THE MAXIMIZATION OF OVERALL REINFORCEMENT RATE ON CONCURRENT CHAINS

ALASDAIR I. HOUSTON, BRIAN H. SUMIDA, AND JOHN M. MCNAMARA¹

UNIVERSITY OF OXFORD AND UNIVERSITY OF BRISTOL

We model behavioral allocation on concurrent chains in which the initial links are independent variable-interval schedules. We also quantify the relationship between behavior during the initial links and the probability of entering a terminal link. The behavior that maximizes overall reinforcement rate is then considered and compared with published experimental data. Although all the trends in the data are predicted by rate maximization, there are considerable deviations from the predictions of rate maximization when reward magnitudes are unequal. We argue from our results that optimal allocation on concurrent chains, and prey choice as used in the theory of optimal diets, are distinct concepts. We show that the maximization of overall rate can lead to apparent violations of stochastic transitivity.

Key words: concurrent chains, relative allocation, reinforcement rate, maximization, diet theory, stochastic transitivity

The concurrent-chains procedure involves presenting an experimental subject with concurrently available variable-interval (VI) schedules (the initial links) which, instead of giving access directly to reinforcement, give access to a terminal link, which in turn gives access to reinforcement. The procedure is outlined in Figure 1 and is described in more detail in the next section.

In this paper we develop a model of optimal allocation on the initial links. Previous models (e.g., Fantino & Abarca, 1985; Staddon, 1983) make the implicit assumption that the animal can respond on both initial links simultaneously. Our model incorporates the process of switching between the initial links.

MODELS OF THE CONCURRENT-CHAINS PROCEDURE

We consider concurrent chains in which the initial links are independent constant-probability variable-interval schedules (e.g., Flesh-

ler & Hoffman, 1962) with programmed reinforcement rate λ_1 on Side 1 and λ_2 on Side 2. Throughout this paper λ has the units s^{-1} , so the mean schedule interval (in s) is $1/\lambda$. When a terminal link is produced on Schedule 1, Schedule 2 becomes inoperative and Side 1 gives the subject a reinforcer of magnitude M_1 with a delay of total mean duration D_1 . This delay is the total mean time from entering the terminal link to returning to the initial links, and because it depends upon the subject's behavior, may be longer than the programmed delay. In terms of overall mean reinforcement rate, there is no distinction between the various ways of scheduling a given mean time until the reinforcer is obtained. Similarly, if an interval times out on Schedule 2 a reinforcer of magnitude M_2 is obtained with an associated total delay D_2 . Without loss of generality, we assume that $D_1 \leq D_2$. The basic cycle is shown in Figure 1.

Models of behavior on concurrent chains can be divided into descriptive models and optimization models. Examples that fall into the first category are the delay-reduction hypothesis of Fantino (1969, 1977, 1981) and the incentive models of Killeen (1982, 1985). Models based on optimization have been proposed by Staddon (1983) and Fantino and Abarca (1985). Both models are strictly valid only when an initial link is either always chosen or always ignored. We summarize these models as an introduction to the model that we wish to develop.

¹ Alasdair I. Houston and Brian H. Sumida are at the University of Oxford; John M. McNamara is at the University of Bristol.

We thank S. E. G. Lea, W. Baum, and L. Green for their helpful comments on an earlier version of this paper. A. I. Houston was supported by a Science and Engineering Research Council grant to J. R. Krebs and J. M. McNamara.

Reprint requests may be sent to A. I. Houston, Department of Zoology, University of Oxford, South Parks Road, Oxford OX1 3PS, England.

Fig. 1. Flow diagram of the concurrent-chains procedure. The initial links are independent VI schedules with schedule rate λ_1 on Side 1 and λ_2 on Side 2. During the choice phase, both keys are white. On the left, a terminal link has been obtained on key 1. That key changes color, key 2 goes dark, and a reinforcer of magnitude M_1 is obtained with an associated delay D_1 . On the right, a terminal link is obtained on key 2.

