CONCURRENT VARIABLE-RATIO SCHEDULES: IMPLICATIONS FOR THE GENERALIZED MATCHING LAW

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Rats' responses were reinforced on concurrent variable-ratio variable-ratio schedules in which responses on one lever incremented the ratio counter and responses on a second lever changed the schedule and correlated stimulus. The relative frequency of reinforcement was varied from .10 to .99. In one set of conditions, responding on the main lever incremented both ratio counters, but reinforcement required a response in the presence of the stimulus correlated with the ratio that had been completed. In a second set of conditions, responses on the main lever incremented only the ratio correlated with the stimulus that was currently present. When main-lever responses incremented both ratio counters, subjects distributed responding and time in a manner consistent with the generalized matching law. When responses on the main lever incremented only the schedule currently in effect, the rats responded almost exclusively on the schedule producing the higher frequency of reinforcement. These results extend the applicability of the generalized matching law to dependent ratio schedules.

Key words: generalized matching law, melioration, variable-ratio schedule, concurrent schedule, momentary maximizing, Findley procedure, lever press, rats

In concurrent schedules in which two responses are reinforced according to variable-interval schedules, the ratio of the response frequencies are systematically affected by changes in the ratio of reinforcement frequencies produced by each of the two responses (Baum, 1974). This, the generalized matching law, has been expressed by Baum as

$$\log(B_1/B_2) = a \log(R_1/R_2) + \log b \quad (1)$$

where B_i represents responding or time spent in Schedule i , and R_i represents reinforcers obtained in each schedule. The parameters b and a have been understood by Baum to represent, respectively, bias toward Schedule 1 and the sensitivity of the organism to changes in the distribution of reinforcements. On linear coordinates, this function plots as a straight line with a slope equal to a and y intercept

equal to b . Behavior consistent with Equation 1 is not expected to be found with concurrent variable-ratio variable-ratio (VR VR) schedules (Herrnstein, 1970), except in a trivial sense. Animals usually respond almost exclusively on the lever producing the higher frequency of reinforcement, and thereby obtain virtually all reinforcers from that lever (Herrnstein, 1970; Herrnstein & Loveland, 1975). In these procedures responses on a lever count only towards the ratio requirement correlated with that lever.

An alternate method of scheduling reinforcers on a concurrent VR VR schedule would arrange for responses on either lever to increment both ratios, but allow reinforcement only when a response occurs on the lever correlated with the completed ratio. This will be called a *dependent* concurrent VR VR schedule, whereas the conventional method of scheduling reinforcers used by Herrnstein and Loveland (1975) will be called an *independent* concurrent VR VR schedule. Because responses on each lever of a dependent schedule increment both ratio counters, this schedule is formally equivalent to a concurrent variable-interval variable-interval (VI VI) schedule. Responses, rather than time, increase the probability that the next response on both the current and the alternate schedule will be reinforced. Consequently, we might expect the be-

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havior of subjects exposed to a dependent concurrent VR VR to conform to the generalized matching relation, as expressed in Equation 1.

Shull and Pliskoff (1971) exposed pigeons to a dependent concurrent VR VR and found a poor approximation to matching. However, they did not vary the relative frequency of reinforcement; they used only one schedule, a dependent concurrent VR 60 VR 60. A response bias, such as one based on color preference, might have masked the matching relation. Had they exposed their subjects to a range of relative frequencies of reinforcement, they could have assessed the sensitivity of their subjects to changes in the distribution of reinforcers and whether there was a response bias (Baum, 1974).

The present experiment assessed whether the generalized matching law could extend to dependent concurrent VR VR schedules. An additional interest was that of examining the functional similarity of dependent concurrent ratio and concurrent interval schedules. In all of the dependent concurrent VR VR conditions, the rats distributed their responses and time in a manner consistent with the generalized matching law (nonexclusive matching). Additional control conditions indicated that allowing responses to increment both ratio counters was necessary for nonexclusive matching to occur.

