ON THE MEASUREMENT OF REINFORCEMENT FREQUENCY IN THE STUDY OF PREFERENCE¹

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In a two-link, concurrent-chain schedule, pigeons' pecks on each key during the initial link occasionally produced a terminal link, during which only that key was operative. Responses in the terminal link were reinforced with food on either fixed-interval or variable-interval schedules. In one experiment, relative amount of responding in the initial link equaled the relative harmonic rate of reinforcement in the terminal links. In a second experiment, the selection of interreinforcement intervals in variable-interval schedules in the terminal links was such that rates of reinforcement based on the harmonic or on the arithmetic means of the interreinforcement intervals predicted opposite preferences in the initial links. The observed preference was consistent with that predicted by the harmonic rather than by the arithmetic rates of reinforcement.

When primary reinforcement is delivered on concurrent variable-interval schedules, differential changes in some dimensions of reinforcement, such as amount or delay, often produce proportional changes in the number of responses on either schedule (Catania, 1963; Chung and Herrnstein, 1967). Autor (1960), Herrnstein (1964a), and Fantino (1967) have extended the concurrent paradigm to study preference for stimuli correlated with different schedules of reinforcement. These investigators used a concurrent-chain procedure, where responses to either of two keys would occasionally produce a stimulus correlated with a schedule of primary reinforcement. While this schedule was in effect on one key, the other key was dark and inoperative. Preference was measured by the relative amount of responding on a key (responses on one key/total responses), during the time that both keys were operative. Preference for a stimulus was found to equal (match) the relative rate of primary reinforcement in its presence. Other aspects of the schedules, such as the relative number of

responses per reinforcement, were poorly or not at all correlated with preference.

Although these studies showed that preference depends on the temporal distribution of reinforcements, there was no consensus as to how reinforcement frequency should be calculated in order to achieve matching. Autor and Herrnstein, who used variable-interval and variable-ratio schedules of primary reinforcement, measured frequency as the reciprocal of the arithmetic mean of the interreinforcement intervals. Fantino, who used fixed-ratio (FR) and mixed-ratio schedules of primary reinforcement, measured frequency as the reciprocal of the geometric mean of the interreinforcement intervals. When Herrnstein (1964b) studied preference for variable-interval (VI) *vs* fixed-interval (FI) schedules, he could find no simple transformation on the distribution of interreinforcement intervals that would cause preference to match relative frequency of reinforcement.

The problem of designating the correct measure of reinforcement frequency is a basic one. It entails first the decision of criteria for a "good" measure, and second, a technique for finding a transformation which most closely satisfies those criteria. Certainly a necessary criterion for any measure of reinforcement frequency, when this is assumed to be the controlling variable, is the following: whenever an organism is indifferent between different schedules of reinforcement, appropri-

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ate measures of reinforcement frequency for these schedules will be equal. The following experiments constituted an attempt to find a transformation on distributions of interreinforcement intervals that will satisfy this criterion.

EXPERIMENT I

The purpose of this experiment was to find, for several VI schedules, those FI schedules that an organism will prefer exactly half the time ($FI_{0.5}$). Once such schedules are obtained, a transformation on the distribution of interreinforcement intervals will be sought that will yield measures that are equal for schedules between which the organism is indifferent, and be valid for different VI schedules. The transformations investigated are the family of power functions, $f(y) = y^r$. Given a VI schedule with N intervals of $y_1, y_2, \dots, y_n$ sec, and an $FI_{0.5}$ schedule of x sec, the following equation will be true for some r :

$$\frac{1}{N}(y_1^r + y_2^r + \dots + y_n^r) = x^r$$

if $y_{min} < x < y_{max}$.

This equation, written the following way,

$$x = M_r = \left[\frac{1}{N} \sum_{i=1}^N (y_i^r) \right]^{1/r} \tag{1}$$

is the formula for one class of generalized means (Hardy, Littlewood, and Polya, 1959). Measures of central tendency such as the root-mean-square, the arithmetic mean, and the harmonic mean are obtained when $r = 2, 1$, and -1 , respectively. As r approaches 0, M_r approaches the geometric mean. The use of this formula permits investigation of not only the more familiar measures of central tendency, but also those measures characterized by a fractional r .

Subjects

Three adult male White Carneaux pigeons, and one adult male homing pigeon (#239), all with previous experimental histories, were maintained at 80% of their free-feeding weight.

Apparatus

The experimental chamber contained two response keys, which required forces of about

20 g to be operated, and a food hopper which occasionally provided 4-sec access to grain. The chamber was illuminated by two 7-w white bulbs, and, except during reinforcement, the response keys were transilluminated at different times with lights of various colors, correlated with various phases of the experiment. White masking noise was continuously present.