A Simple Optimality Model

The argument used by Staddon (1983) and by Fantino and Abarca (1985) runs as follows. We start by assuming that $M_1 = M_2 = 1$ and $D_1 < D_2$. Two possible strategies are to respond exclusively to Side 1 or to alternate between the initial links. The reinforcement rate that results from the first of these strategies is clearly $1/(1/\lambda_1 + D_1)$. To obtain a rate for the second strategy, assume that alternation is so rapid that each terminal link is entered as soon as it is set up. Then the overall rate of entry into the terminal links is $\lambda_1 + \lambda_2 = \lambda$. A proportion λ_1/λ of these are on Side 1 and a proportion λ_2/λ are on Side 2, so that the resulting rate is $1/(1/\lambda + \lambda_1 D_1/\lambda + \lambda_2 D_2/\lambda)$. By comparing these rates it can be seen that responding exclusively on Side 1 produces the higher rate of reinforcement if and only if $1/\lambda_1 + D_1 < D_2$. This is the conclusion reached by Staddon (1983). Fantino and Abarca (1985) use the same sort of argument but also consider unequal reinforcers. Given that $M_1/D_1 > M_2/D_2$, they conclude that only Side 1 should be chosen if and only if

$$\frac{1}{\lambda_1} + D_1 < \frac{M_1}{M_2} D_2. \quad (1)$$

As they point out, this is the standard condition for specialization on the more profitable prey type in models of diet selection (Charnov, 1973, 1976; Pulliam, 1974). Each terminal link, with its associated reinforcer and delay, corresponds to a prey type with an associated energy and handling time.

In the prey-choice paradigm and in some operant procedures (e.g., Abarca & Fantino, 1982; Lea, 1979) it is possible to search for both prey types (or work for both terminal links) simultaneously. This is not true on the concurrent-chains procedure; the animal can only respond on one initial link at a time. It follows that there is a delay between a terminal link being set up at a side on which the animal is not responding and that link being entered. This delay, which is ignored in the models of Staddon (1983) and Fantino and Abarca (1985), will depend (among other things) on the animal's switching pattern. Ignoring this delay amounts to assuming that the animal can be responding on both initial links simultaneously. It follows that the models of Staddon (1983) and Fantino and Abarca (1985) only give an approximation of the overall rate when the animal frequently switches between the initial links. Staddon (1983) acknowledges this, but states that an optimality analysis could be extended in several ways to account for nonexclusive choice, "none of them inviting." We have adopted a relatively simple approach that treats the concurrent-chains procedure as a concurrent VI VI with unequal rewards and delays.

The Model

It is clear from our discussion of the models of Staddon (1983) and Fantino and Abarca (1985) that relative allocation on the initial links is an insufficient characterization of behavior if reinforcement rate is to be calculated. The frequency of switching, or alternatively the distribution of stay times, must also be specified. We consider a model based on the random-switching model of Heyman and Luce (1979). The animal stays for a random time on each initial schedule, subject to a constraint on the maximum switching rates. Like Houston and McNamara (1981) and Heyman and Luce (1979), we characterize behavior purely in terms of bout durations. The assumption is that the subject is on a variable-time schedule rather than a variable-interval schedule, or that

it is responding so rapidly that there is no delay between a terminal link being set up on the side where the subject is and the link being entered. Assuming that the local response rates on the initial links are equal, the proportion of time on an initial link thus corresponds to the proportion of responses made on that initial link. The animal switches from Side 1 to Side 2 at a rate μ_1 and from Side 2 to Side 1 at a rate μ_2 . These rates are subject to the constraint $\mu_1 + \mu_2 = 1/I$, where I is half the harmonic mean of time between switches. (The global optimum involves fixed stay times—cf. Houston & McNamara, 1981. For a discussion of global and constrained optima, see Houston & McNamara, 1985). We also include a delay τ that must elapse between leaving one side and obtaining a terminal link on the other side. The delay can be either a conventional programmed changeover delay or the minimum time taken in moving from one side to the other. Once a terminal link on Side i has been completed, the subject returns to the initial link on Side i . We use $p = \mu_2/(\mu_1 + \mu_2)$ to denote the relative allocation of time to the initial link on Side 1. In Appendix 1 we derive the equation for the long-term reinforcer rate γ as a function of p . The value of p that maximizes γ is denoted by p^* . In general p^* has to be found numerically, but the way in which p^* depends on the schedule parameters in some special cases is presented in Appendix 2.

Some studies (Davison, 1983; MacEwen, 1972; Snyderman, 1983; Williams & Fantino, 1978) use nonindependent VI schedules for the initial link, because when independent VI schedules are used, the subject's behavior can influence the proportion of entries into a given terminal link. We quantify this effect in Appendix 1. The probability r of obtaining a terminal link on Side 1 as a function of relative allocation for Side 1 is illustrated in Figure 2.