METHOD

Subjects

Four male albino rats obtained from Holzman Co. (Madison, Wisconsin) were the subjects. At the beginning of the experiment they were experimentally naive and approximately 100 days old. Throughout the experiment they were maintained at 80% of their free-feeding weights. They were housed individually in a constantly illuminated room with water continually available.

Apparatus

The apparatus consisted of a two-lever conditioning chamber located within a sound-attenuating, light-controlled enclosure. The chamber was 23 by 20 by 19 cm. Each lever was 1 by 5 cm, protruded 1.2 cm, and was located 7 cm above the floor and 1 cm from the adjacent wall. A minimum force of approximately 0.35 N was required to operate

each lever. The food cup was 4.5 by 4.5 cm, centered between the levers, and 2 cm above the floor. A 6-W, 110-VAC stimulus light was centered 7 cm above each lever. Only the light above the left (main) lever was used. A fan provided masking noise and ventilation. The reinforcers consisted of 45-mg pellets (BioServ Inc.).

Control of the contingencies and stimuli and recording of data were provided by a PDP-8a® computer (Digital Equipment Corp.) with SUPERSKED® software and appropriate interfacing located in an adjacent room.

Procedure

Overview. The rats were exposed to concurrent VR VR schedules using a one-lever procedure (Findley, 1958) in which both ratio counters were incremented by responses on one lever (the main lever); responses on the second lever (the changeover lever) turned a stimulus light on or off and changed the ratio schedule whose reinforcers could be delivered by responses on the main lever. In one set of conditions, dependent concurrent schedules were programmed such that responses on the main lever simultaneously advanced the counters for both ratio schedules. Subsequently, to assess whether the observed patterns of behavior had resulted from the dependent feature of the schedules, 2 subjects were each exposed to two conditions of independent concurrent ratio schedules in which responses incremented only the counter correlated with the schedule currently in effect.

Specifics. In the dependent conditions, responses that completed a ratio and occurred in the presence of the stimulus correlated with that ratio were reinforced. If the ratio that was satisfied was not correlated with the current stimulus, reinforcement was withheld until a changeover response produced the appropriate stimulus followed by a response on the main lever. The next ratio on that counter was not advanced until that reinforcer was collected. In the independent conditions, a response during light-on or during light-off incremented only the ratio correlated with the current stimulus. Because only the current counter was being incremented, as soon as its value was reached a reinforcer was delivered. All other contingencies remained unchanged.

The houselight was on throughout the session, which terminated after 100 reinforce-

ments had been obtained. During all conditions, each changeover response produced the alternate stimulus and correlated schedule. There were three reasons for not using a changeover delay. First, because no one has reported data from dependent concurrent ratio schedules, an appropriate duration was not obvious. Second, high response and reinforcement rates were expected; these made it difficult to estimate appropriate durations from the literature on concurrent interval schedules. Third, a possible use of this procedure is in the experimental analysis of the role of momentary maximizing (Shimp, 1966) in concurrent schedules. The use of a changeover delay seriously complicates, if not negates, analyses based on momentary maximizing.

Table 1 presents for each subject the specific sequence of conditions and number of sessions of exposure to each condition. After magazine training, pressing of the main lever was shaped. After one session in which each response in the presence of either stimulus produced a reinforcer, the schedule was rapidly increased to dependent concurrent VR 50 VR 50 or concurrent VR 100 VR 100. During these sessions, identical schedules accompanied each stimulus. A condition remained in effect for a given subject for at least 10 sessions and until there was no apparent upward or downward trend in the distribution of response and of time. Subject 7 died after 7 days exposure to the fourth condition.

