THE GENERATION OF RANDOM INTERVAL SCHEDULES¹

FOGLE C. CLARK AND LARRY D. HULL

EVANSVILLE STATE HOSPITAL

In a random interval schedule of reinforcement the probability (P) of a reinforcement becoming available in a small time interval (T) is constant and independent of the number of intervals since the last reinforcement availability. Under these conditions, inter-reinforcement availability times follow the geometric distribution with $P(N) = PQ^{N-1}$, where $P(N)$ is the probability of exactly N intervals T between reinforcement availabilities, and $Q = (1-P)$. The minimum inter-reinforcement availability time is T and the mean is T/P . The probability of more than N intervals between reinforcements follows the cumulative distribution $P_{>N} = Q^N$. If the minimum inter-reinforcement availability time becomes infinitesimal, the geometric distribution approaches the exponential with $P(t) = re^{-rt}$ and $P_{>t} = e^{-rt}$ where t is the time between reinforcement availabilities and r is the rate of reinforcement. These relations are discussed in Feller (1957), Mueller (1950), and McGill (1963). Random interval schedules with P less than one and T on the same order of magnitude as an inter-response time give rise to a distribution of inter-reinforcement availabilities which constitute a special class of variable-interval schedules.

Random interval schedules have been studied by Millenson (1963), and by Farmer (1963). Millenson made use of radioactive cobalt emission as a source of random pulses for programming purposes. This method yields a satisfactory distribution of inter-reinforcement availability times, but is cumbersome and somewhat expensive. Farmer programmed random intervals by feeding a train of pulses with

a fixed period to a random-ratio generator which controlled the probability of an output pulse, given an input. Although not all deviations from a geometric distribution will have effects on behavior, the use of periodic clock pulses as inputs to currently available random-ratio generators produces a distribution of outputs which deviates systematically from the geometric. This is because, with these units, the probability of an output is not independent of the arrival time of input pulses for sufficiently small intervals of time. Hence, a constant probability of an output is maintained only if input pulses are aperiodic.

This discussion will be confined to the BRS Electronics Precision Probability Unit, Model PP-1.² Although circuit details differ somewhat, the same considerations apply to other similar equipment such as the random-ratio generator described by Schoenfeld, Cumming, Snapper and Haas (1960). In these units, input pulses pass through a gate which is controlled by an internal train of pulses with a fixed repetition rate and an adjustable ratio of "on" time to total cycle length. During the "on" portion the gate is enabled and an input will produce an output; during the "off" portion no output can occur. The probability of an output is set by adjusting the ratio of "on" time to total cycle duration in the pulse train. In the PP-1 this is accomplished by an internal multivibrator in combination with counters. One gating cycle consists of 100 pulses from the internal clock. The gate is enabled by the first pulse in each cycle, and disabled by the n th pulse, n being a preset number between 1 and 100. If, for example, a probability of 0.50 is set on the unit, then during each gating cycle, the gate will be enabled for the first 50 pulses of the internal clock and disabled for

¹Supported by Grant NsG-446 from the National Aeronautics and Space Administration and Grants MH-04335 and MH-05863 from the National Institutes of Mental Health. Reprints may be obtained from the authors at Research Department, Evansville State Hospital, Evansville, Indiana.

²Manufactured by BRS Electronics, 5451 Holland Drive, Beltsville, Maryland.

the second 50. The frequency of the internal multivibrator is about 3 kc. The period of the internal clock is, then, about 0.33 msec, and the duration of a gating cycle is 33 msec. The error involved in this cycle is on the order of 10% of the internal multivibrator period. From this it is obvious that if external pulses are fed to the unit with random variations of as much as 30 or 40 msec in their arrival times, the probability of any pulse producing an output will be independent of the outcome of the last previous pulse. On the other hand, if external pulses with a fixed frequency and very little error (say, 4% of 1 sec or less) are fed to the unit, then some degree of synchrony will be established between the external clock and the internal gating cycle. Beating between the two clocks will occur, depending upon their harmonic relationship, and the probability of an output will not be independent of the outcome of the last previous pulse. Further, since successive external pulses may tend to fall near the same portion of internal gating cycles, bias in the probability of an output can be introduced and will vary according to the setting on the unit. With small probability settings, the probability of an output will be even less than the preset value. With large probability settings, the probability of an output will be greater than the setting.