Procedure

At the start of each session, both keys were illuminated with blue light. Responding on either key was reinforced, according to independent VI 1-min schedules, by a change of key-light color. A response on the left blue key was reinforced by a change of that key color to red, with the other key going dark. Responding on the left red key was then reinforced with grain according to an FI schedule. After one such reinforcement the program reverted to the original state, with both keys blue. Responses to the right blue key were reinforced by a change of that key-light color to green, with the other key going dark. Responding to the right green key was then reinforced with grain according to a VI schedule, after which the program reverted to the original state. All responses to illuminated keys resulted in an audible feedback click.

Table 1
Duration of Experimental Conditions

Terminal link		Birds	Number of Sessions
Left Key	Right Key		
FI (sec)	VI (sec)		
5	23	276, 277	39
7.5		239, 321	39
10		276, 277	28
13		239, 321	28
7.5	54	239, 276	50
10		277	50
10		321	40
15		239	40
15		321	50
20		276	40
25		277	40
10	31	277	48
10		321	50
15		239, 276	50
20		276	48
20		277	50
25		239	25
25		321	48

Table 2
Intervals for VI Schedules in Exp. I and II (All Values in Seconds)

Experiment	VI	Nominal Function Rule	Programmed Intervals
Link 1			
I, II	56	Arbitrary	80, 9, 94, 85, 43, 100, 5, 10, 87, 51
Link 2			
I	23	Geometric progression	2.7, 52, 16.4, 5.2, 23.7, 10.1, 4.1, 74.8, 34.8, 7.7
I	54	Geometric progression	5.7, 3.7, 74.5, 228, 2.7, 15.2, 9.0, 130, 25.3, 43.3
I	31	Arithmetic progression	29, 51.9, 39.9, 16.4, 10.3, 45.9, 56.7, 22.2, 34.1, 4.3
II (right key)	40	Arithmetic progression	56.4, 60, 18, 36.6, 66.3, 48, 6.8, 30.5, 42.4, 12.8, 24.5, 76
II (left key)	80	Geometric progression	4, 35.5, 50.5, 3.1, 25.3, 14.7, 124, 394, 5.7, 8.8, 74.6, 217

Sessions terminated after 40 reinforcements with grain, and schedules were changed when preferences appeared stable from day to day.

This procedure consisted of two chained schedules, one for each key. The first links, correlated with the blue key-lights, were always identical concurrent VI 1-min schedules, running in the same direction but out of phase with each other. The second links were mutually exclusive VI and FI schedules, as listed in Table 1. The sequences of programmed

intervals for the VI schedules are given in Table 2.

Results

Table 3 contains the rates of responding in the initial and terminal links for each bird. Each entry is the geometric mean of the rates from the last five days on each schedule. Figure 1 shows the median relative number of responses on the FI key over the last five sessions, as a function of the rate of reinforce-

Fig. 1. Relative amount of responding on the FI key during the first link as a function of the absolute rate of reinforcement for the FI key in the second link. The linear regression lines and corresponding equations are shown on each graph. If preference matches the relative harmonic rate of reinforcement, the points should fall on the dotted lines.

Table 3
Responses per Minute on Left and Right Keys

Experiment I						
Bird	Schedule		Rate Link 1		Rate Link 2	
	Left	Right	Left	Right	Left	Right
276	FI 5-sec	VI 23-sec	23.2	16.9	79.1	72.4
277			43.2	18.2	79.3	91.9
239			31.0	29.3	37.5	79.1
321	7.5		33.2	35.6	114.2	78.3
276			25.9	26.1	47.1	51.1
277			23.1	25.1	92.3	83.6
239	13		25.1	35.8	57.5	100.7
321			20.3	39.6	102.9	59.1
239	7.5	VI 54	32.0	24.8	51.2	71.9
276			26.1	19.2	116.5	58.0
277			28.4	38.4	62.4	84.4
321	10		28.7	34.4	82.8	56.6
239			19.9	28.5	61.1	65.7
321			22.3	37.8	51.3	56.0
276	20		16.2	35.5	55.9	64.0
277			10.4	45.0	70.9	80.5
277	10	VI 31	35.3	13.1	49.2	60.8
321			39.8	22.8	56.3	61.6
239			30.8	23.7	41.2	60.4
276	15		20.4	22.3	64.7	40.3
276			11.6	29.6	63.3	41.3
277			43.4	32.5	43.8	79.8
239	25		25.8	39.9	65.7	75.2
321			28.7	35.6	34.9	73.5
Experiment II						
365	VI 80	VI 40	35.2	17.7	72.0	61.4
366			48.1	14.4	71.9	98.9
468			25.7	14.7	41.6	40.6