The sequences of values in all schedules were exponentially distributed and were obtained from the method suggested by Fleshler and Hoffman (1962), treating seconds as responses and always using 20 values. The only exception was Rat 4 VR 362.8, whose variable-ratio schedule was comprised of the following 23 values: 4, 14, 26, 38, 50, 63, 78, 93, 110, 128, 148, 170, 195, 224, 257, 298, 348, 416, 521, 798, 1,041, 1,304, and 1,997. Values from the set of 20 (or 23) were randomly selected without replacement until all values were used, and then random selection began again using all 20 (or 23) values. During dependent concurrent schedules, the mean ratio requirements for a pair of ratio schedules were selected to produce a constant programmed overall response-reinforcement ratio, which was calculated according to Equation 2:

$$1/PR_i = 1/PR_1 + 1/PR_2 \quad (2)$$

where PR_i represents the overall programmed response reinforcer ratio and PR_i represents the response-reinforcer ratio for each member of a pair of schedules. This is analogous to the method used by Herrnstein (1961) in calculations of the programmed overall reinforcement rate (reinforcers per minute). The obtained overall response-reinforcer ratio was calculated by dividing total responses by total reinforcers. The programmed overall response-reinforcer ratio was 25:1 for Rats 6, 7, and 10, and either 25:1 or 50:1 for Rat 4. The only exception was Rat 4, for which VR 66.7 and VR 362.8 yielded a ratio of 56.3:1.

Rats 6 and 10 were exposed subsequently to an independent concurrent VR 33.3 VR 100 schedule. Responding exclusively on the VR 33.3 schedule or distributing responses between the two schedules (in any proportion) results in an obtained response-reinforcer ratio of at least 33.3:1. Because the effect of this change to a less favorable ratio was not known, these rats were placed on a second independent concurrent schedule in which the obtained response-reinforcer ratio would not be less favorable than that of the dependent schedules. On this schedule, an independent concurrent VR 16.7 VR 50, if subjects respond exclusively on the VR 16.7 the obtained ratio is 16.7:1. As long as subjects distribute responses such that at least 75% of responses (equal to the programmed distribution of reinforcers) occur on the VR 16.7 schedule, the obtained response-reinforcer ratio on this procedure will be at least as favorable as 25:1.

RESULTS

All results presented here are calculated from the data obtained from the last five sessions in each condition. Those data indicate that dependent concurrent VR VR schedules maintain behavior consistent with the generalized matching law, as formulated by Baum (1974). Table 1 presents, for each rat in each condition, mean numbers of responses during light-off and during light-on, reinforcers obtained during light-off and light-on, time spent in light-off and light-on, and number of changeovers. The standard deviation and range for each dependent variable are also included. During dependent concurrent VR VR conditions, responses and time were distributed between both schedules. The only exception

Table 1

For each rat, the sequence of experimental conditions (with various schedules accompanying light-on and light-off); the number of sessions in each condition; the mean, standard deviation, and range of the responses; time allocated to and reinforcers obtained in light-off and light-on; and the changeover frequency.