Maximizing Overall Rate

Before comparing the predictions of rate maximization with the data, we must draw attention to some general points about optimality models and some specific points about the model presented here. We are not suggesting that animals are "literal maximizers" (Herrnstein & Vaughan, 1980). A literal maximizer evaluates the reinforcement rate as a function of its behavior and chooses the behavioral allocation with the highest rate. In

Fig. 2. The relationship between the probability of producing a terminal link, r , and the preference p for the same initial link. $\lambda_1 = \lambda_2 = 1/60$, $\tau = 0$. The effect of the switching index I is revealed by comparison of the two curves, corresponding to index values of 3 and 20.

contrast to this view, we have argued that animals behave in orderly ways that result in high reinforcement rates under certain circumstances (e.g., Houston, 1983, 1987; Houston, Kacelnik, & McNamara, 1982; Houston & McNamara, 1981, 1984; McNamara & Houston, 1980). We are not trying to prove that animals are optimal, but rather to see whether the observed trends in behavior can be understood in terms of criteria based on adaptation. In this context, deviations from the predictions of optimality theory can be as instructive as agreements.

RESULTS

In estimating parameters for our models it is important to remember that τ includes both a programmed changeover delay (if any) and the time required to move from one initial link to the other. The delays D_1 and D_2 include both the programmed delays until the reinforcer becomes available, the reinforcement time, and the time to move from the location of the reinforcer (e.g., the feeder) to the initial link. We show in Appendix 2 that when the reinforcement magnitudes are equal, optimal allocation between initial links depends only on the difference in the delays, and so in this case only the difference in the programmed delays should matter. When terminal links differ in both delay and magnitude, the total delays need to be considered.

The results depend partly on the switching index I . There is no information on whether

Table 1

Optimal relative allocation, p^* , compared with the data from Fantino and Davison (1983). Initial links are VI 60 s (i.e., $\lambda_1 = \lambda_2 = 1/60$), reinforcement magnitudes: $M_1 = M_2 = 1$, $\tau = 0$. γ^* is the rate that results from adopting p^* expressed in units of reinforcements per minute. The table shows the effect of the delays D_1 and D_2 and the switching index I .

D_1 (s)	D_2 (s)	I (s)	p^*	γ^*	p (data)
5	55	3	.78	0.988	.95
		12	.83	0.958	
		20	.87	0.941	
10	50	3	.70	1.002	.89
		12	.73	0.943	
		20	.76	0.918	
15	45	3	.64	0.981	.86
		12	.66	0.934	
		20	.68	0.904	
20	40	3	.59	0.979	.73
		12	.60	0.928	
		20	.61	0.896	

the assumption of constant switching rates is reasonable in the case of concurrent chains, much less on the value of I . For pigeons on concurrent VI schedules, I seems to lie between 1 and 10 s (Houston, 1983).

Equal Reward Magnitudes ($M_1 = M_2$)

When $\lambda_1 = \lambda_2$ and $D_1 < D_2$ and $D_1 + D_2 =$ constant, relative allocation to Side 1 increases as D_1 decreases (Fantino & Davison, 1983). This is predicted by the model, as is shown in Table 1, but p^* is smaller than the observed p . In other words, the animal's allocation to the side with the shorter delay is greater than the allocation that maximizes overall rate. It can be seen that increasing I increases the relative allocation.

Table 2

Optimal preference when $M_1 = M_2$ and $D_1 = D_2$. $I = 3$ throughout. "Data" gives the average behavior of pigeon P1, Squires and Fantino (1971). Initial-link schedule values (1/s) are indicated by λ_1 and λ_2 . The table shows the effect of the initial link for two values of the delay τ , between a subject's leaving one initial-link schedule and obtaining access to the other initial-link schedule.

λ_1	λ_2	$\tau = 3$	$\tau = 0$	Data
.0167	.00167	.95	.91	.77
.0167	.0033	.90	.84	.62
.0167	.0083	.72	.67	.56
.0333	.0167	.73	.67	.66

Fig. 3. The dependence of overall mean reinforcer rate on relative allocation, illustrating the effect of the initial links. $M_1 = 1$, $M_2 = 1$, $D_1 = 30$, $D_2 = 90$, $\tau = 0$, $I = 3$ throughout. The curves are, from top to bottom, $\lambda_1 = 1/40, 1/120, 1/600$.