ment for that key. The linear regression lines for these points are also shown. The use of straight lines to indicate the locus of these points is misleading, because it implies that preference for a schedule is proportional to the rate of reinforcement for that schedule. It is more probable that rates of reinforcement on the two schedules interact to determine preference. However, the linear regression line will provide a first approximation to the true locus, and permit a tentative interpolation to find $FI_{0.5}$. Setting $y = 0.5$ and solving the equations gives $FI_{0.5}$ values of 8.4, 9.6, and 16.9 sec for VI's 23, 54, and 31, respectively. Attempt now to find the value of r such that $M_r = FI_{0.5}$. Equation 1 was solved for 36 values of r , ranging from 1.5 to -2.0 in steps of 0.1, and then for another 21 values of r from -0.90 to -1.10 in steps of 0.01. The deviation between M_r and $FI_{0.5}$ reached a minimum for all schedules when r was between -0.93 and

-1.04 . When r was -1.0 , M_r was 8.3, 9.3, and 17.1 for VI's 23, 54, and 31, respectively.

An exponent of -1.0 in Equation 1 would indicate that the appropriate measure of central tendency for distributions of interreinforcement intervals is the harmonic mean. This measure is obtained by taking the reciprocal of the average of the reciprocals of the interreinforcement intervals. Whenever two schedules have equal harmonic means, a pigeon would be expected to be indifferent between them. This conclusion does not depend on any assumptive relation between preference and conditions of reinforcement, such as the matching relation. It is interesting, however, to note how the harmonic transformation affects the relation of preference to relative frequency of reinforcement. Figure 2 shows the relative number of responses to the VI key in the initial link as a function of the relative harmonic rate of reinforcement on

Fig. 2. Relative amount of responding on the VI key during the first link as a function of the relative harmonic rate of reinforcement for the VI schedule during the second link.

that key in the terminal link, for the three VI's in this study, and for Herrnstein's (1964b) data. The relative harmonic rate of reinforcement is calculated in the following way:

Let y_i = the value of the i^{th} interval on the VI schedule,

x_1 = the value of the FI schedule,

N = the number of intervals on a schedule, and

$$v(z) = \frac{1}{N} \sum_{i=1}^N \frac{1}{z_i}.$$

Then the relative harmonic rate of reinforcement for the VI schedule = $\frac{v(y)}{v(y) + v(x)}$.

As can be seen in Fig. 2, preference for a stimulus equals the relative harmonic rate of reinforcement in its presence. These graphs may be compared with those of Fig. 3, which shows preference as a function of the relative arithmetic rate of reinforcement.

If, as seems to be the case, preference matches the relative harmonic rate of reinforcement, the points in Fig. 1 should fall along the dotted lines. The linear regressions are close enough to these dotted lines in the range where data were collected to justify their use in calculating $FI_{0.5}$.

The data from the four studies, averaged across birds, are presented in Fig. 4. Although the distributions of interreinforcement inter-

Fig. 3. Relative amount of responding on the VI key during the first link as a function of the relative arithmetic rate of reinforcement for the VI schedule during the second link.

vals for the four VI's were quite different, there is no systematic deviation from the 45-degree line that is correlated with the VI schedules. In these experiments, therefore, the harmonic transformation preserves all the important information about the distributions. Preference depends on other temporal

Fig. 4. Relative amount of responding on the VI key during the first link as a function of the relative harmonic rate of reinforcement for the VI schedule during the second link. Data are averaged across birds from the three studies of Exp. I, and from Herrnstein's (1964b) study. Solid points represent single observations.

aspects of the schedules, such as the variance or the skewness of the interreinforcement intervals, only insofar as these aspects affect the harmonic rate of reinforcement.

EXPERIMENT II

As the value of the exponent r in Formula 1 decreases, the value of the corresponding generalized mean is increasingly determined by the smaller values in the set $\{y\}$. It is therefore possible to construct two VI schedules such that the arithmetic mean of the first is greater than that of the second, while the harmonic mean of the first is less than that of the second. This condition would obtain if the first schedule contained a sufficiently greater proportion of short intervals than the second. If preference matches the relative arithmetic rate of reinforcement, the second schedule should be preferred, whereas if preference matches the relative harmonic rate of reinforcement, the first schedule should be preferred. Such an experiment would provide a strong test of the adequacy of the harmonic rate of reinforcement as the appropriate measure of reinforcement frequency.

Subjects

Three adult, male White Carneaux pigeons were maintained at about 80% of their free-feeding weight. Each pigeon had been used in other concurrent-chain experiments, but not in Exp. I of this study.