Light	Schedule	Type	Ses- sions	Responses			Time (min)		
				Mean	SD	Range	Mean	SD	Range
Rat 4									
On	VR 100	dependent	14	2,577	59	2,506–2,636	21.9	2.4	17.9–23.8
Off	VR 100			2,592	60	2,494–2,641	22.9	2.1	19.7–25.1
On	VR 250	dependent	16	230	71	108–288	2.2	0.6	1.2–2.8
Off	VR 27.8			2,498	130	2,385–2,712	19.0	1.3	17.5–21.1
On	VR 27.8	dependent	14	2,430	94	2,299–2,517	17.4	0.8	16.5–18.5
Off	VR 250			254	47	178–305	2.4	0.3	1.9–2.8
On	VR 66.7	dependent	20	3,964	178	3,676–4,111	26.9	1.2	25.1–27.9
Off	VR 200			1,370	125	1,226–1,569	11.4	1.2	10.3–13.4
On	VR 500	dependent	15	583	81	461–656	5.2	0.9	4.1–6.2
Off	VR 55.5			4,633	170	4,478–4,904	30.8	1.2	29.1–32.0
On	VR 362.8	dependent	15	1,155	43	1,104–1,205	11.5	1.8	9.7–14.0
Off	VR 66.7			4,812	201	4,691–5,165	34.5	1.8	31.9–36.6
On	VR 100	dependent	10	2,296	172	2,114–2,549	21.0	2.5	17.8–23.8
Off	VR 100			3,231	192	3,019–3,472	26.4	3.0	22.8–29.6
Rat 6									
On	VR 50	dependent	12	1,551	88	1,436–1,680	11.7	2.4	9.6–15.8
Off	VR 50			1,319	42	1,255–1,361	13.5	8.9	9.2–29.3
On	VR 33.3	dependent	11	2,058	62	1,960–2,122	12.8	0.7	11.8–13.6
Off	VR 100			683	80	560–773	5.8	0.5	5.1–6.4
On	VR 100	dependent	24	996	111	850–1,111	6.5	0.7	5.7–7.5
Off	VR 33.3			1,814	87	1,689–1,891	11.8	0.7	10.8–12.8
On	VR 27.8	dependent	21	2,232	173	1,953–2,409	17.6	1.2	15.6–18.7
Off	VR 250			474	65	426–586	5.1	0.5	4.5–5.7
On	VR 33.3	dependent	10	1,985	78	1,883–2,086	15.5	1.2	14.6–17.5
Off	VR 100			762	105	632–887	7.0	0.8	5.9–7.8
On	VR 16.7	independent	15	1,639	27	1,607–1,679	14.0	1.0	13.1–15.4
Off	VR 50			51	37	12–90	0.6	0.5	0.2–1.4
On	VR 100	independent	27	240	123	80–393	1.9	0.9	0.6–2.8
Off	VR 33.3			3,299	54	3,216–3,362	28.5	2.2	25.6–31.1
Rat 7									
On	VR 50	dependent	14	1,607	100	1,464–1,716	19.9	2.3	13.9–20.3
Off	VR 50			1,223	29	1,192–1,254	14.6	2.3	11.9–19.9
On	VR 100	dependent	27	701	83	571–772	7.0	1.0	5.7–8.3
Off	VR 33.3			1,989	122	1,888–2,194	15.9	0.9	14.8–17.1
On	VR 250	dependent	19	378	95	280–537	2.9	0.6	2.2–3.6
Off	VR 27.8			2,362	158	2,109–2,498	13.8	1.4	11.8–15.5
Rat 10									
On	VR 50	dependent	14	1,467	92	1,365–1,592	18.1	3.9	14.4–24.3
Off	VR 50			1,425	151	1,232–1,645	18.2	3.5	13.3–21.9
On	VR 33.3	dependent	12	2,260	125	2,149–2,460	26.2	3.8	22.0–32.3
Off	VR 100			470	101	355–611	8.2	2.5	5.8–12.1
On	VR 50	dependent	18	1,479	80	1,393–1,594	18.8	2.1	16.8–22.0
Off	VR 50			1,311	32	1,271–1,346	17.5	1.7	15.2–19.7
On	VR 250	dependent	18	517	121	391–690	6.9	1.6	5.7–9.5
Off	VR 27.8			2,337	84	2,211–2,437	31.9	1.7	29.0–33.1
On	VR 100	dependent	15	840	76	735–906	15.3	3.0	13.4–20.5
Off	VR 33.3			2,000	146	1,846–2,179	34.5	4.6	29.2–40.2
On	VR 100	independent	17	225	42	173–284	3.2	0.9	2.3–4.4
Off	VR 33.3			3,316	71	3,270–3,439	50.5	2.6	46.8–53.5
On	VR 16.7	independent	34	1,662	16	1,639–1,684	29.6	2.4	27.1–32.3
Off	VR 50			95	63	0–162	1.4	1.1	0.0–2.6
On	VR 2,500	dependent	14	116	67	26–166	1.5	1.0	0.2–2.6
Off	VR 25.3			2,442	51	2,364–2,500	39.2	2.5	36.6–43.3