Squires and Fantino (1971) used concurrent chains in which $\lambda_1 > \lambda_2$ and $D_1 = D_2$. They showed that relative allocation to Side 1 increases as the ratio of λ_1 to λ_2 increases. Optimal behavior produces the same effect, as can be seen in Table 2. For given schedule parameters, p^* increases with τ .

Fantino (1969) showed that, when $D_1 < D_2$ and $\lambda_1 = \lambda_2 = \lambda$, relative allocation decreases towards .5 as λ decreases. In Figure 3 we plot reward rate as a function of relative allocation for three values of λ . It can be seen that optimal p decreases when λ is reduced. The figure also shows that the curve of rate as a function of relative allocation is very flat when λ is small. Table 3 gives the resulting optimal relative allocations, together with the data from Fantino (1969). The effect of I is also shown. When I is increased (i.e., switching rate is decreased) the optimal value of p increases.

Effect of Reward Magnitude and Delay

Navarick and Fantino (1976) presented pigeons with reinforcer magnitudes of 1.5 and 4.5, with the bigger magnitude having a delay

Table 3

Optimal relative allocations, p^* , compared to the allocations found by Fantino (1969). The initial links are equal ($\lambda_1 = \lambda_2 = \lambda$), $D_1 = 30$ s, $D_2 = 90$ s, $M_1 = M_2 = 1$, $\tau = 0$.

$1/\lambda$	$I = 3$	$I = 12$	Data
40	1	1	.95
120	.64	.65	.81
600	.53	.53	.60

Table 4

Optimal relative allocation on Side 1, p^* , indicated for various combinations of reinforcement magnitude and delay. Data are the average allocations to the side with the shorter delay in the experiments of Green and Snyderman (1980). $\lambda_1 = \lambda_2 = 1/60$, $M_1 = 2$, $M_2 = 6$, $I = 3$, $\tau = 0$. (The column on the extreme right shows p^* for $M_1 = 1$, $M_2 = 2$.)

D_1 (s)	D_2 (s)	p^* (1:3)	Data	p^* (1:2)
$D_2 = 6D_1$				
2	12	.37	.55	.43
4	24	.38	.72	.45
10	60	.43	.86	.53
20	120	.49	.93	.66
30	180	.56		1.00
50	300	.73		
56	336	1.00		
$D_2 = 3D_1$				
2	6	.35	.39	.41
10	30	.35	.59	.44
20	60	.35	.72	.47
40	120	.35	.86	.53
$D_2 = 1.5D_1$				
4	6	.34	.38	.40
20	30	.26	.31	.37
40	60	0	.33	.34
80	120	0	.10	.24

that was 10 s longer than the delay of the smaller magnitude. Four values of the delays were used, ranging from 2 s versus 12 s to 30 s versus 40 s. The 3 birds that had fixed-interval delays as opposed to black-outs showed some evidence of an increase in relative allocation with increasing delay duration. Our model shows the same trend, but the variability in the data makes an exact comparison difficult.

Green and Snyderman (1980) provide the most extensive data set on the interaction of reinforcer magnitude and delay. The longer delay always resulted in 6-s access to food, whereas the shorter delay resulted in 2-s access to food. As we have mentioned above, the feeder times should be included in the delays D_1 and D_2 . For the parameters used by Green and Snyderman (1980), the effect of including these times is either very small or zero, so we have not included them. Following our previous convention, the side with the shorter delay will be called Side 1. Now that the reinforcer magnitudes are unequal, it is no longer necessary for the optimal value of p to be greater than .5.