Apparatus

The experimental chamber was the same as in Exp. I. The response keys were adjusted so that they required forces of 15 g to be operated, and the duration of access to grain was reduced to 3.5 sec.

Procedure

The concurrent-chain procedure was basically the same as in Exp. I, but now VI schedules were used in both terminal links. The intervals for these schedules are given in Table 2. The arithmetic and harmonic means for these schedules are respectively, left key: 79.8, 11.5; right key: 39.9, 24.6. All responses to illuminated keys resulted in both an audible feedback click, and a brief (35-msec) flicker of the key lights. Sessions were terminated after 48 reinforcements with grain. All birds performed daily for 37 sessions.

Results

The relative arithmetic rate of reinforcement and the relative harmonic rate of reinforcement for the left key were, respectively, 0.33 and 0.68. The median preferences over the last five sessions for the schedule on the left key were 0.68, 0.75, and 0.65. Averaged across birds, the mean preference of 0.69 is very close to that predicted by the relative harmonic rate of reinforcement, and obviously discrepant from that predicted by the relative arithmetic rate of reinforcement. (The obtained rates of responding in each link for each bird are shown at the bottom of Table 3.)

DISCUSSION

Chung and Herrnstein (1967) measured the relative amount of responding on concurrent VI schedules in which various delays of reinforcement were associated with each schedule. Their procedure may be viewed as a concurrent-chain schedule where reinforcement in the terminal link was not conditional on responding. They found that preference matched the relative immediacy of reinforcement associated with each schedule, immediacy being defined as the reciprocal of the delay of reinforcement. These results elucidate the findings of the present study. Behavior is often more easily analyzed in terms of relevant psychological dimensions, rather than arbitrary physical dimensions (Blough, 1965; Stevens, 1955). Thus, in predicting where a human subject will bisect the loudness of two tones, it is better to average the *sone* values of these tones than to average their decibel levels. Similarly, if preference depends on the immediacy of reinforcement, when more than one value of delay is associated with a schedule, it would seem more appropriate to average immediacies than to average delays. Average immediacy of reinforcement is, of course, the harmonic rate of reinforcement. By averaging the reciprocals of delays, this measure gives more weight to shorter delays than does the arithmetic rate of reinforcement, and reflects more faithfully the inverse relation between delay and efficacy of reinforcement.

In the present experiment the harmonic transformation was employed because it satisfied an explicitly defined criterion. Since most experiments in the analysis of behavior are of a more exploratory nature, they generally

lack such criteria, and transformations of the data are treated more as a matter of style than of necessity. Logan (1960) systematically converted latencies of exit from a start box to their reciprocals before averaging them, presumably to obtain measures with a more normal distribution. Transformations to achieve homogeneity of variance are useful, since combination of several distributions of scores into a single distribution effectively weights the separate distributions in proportion to their variability (Mueller, 1949). Clark (1959), in his study of time-correlated reinforcement schedules, found that the standard deviation of response rate was proportional to the mean response rate. Here a logarithmic transformation on response rate would tend to equalize the variance for different rates. Thus, in combining rates within or across birds, it is the logarithm of rate that should be averaged, perhaps by use of the geometric mean.

The harmonic transformation may prove useful in analyzing the results of other experiments. McDiarmid and Rilling (1965) measured ordinal preferences for different VI schedules, and found that a scale based on the harmonic rate of reinforcement was useful in accounting for their data. Gollub (1958), in his study of second-order schedules, found that much higher rates of responding were maintained in the early links of a chain FR 5 (VI 1) than in the early links of a chain FR 5 (FI 1). This finding is consonant with the fact that a VI 1-min schedule has a greater harmonic rate of reinforcement than an FI 1-min schedule.

From the present results it is not clear whether reinforcement in the presence of a stimulus confers upon that stimulus a reinforcing strength of its own, which mediates behavior in the first link, or whether reinforcement acts directly on responses in the first link with an effectiveness inversely proportional to its delay. That question may be settled by an experiment employing more than one reinforcement in the terminal links. If first-link behavior is maintained by the change in key-light color, relative harmonic rate of reinforcement should predict preference. If, on the other hand, reinforcement in the terminal link acts directly on first-link behavior, preference should match the relative immediacy of reinforcement on a key, where

immediacy is measured from the last response in the first link to each reinforcement separately, and then summed.

Experiments which report matching to some other scale of reinforcement frequency (e.g., Herrnstein, 1964a), are not necessarily inconsistent with the present results. If the inter-reinforcement intervals of one schedule are proportional to those of another, all generalized means of the type corresponding to Formula 1 will predict the same preference between these schedules. It is only when this proportionality between schedules is relaxed, as it was in Exp. I and II, that it becomes possible to determine the correct transformation on reinforcement frequency.

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