Table 1 (Continued)

Reinforcements			Changeovers		
Mean	SD	Range	Mean	SD	Range
49.4	1.1	48-51	405	48	335-467
50.6	1.1	49-52			
6.2	2.4	3-9	53	28	11-82
93.8	2.4	91-97			
92.4	2.6	89-96	55	13	37-68
7.6	2.6	4-11			
78.0	1.4	76-79	203	18	181-223
22.0	1.4	21-24			
8.0	2.1	6-11	119	9	106-128
92.0	2.1	89-94			
14.4	2.9	10-17	187	10	178-201
85.6	2.9	83-90			
47.6	1.3	47-50	303	44	245-360
52.4	1.3	50-53			
51.0	1.0	50-52	158	7	148-163
49.0	1.0	48-50			
77.0	1.0	76-78	135	21	108-165
23.0	1.0	22-24			
24.8	0.8	24-26	119	17	97-136
75.2	0.8	74-76			
90.6	3.2	86-95	124	10	108-134
9.4	3.2	5-14			
76.0	1.0	23-25	155	18	131-180
24.0	1.0	75-77			
99.0	1.0	98-100	7	4	3-13
1.0	1.0	0-2			
1.8	1.5	0-4	16	5	10-22
98.2	1.5	96-100			
50.2	2.9	47-59	217	38	173-275
49.8	2.9	46-53			
24.0	2.1	22-27	193	13	176-207
76.0	2.1	73-78			
9.0	3.1	6-14	105	25	66-133
91.0	3.1	86-94			
49.8	1.3	48-51	135	27	108-176
50.2	1.3	49-52			
79.4	1.7	78-82	107	31	77-154
20.6	1.7	18-22			
50.0	2.1	47-52	125	18	104-154
50.0	2.1	48-53			
8.0	2.3	6-12	90	2	87-91
92.0	2.3	88-94			
21.4	2.3	19-25	87	16	75-115
78.6	2.3	75-81			
1.6	0.9	1-3	14	3	9-17
98.4	0.9	97-99			
98.8	1.8	96-100	3	4	0-9
1.2	1.8	0-4			
0.2	0.4	0-1	10	7	2-20
99.8	0.4	99-100			

(Rat 10, Condition 8) was one in which 99% of the reinforcers were programmed to occur in one schedule. In all four conditions in which independent concurrent VR VR was programmed, responding and time were allocated almost exclusively to the schedule with the smaller ratio requirement.

Table 1 also shows that the frequency of changing over was affected by the dependent or independent scheduling of reinforcers. Four of the five conditions with the lowest frequencies of changing over were conditions with independent scheduling. In the fifth condition, reinforcers were dependent but again 99% of the reinforcers were scheduled for delivery on one schedule.

Figure 1 presents the log of the response ratio and the log of the time ratio each as a function of the log of the ratio of obtained reinforcers for individual rats. Response and time data for the dependent concurrent VR VR conditions were fitted separately to Equation 1 by the method of least squares. When conditions were replicated, both the original and redetermined values were used in the calculations. The sensitivity a , bias b , and their standard errors are also presented in Figure 1. Equation 1 was a good fit to both response and time data, accounting for at least 97% of the variance of each subject's response and time data. The bias b approximated zero, indicating that no subject preferred light-on to light-off (or vice versa) irrespective of their correlated schedules. Focusing first on response measures, for 3 subjects sensitivity was less than 1.0 (undermatching); for the 4th subject sensitivity approximated 1.0. In terms of time measures, for all 4 rats the sensitivities were much lower than 1.0 (undermatching).

An interesting relation is seen when sensitivities for time and response ratios are compared. Sensitivity for response ratios is always greater than sensitivity for time ratios. This difference was large for 3 of the 4 rats.