Table 4 compares the average relative al-

Fig. 4. The dependence of overall mean rate of reinforcement on relative allocation in the $D_2 = 6D_1$ condition of Green and Snyderman (1980). $\lambda_1 = \lambda_2 = 1/60$, $M_1 = 2$, $M_2 = 6$, $I = 3$, $\tau = 0$ throughout. From top to bottom, $D_1 = 2, 4, 10, 20$. The optimal allocation in each case is indicated by the filled circles and the averaged results obtained by Green and Snyderman by the vertical lines.

locations found by Green and Snyderman (1980), along with the optimal relative allocations indicated by our model. (The table also includes some parameter values not used by Green and Snyderman.) In each condition the ratio of the programmed delays is constant. The optimal relative allocations in the $D_2 = 1.5D_1$ condition have some resemblance to the data, but in the other two conditions predictions of the model are dramatically different from the behavior of the pigeons. The $D_2 = 6D_1$ condition is especially interesting. When the delays are 20 s and 120 s the optimal value of p is slightly less than .5, whereas the birds virtually ignored the initial link with the long delay. It appears that the birds do not give sufficient weight to magnitude as opposed to delay. Green and Snyderman (1980) drew attention to this fact, which seems to be general (e.g., Lea, 1981). The effect of relative allocation on reinforcement rate for a particular choice of switching rate is shown in Figure 4. The difference between the model and the data increases with the increasing flatness of the curve describing rate of reinforcement as a function of relative allocation. It does not necessarily follow that the flatness of this curve is controlling behavior. What can be said is that in terms of rate maximization the birds tend to avoid costly mistakes (see Houston, 1987). We have calculated the optimal behavior in the $D_2 = 6D_1$ condition for absolute delays greater than those used by Green and

Fig. 5. Equivalent combinations of delays. For $M_1 = 1$, $M_2 = 2$, $\lambda_1 = \lambda_2 = 1/60$, $I = 3$, and $\tau = 0$, we plot the combinations of D_1 and D_2 that result in a given value of p^* . These are, from left to right, $p^* = .55, .5, .45, .4, .3$. Preference reversal is also illustrated. At the point $D_1 = 10$ s, $D_2 = 75$ s, $p^* > .55$. Adding a delay of 40 s to both $\hat{D}_1$ and $\hat{D}_2$ (shown by the broken lines) results in a new combination of delays for which $p^* \approx .45$.

Snyderman (1980). It can be seen from Table 4 that eventually it is optimal to ignore the longer delay. The trend in this condition reflects the importance of obtaining the larger reward when the delays are short and the importance of obtaining the shorter delay when the delays are long.

In the $D_2 = 3D_1$ condition the optimal relative allocation is the same for all values of delay. This is an example of a general property which can be expressed as follows: if $M_1/M_2 = D_1/D_2 = k$, then the optimal relative allocation depends on k but not on the absolute values of the magnitudes and delays (see Appendix 2 for a proof).

One possible reason for this discrepancy between the model and the data is that the ratio of feeder times (3:1) may not reflect the ratio of effective magnitudes of reinforcement. Baum (1974) analyzed data from Fantino, Squires, Delbrück, and Peterson (1972) in which the feeder times were 1.5 s and 6 s and found that the data suggested a relative value of between 1.5 and 3.2 to 1 (see also Killeen, 1985). Epstein (1981) found that the amount of food consumed is an increasing but decelerating

function of the duration for which food is made available. To investigate the effect of a discrepancy between the ratio of feeder times and the ratio of reinforcer magnitudes, we have computed the optimal behavior for the Green and Snyderman (1980) conditions but with a 2:1 ratio of reinforcer magnitudes. The results (extreme right column in Table 4) are closer to the data than those obtained from the 3:1 ratio, but are still not a very good fit.

Navarick and Fantino (1976) found some evidence in their data for a form of preference reversal. The effect can be illustrated as follows. For fixed reinforcement magnitudes, there are delays $\hat{D}_1$ and $\hat{D}_2$ such that the relative allocation to Schedule 1 is greater than .5. By adding the same delay to both $\hat{D}_1$ and $\hat{D}_2$, it is possible to make the relative allocation to Schedule 1 less than .5. Such an effect can occur if overall rate is being maximized. It is shown in Appendix 2 that for fixed M_1 and M_2 , combinations of D_1 and D_2 that result in constant values of p^* are given by $D_2 = (M_2/M_1)D_1 + \text{constant}$. For the parameters used by Navarick and Fantino (1976), the constant is about 62 s, which means that the pigeons should have preferred the longer delay (leading to the larger reinforcer) in all the conditions of this experiment. As in the case of the Green and Snyderman (1980) data, an analysis based on a reinforcer ratio of 2:1 rather than 3:1 provides a better fit. Figure 5 shows the lines of constant p^* in this case, together with an example of preference reversal. Adding a constant delay to the delays $\hat{D}_1 = 10$ s and $\hat{D}_2 = 75$ s changes p^* from being greater than .5 to less than .5. (The analysis of Staddon, 1983, pp. 230–240, can be seen as a special case of this result.)

