0
	6	23.0	37.0	23.0	16.0
	7	32.0	28.0	27.0	16.0
Aide/on-task	1	13.25	46.75	1.17	0
	2	0.82	59.18	0	3.27
	3	5.06	54.94	1.0	8.25
	4	5.69	54.31	2.0	12.56
	5	7.54	52.46	4.31	17.77
	6	5.73	54.27	2.07	22.80
	7	5.60	54.40	2.40	28.40
	8	5.09	54.91	1.27	32.64
	9	0.80	59.20	0.33	37.0
	10	0.25	59.75	0.25	41.75
	11	2.71	57.29	1.29	47.86
	12	1.50	58.50	0.50	53.30
	13	1.33	58.67	0.67	58.67