Figure 2 presents, for each rat, the changeovers per response (analogous to changeover rate) as a function of the log of the ratio of responses. Also shown is the changeover per response predicted by momentary maximizing (see Discussion for explanation of how these predictions were made). For all subjects the maximum obtained ratio of changeovers per response and the maximum predicted by momentary maximizing occurred when responding was approximately equally distributed be-

Fig. 1. For each subject, the log of the response ratio (light-off/light-on; squares show dependent ratio schedules, and plus symbols show independent ratios) and the log of the time ratio (light-off/light-on; triangles, dependent; X, independent) as a function of the log of the reinforcement ratio (light-off/light-on). All logs are base 10. Also shown are the best fitting lines.

tween the two alternatives. These ratios decreased systematically, approaching zero, as the log of the response ratio approached its maximum or its minimum.

The differences between local reinforcement rates plotted as functions of the log of the response ratios allow an assessment of melioration. This is shown, for individual rats, in Figure 3. Also plotted are the differences between the local reinforcer-response ratios. Local reinforcement rates were obtained by dividing the frequency of reinforcement in light-off and light-on by the time in minutes spent in light-off or light-on, respectively. Local reinforcer-response ratios were obtained by dividing the reinforcers in light-off or light-on by the responses in light-off or light-on. The differences in reinforcement rates and reinforcer-response ratios are generally not equal to zero; rather, there is a positive relationship between these differences and the log of the

response ratio. This effect is more pronounced for the reinforcement rate differences than for the reinforcer-response ratio differences.

DISCUSSION

These data clearly show that the generalized matching law can apply to conditions in which responses are reinforced on concurrent variable-ratio schedules. The use of dependent schedules appears to be essential for nonexclusive matching in concurrent variable-ratio schedules, as can be seen by comparing the results obtained from conditions in which dependent concurrent schedules were used with those from conditions in which independent schedules were used. Only in dependent concurrent schedules did the rats distribute responses and time between the two schedules (see Table 1). These distributions conform to the generalized matching law (Figure 1).

Response and time ratios both reflected consistent undermatching; this was not expected. Baum (1979) reviewed the literature, concluding that matching of time and reinforcement ratios, $a = 1$, and undermatching of response and reinforcement ratios, $a < 1$, are generally found. Thus, it is important to distinguish between strict matching (Herrnstein, 1970) (i.e., conforming to Equation 1 with $a = 1$) and generalized matching (Baum, 1979) (i.e., conforming to Equation 1 with $a \neq 1$). The present results are well described by the generalized matching law; at least 97% of the variance is accounted for by Equation 1 when $a < 1$. These results are not well described by the strict matching law.

The extension of the generalized matching law to dependent concurrent-ratio schedules is independent of the response-reinforcer ratios within the limits tested in the present experiment. Data consistent with the generalized matching law were obtained when response-reinforcer ratios of 25:1 and 50:1 were used. The robustness of this effect may be seen by examining the data of Rat 4. The generalized matching law fit the data when this rat was exposed to response-reinforcer ratios of 25:1, 50:1, and 56:1 in a mixed sequence of conditions.

The results reported here are consistent with the data reported by Green, Rachlin, and Hanson (1983) in which pigeons were exposed to combined independent-dependent concurrent-ratio schedules. In their experiment, one counter was incremented by responses on either key (dependent counter) and the other counter was incremented only by responses on one key (independent counter). This produced a concurrent schedule formally equivalent to a concurrent VR VI, which they called a concurrent VR "VR." Responses on the key that incremented only the dependent counter were reinforced on the "VR" schedule. In general, they found that their data were described by the generalized matching law. They also exposed pigeons to independent concurrent-ratio schedules and reported pigeons consistently responded exclusively, or almost exclusively, on the key correlated with the more favorable schedule.