Prey Choice and Optimal Allocation

Throughout the $D_2 = 6D_1$ condition, $M_1/D_1 > M_2/D_2$. Despite the fact that the "rate" of the terminal link on Side 1 (i.e., reinforcer magnitude over delay) is always greater than that of the terminal link on Side 2, it is optimal to allocate more time to the terminal link on Side 2 if the delay of the terminal link on Side 1 is less than about 20 s. The ratio M/D corresponds to the ratio E/h (energy over handling time = "profitability") that is used to characterize prey types in foraging theory. According to the classical theory of prey choice (e.g., Charnov, 1973, 1976; Pulliam, 1974)

both prey types should be taken in all the $D_2 = 6D_1$ conditions. It can be seen from Table 4 that Side 2 is not used when $D_1 = 56$ and $D_2 = 336$ even though classical prey choice requires both prey types to be accepted.

All that foraging theory says about a prey type is that it is either accepted or rejected. In the concurrent-chains procedure, even when both "types" (i.e., links) are accepted, the relative allocation may depend on the schedule parameters. Although the terminal link on Side 1 has the higher profitability throughout, the relative allocation to the terminal link on Side 2 is higher when the delays are small. In the theory of optimal diets, the encounter rate with the prey type with the lower profitability does not have any effect on whether or not this prey type should be eaten (see Inequality 1 above). Fantino and Abarca (1985) argued from this that the initial link on the schedule with the lower relative allocation should not influence optimal allocation. Our results show that a model for optimal time allocation (as opposed to optimal acceptance or rejection of a terminal link) is sensitive to the value of both initial links.

Scaling Rewards and Delays

The concurrent-chains procedure can be seen as a way of measuring how an animal evaluates various combinations of rewards and delays. Navarick and Fantino (1974, 1975) point out that many theories of choice assume that choice is determined by one scale of value, or "dimension." They go on to discuss how concurrent-chains experiments can test this assumption. We outline their argument and then show that it can run into difficulties.

Let S be a set of alternatives denoted by $a, b, c, \dots$ and let $P(a, b)$ be the probability that a is chosen over b . A set of choice probabilities satisfies simple scalability if there exist real-valued functions F and v such that for all a and b in S

$$P(a, b) = F[v(a), v(b)]$$

where F is strictly increasing in its first argument and strictly decreasing in its second (Tversky & Russo, 1969).

Tversky and Russo (1969) show that simple scalability, strong stochastic transitivity, substitutability, and independence are all equivalent. Following Navarick and Fantino (1975) we concentrate on substitutability. The sub-

Table 5

Violation of substitutability. $\lambda_1 = \lambda_2 = 1/60$, $\tau = 0$, $I = 3$ throughout. a, b , and c are possible terminal links. c is the baseline, and the parameters have been chosen so that the optimal relative allocation when a and c are the terminal links is the same as when b and c are the terminal links. Substitutability requires that when a and b are the terminal links, the optimal relative allocation should be .5, but in fact more time is spent on the side that leads to b .

	Terminal links	
	Magnitude	Delay
a	15.826	120
b	5.811	1
c	1	5
Optimal preferences		
a versus c	.8	
b versus c	.8	
b versus a	.606	

stitutability condition can be given as follows (Tversky & Russo, 1969):

- (a) $P(a, c) > P(b, c)$ implies $P(a, b) > .5$
- (b) $P(a, c) = P(b, c)$ implies $P(a, b) = .5$

In applying condition (b) Navarick and Fantino (1975) identify the alternatives $a, b, c, \dots$ with various possible terminal links and take the choice probabilities to be the same as the relative allocations on the initial links. Under this interpretation, we show that both conditions can be violated by consistently behaving in ways that maximize overall rate of reinforcement. We consider three terminal links denoted by a, b , and c , and described in Table 5. It can be seen from the table that although a and b are equivalent in terms of a comparison against c , they do not produce a relative allocation of .5 when offered as alternatives, which means that condition (b) is violated. Now consider a new terminal link a' with a reinforcement magnitude of 16 and a delay of 99. $P(a', c) = .804$, $P(b, c) = .8$ and $P(a', b) = .491$, which violates condition (a). The reason for this apparent lack of substitutability is that the optimal allocation of initial links does not involve a comparison of the terminal links, but a comparison of all possible relative allocations. The relative allocations are assessed on the basis of a unidimensional scale (overall rate), but if this optimal choice is interpreted as an assessment of the terminal links, then it will be concluded that choice is not based on a unidimensional scale. We note that 2 of the

5 pigeons studied by Navarick and Fantino (1975) showed violations of transitivity, which might be expected from the fairly small effect shown in Table 5. It must be pointed out, however, that other models can also produce violations of transitivity.