Taylor and Davison (1983) also reviewed the literature and concluded that strict matching of time ratios and reinforcement ratios is approximated ($a = 1$) when reinforcers are

Fig. 2. The ratio of changeover responses to total main-lever responses as a function of the log of response ratio. Data for each subject are represented by a distinctive open symbol. Also shown are predicted changeovers per total main-lever responses based on momentary maximizing.

exponentially distributed on variable-interval schedules. Because the present experiment used VR schedules whose values were exponentially distributed, there is no reason to believe that undermatching was produced by ratio schedules per se or by the use of rats as subjects. Approximations to strict matching ($a = 1$) have been reported by Logue and de Villiers (1978), Norman and McSweeney (1978), and Poling (1978). Rather, undermatching was probably a result of not using a changeover delay (COD) and of requiring one response to effect a changeover. The exact function of a COD is unclear. Using pigeons and increasing the COD from zero to a few seconds when concurrent-interval schedules are used tend to reduce undermatching, whereas further increases in the COD produce matching (Shull & Pliskoff, 1967). When an independent concurrent-ratio schedule was employed, Herrnstein and Loveland (1975) reported a small change in the response proportions (less than 5%) when the COD was increased from 0 to 1.5 s. When rats are exposed to concurrent schedules, a COD of 5 to 10 s is frequently

Fig. 3. For each subject, the difference between the local reinforcers per minute (light-on minus light-off; triangles) as a function of the log of response ratio (light-on/light-off), and the difference between the local reinforcers per response (squares) as a function of the log of response ratio. To make the difference in reinforcers per response and difference in reinforcers per minute fit on the same scale, the reinforcers per response were multiplied by 100.

employed. Pliskoff and Fetterman (1981) have shown that in the absence of a COD, a change-over requirement of one response results in undermatching, and increasing the changeover requirement results first in matching and then overmatching.

Melioration, as discussed by Herrnstein and Vaughan (1980), makes a prediction that is inconsistent with a result of the present research. Melioration proposes that subjects respond on the schedule whose local reinforcement rate is highest. Responding in that pattern would result in equal local reinforcement rates on concurrent-interval and on dependent concurrent-ratio schedules. Figure 3 shows that this did not occur. Rather, the greater the difference between local reinforcement rates the larger the log ratio of responding. These rates differed by as much as five reinforcers per minute, a deviation from zero that is sufficiently large to make untenable the argument that the differences were not discriminable by the rats.

It might also be argued that applying melioration to concurrent ratios scheduled according to a one-lever procedure is inappropriate. For example, the time taken to change over contributes to both local response and reinforcement rates, and when one alternative is preferred, the brief time spent in the non-preferred alternative is artifactually increased by the time it takes to change out of that alternative. Using an analysis parallel to that presented by Herrnstein and Vaughan in the development of melioration, one can extend melioration to concurrent-ratio schedules by the following equation:

$$d = (R_1/B_1) - (R_2/B_2). \quad (3)$$

Here the comparison is the local reinforcer-response ratio, rather than local reinforcers per minute. The animal will respond on the lever with a greater local reinforcer-response ratio. Melioration makes the same predictions for dependent ratio schedules: The difference in the local reinforcer-response ratios will be zero.

Figure 3 also presents these differences as a function of the log of the response ratio. Although the absolute differences are small, ranging from .0004 to .04, the absolute reinforcer-response ratios are also small, ranging from .001 to .04. Thus, the relative differences are not small. There is a positive relationship between differences in local reinforcer-response ratios and the log of the response ratios that is not consistent with this application of melioration to dependent ratio schedules.

An alternative view of the inconsistency between the prediction based on melioration and the results in Figure 3 is that the results may be due to not using a COD and to the requirement of only one response to effect a changeover; the inconsistency is not a failure of theory but of the procedure used. Unequal local reinforcer-response ratios are a consequence of undermatching, and as previously discussed, undermatching was probably the result of not using a COD and requiring one response to effect a changeover. However, melioration is silent with respect to the necessity of using a COD and/or the size of the changeover requirement for its predictions to be confirmed.