DISCUSSION

When the terminal links provide reinforcers of equal magnitude, the trends in relative allocation are correctly predicted by the maximization of overall rate. When the initial links are equal, it is optimal to allocate more time to the side with the shorter delay, but allocation tends towards .5 as the rate at which terminal links are set up decreases. This trend was found by Fantino (1969). When delays are equal, but the initial links are unequal, it is optimal to allocate more time to the initial link with the higher λ . The optimal allocation increases with increasing difference between the initial links, in agreement with the data of Squires and Fantino (1971).

When rewards and delays are unequal, the optimal allocation, p^* , is not very close to the observed allocations. In particular, the $D_2 = 6D_1$ and $D_2 = 3D_1$ conditions of Green and Snyderman (1980) revealed dramatic discrepancies between pigeons' relative allocations and the relative allocations that are required by rate maximization. It should be pointed out, however, that the trends in the data are the same as those in p^* , and that p^* is closer to the data if the effective magnitude of the larger reinforcement is reduced. Although we have justified this reduction in terms of the nonlinear relationship between feeder time and reinforcement magnitude, our results suggest that another effect can produce the same sort of discrepancy between p^* and the data. When reinforcement magnitudes are equal, p^* tends to be smaller than the observed allocation, which indicates that the animals give more weight to the shorter delay than is required for the maximization of reinforcement rate. When the reinforcement magnitudes are unequal, this effect acts in the same direction as giving less weight to the bigger reinforcer.

The standard theory of optimal prey choice (Charnov, 1973, 1976; Pulliam, 1974) assumes that all prey types can be searched for simultaneously. On the concurrent-chains procedure the animal searches for one terminal link at a time by responding on one initial link

at a time. It follows that standard prey-choice theory cannot be applied directly to relative allocation on concurrent chains (Houston, 1985). When the principle of rate maximization is applied to the concurrent-chains procedure, the relative allocation that maximizes rate depends on the VI schedules of both the initial links (i.e., on both encounter rates, in the terminology of foraging theory).

Our analysis suggests that pigeons do not calculate overall long-term rate and then choose the behavior that maximizes this rate. We find such literal optimization unlikely on a priori grounds, and there are further objections to it in the context of concurrent chains. The rather flat curves of rate against relative allocation would make such a procedure difficult to implement, but even if this problem is ignored there is one feature of behavior that is incompatible with literal optimization. In the calculation of overall rate the terminal links are completely specified by their mean reinforcer magnitude and mean delay. It is well known that animals are sensitive to the variance in delay (e.g., Davison, 1972; Herrnstein, 1964; Hursh & Fantino, 1973; Killeen, 1968); therefore, animals are not just calculating mean reinforcement rates.

We have shown that the maximization of long-term rate can result in relative allocations that appear to violate the substitutability condition. Even though long-term rates are not being maximized on concurrent chains, the example is instructive. It illustrates that choice can be based on a unidimensional scale and yet appear to violate stochastic transitivity. Thus we cannot conclude from the data of Navarick and Fantino (1975) that animals do not use a unidimensional scale. It could be that choice is unidimensional, but that relative allocation is not a measure of the value of the terminal links.

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Received July 27, 1985

Final acceptance March 16, 1987

APPENDIX 1

REINFORCEMENT RATE

A visit to a schedule starts when the subject switches to it from the other schedule ("arrives") and ends when it switches away. Thus a visit to a schedule may be interrupted by entries into the terminal link but not by visits to the other schedule. A visit to Schedule i lasts an exponential time X_i with mean $1/\mu_i$.

Let c_i be the probability of producing a terminal link on arrival at Schedule i . On arrival at Schedule 1, the time since the subject was last on Schedule 1 is $X_2 + 2\tau$. Thus

$$\begin{aligned} c_1 &= 1 - E(e^{-\lambda_1(X_2 + 2\tau)}) \\ &= 1 - \frac{\mu_2}{\lambda_1 + \mu_2} e^{-2\lambda_1\tau}. \end{aligned} \quad (1.1)$$

The analogous formula holds for c_2 .