Applying momentary maximizing (Shimp, 1966) to dependent concurrent VR VR allows one to predict the changeover ratio, the number of changeovers divided by total main-lever responses. Momentary maximizing states the animal will emit the response that is most likely to be reinforced at a given moment. Because a one-lever procedure was used, two responses were required for a changeover to lead to a reinforcer—a changeover response followed by a main-lever response. Applying momentary maximizing to this situation results in the following rule: The subject stays on the current schedule until the probability of reinforcement for one of the next two responses on the present schedule is less than the reinforcement probability for the first response on the other schedule. Calculating the sequence that would satisfy momentary maximizing according to this rule, the changeover ratio was obtained by summing the responses in one cycle, responding on one schedule then the other, and then returning to the first. This sum was then divided by two and the inverse is presented in Figure 2. The predicted shape and maxima are consistent with the present data but the predicted values exceed the obtained values by a factor

of four. The lack of correspondence between theory and data may be attributable to the one-lever procedure.

Comparing the probability of reinforcement of one of the next two responses on the current schedule with the first response on the other schedule following a changeover ignores differences in response topography. Two successive responses on one lever differ from two responses on two levers. In the latter situation the rat moves from the main lever, presses the changeover lever, and moves back to and presses the main lever. This topographical difference functions to reduce the predicted changeover rate. This change will reduce the difference between the predicted and obtained changeover rates. Unfortunately the magnitude of this change cannot be quantified. Thus, the divergence between predicted and obtained changeover rates should be interpreted with caution.

As discussed in the Introduction, dependent concurrent-ratio schedules are formally equivalent to concurrent-interval schedules. The present results suggest they are functionally equivalent as well. First, the generalized matching law describes data from each procedure. Second, there is an inverted U-shaped function relating changeovers per response (or per minute) and response allocation. Dependent concurrent-ratio schedules allow the precise manipulation of the momentary reinforcement probabilities. Because of this feature and their formal and functional equivalence to concurrent-interval schedules, their use may help to clarify the role of moment-to-moment changes in the probability of reinforcement and behavior allocation.

An inspection of the slopes relating the time and response ratios to reinforcement ratios shows that the slopes of response ratios are consistently closer to 1.0 (i.e., strict matching). This result is the opposite of that reported by Baum (1979, 1983) after reviewing the literature on concurrent VI VI schedules, but has been previously reported for concurrent schedules with ratio components (Davison, 1982; Rider, 1979). The use of concurrent-ratio rather than concurrent-interval schedules in the present experiment is probably responsible for this difference. In dependent concurrent VR VR schedules, the probability of reinforcement is incremented by responding. Thus, responding affects reinforcement rate and indirectly affects reinforcement ratios as well as

response ratios: Time has no effect on either ratio. Because of the partly confounded influence of responding, response and reinforcement ratios are interrelated. Thus, the slope relating response and reinforcement ratios will be closer to 1.0 than the slope relating time and reinforcement ratios.

A similar argument may be made for concurrent-interval schedules. Time increments the probability of reinforcement. Thus, time influences reinforcement rate, and indirectly influences reinforcement ratios as well as time ratios: Responding has little effect on reinforcement ratios and no effect on time ratios. Now, time and reinforcement ratios are interrelated. The slope of the line relating time and reinforcement ratios will be closer to 1.0 than is the slope of the line relating responding and reinforcement. Phrased differently, the closer approximation to strict matching produced by time ratios on concurrent-interval schedules, and the closer approximation to strict matching produced by response ratios on concurrent-ratio schedules, are consequences of the schedule used. This suggests that the closer approximation to strict matching with time ratios using interval schedules (Baum, 1979) has no special theoretical significance; this relationship does not support the proposition that time allocation is the fundamental principle by which behavior is distributed. The apparent fundamental nature of time allocation may be a result of an overdependence on interval schedules in investigations of behavior allocation.

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