Let $E(N_i)$ be the expected number of terminal links produced on a visit to Schedule i . Since the mean time on schedule is $1/\mu_i$ and terminal links are produced at rate λ_i ,

$$E(N_i) = c_i + \lambda_i/\mu_i. \quad (1.2)$$

We consider a cycle that consists of a visit to Schedule 1 followed by a visit to Schedule 2. The expected reinforcement, $E(R)$, obtained on a cycle is

$$E(R) = M_1 E(N_1) + M_2 E(N_2). \quad (1.3)$$

The expected duration of the cycle, $E(T)$, is

$$E(T) = 2\tau + 1/\mu_1 + 1/\mu_2 + D_1 E(N_1) + D_2 E(N_2). \quad (1.4)$$

By renewal theory, the long-term rate γ is given by the expected number of reinforcers for a cycle divided by the expected duration of a cycle (Johns & Miller, 1963). Thus,

$$\gamma = E(R)/E(T). \quad (1.5)$$

This γ is a function of μ_1 and μ_2 . Following the approach of Heyman and Luce (1979) we assume that the sum of these switching rates is constrained by the equation

$$\mu_1 + \mu_2 = 1/I. \quad (1.6)$$

We then maximize γ by choice of μ_1 for a given I . From the value μ_1^* of μ_1 that maximizes γ , the optimal relative allocation p^* is found by

$$p^* = \frac{\mu_2^*}{\mu_1^* + \mu_2^*} = 1 - \mu_1^* I. \quad (1.7)$$

The proportion r of terminal links that are obtained on Side 1 is given by

$$r = E(N_1)/(E(N_1) + E(N_2)). \quad (1.8)$$

APPENDIX 2

The dependence of the optimal allocation (of time, to the initial link on Side 1) on terminal link parameters in some special cases.

By the results of Appendix 1,

$$\gamma = \frac{E(R)}{E(T)} = \frac{M_1 E(N_1) + M_2 E(N_2)}{2\tau + \frac{1}{\mu_1} + \frac{1}{\mu_2} + D_1 E(N_1) + D_2 E(N_2)}. \quad (2.1)$$

We consider four special cases of the optimal allocation. (a) For given D_1 and D_2 the optimal allocation depends on only the ratio M_1/M_2 . This follows a scaling argument based on measuring magnitude in multiples of the smaller reinforcer magnitude. (b) When $M_1 = M_2$,

$$\frac{1}{\gamma} = \frac{\left[2\tau + \frac{1}{\mu_1} + \frac{1}{\mu_2}\right]}{[E(N_1) + E(N_2)]} + D_1 + \frac{E(N_2)(D_2 - D_1)}{E(N_1) + E(N_2)}.$$

The term D_1 cannot be influenced by the animal's behavior, and so the optimal allocation depends on just $D_2 - D_1$ and τ . (c) When $M_1/M_2 = D_1/D_2 = k$

$$\frac{1}{\gamma} = \frac{\left[2\tau + \frac{1}{\mu_1} + \frac{1}{\mu_2}\right]}{M_2 \beta} + \frac{D_2}{M_2}$$

where

$$\beta = kE(N_1) + E(N_2).$$

The term D_2/M_2 cannot be influenced by the animal's behavior, so the optimal allocation does not depend on the absolute magnitudes M_1 , M_2 , D_1 , D_2 but only on the ratio k and τ . (d) We fix M_1 and M_2 and find the combinations of D_1 and D_2 that produce a constant optimal allocation p^* .

As $\mu_1 + \mu_2 = 1/I$, γ can be expressed as a function of just μ_1 . Differentiating with respect to μ_1 and setting the result equal to zero, it is found that the expression $(M_1 D_2 - M_2 D_1)$ is equal to an expression that depends on M_1 , M_2 , λ_1 , λ_2 , τ and μ_1^* (and hence p^*). It follows that for fixed M_1 , M_2 and p^* , $M_1 D_2 - M_2 D_1 =$ constant, that is lines of constant p^* are given by

$$D_2 = \frac{M_2 D_1}{M_1} + \text{constant}.$$