The four factors contributing to this state of affairs are the error or drift of the external clock, the error in the internal gating cycle, the harmonic relation between the two clocks, and the probability setting. The condition for obtaining a geometric distribution of inputs between outputs is that the variation in input arrival times be greater than the duration of a gating cycle. Since it is generally undesirable to build error into clock circuits, the most obvious solution is to decrease the duration of the internal gating cycle. With circuits such as that in the PP-1 this may be done by increasing the frequency of the internal multivibrator. Figure 1 compares the cumulative distribution of input pulses between outputs obtained from an unmodified PP-1 with a 3 kc internal clock, and the distribution obtained from the same unit with the internal multivibrator frequency raised to 13 kc. In both cases the input pulse frequency was 1 per second and the probability of an output was set at 0.10. Both distributions are based on

Fig. 1. Cumulative distribution of input pulses between outputs obtained with a BRS PP-1 unmodified and modified to increase the internal clock frequency from 3 kc to 13 kc. The input pulse rate was 1 pps out of a precision clock (BRS MV-4) and the probability of an output was set at 0.10. Both samples contained 100 outputs. The smooth curve gives the cumulative geometric distribution $P_{>N} = Q^N$.

samples of 100 outputs. From the point of view of the error in the external clock these are "worst case" data since inputs were supplied from a precision clock controlled by a 1 kc tuning fork with an error on the order of

Fig. 2. Obtained cumulative proportions $p_{>N}$ as a function of the corresponding predicted proportions of the geometric distribution. The parameters are the input pulse period (T) and the preset probability of an output (P). All data are based on samples of 100 outputs and were obtained with a PP-1 with an internal clock frequency of 13 kc.

0.1% (BRS Electronics Precision Clock, Model MV-4). The stepwise deviations from the theoretical distribution give clear evidence of beating in the data obtained with a 3 kc internal multivibrator. On the other hand, the distribution obtained with a 13 kc internal clock conforms well with the geometric distribution. This is particularly true for the short inter-reinforcement availability times, which are most likely to influence behavior. Data obtained with a 13 kc internal clock and several additional input pulse rates and probability settings are presented in Fig. 2. In this case the obtained cumulative proportions $p_{>N}$ have been plotted against the proportions predicted for the geometric distribution with the corresponding values of T and P. These data are all based on samples of 100 outputs. One distribution in Fig. 2 deviates above the line and one below the line. This corresponds to variation in the mean inter-reinforcement availability time which appears to be due to sampling, since a number of samples of the same size obtained under the same conditions varied in either direction about equally often. With the internal multivibrator running at 13 kc, all distributions of inputs per output obtained with the values of T and P shown in Fig. 1 and 2 conformed well to the geometric distribution.

The modification described above for the PP-1 may be made by replacing the .005-mfd capacitors in the multivibrator circuit with

.002-mfd capacitors. This raises the multivibrator frequency to about 13 kc, giving a period of .077 msec and a gating cycle about 8 msec in duration. With other electronic probability gate circuits, analogous changes can be made to reduce the duration of gating cycles. In the case of the PP-1, the multivibrator frequency should not be raised much above 13 kc since the operation of the counting circuits which control the gating cycle becomes marginal at about 16 kc. With higher multivibrator frequencies failure of the counting circuits occurs because of variations in the arrival time of pulses at and-gates.

REFERENCES

- Farmer, J. Properties of behavior under random interval reinforcement schedules. *J. exp. Anal. Behav.*, 1963, 6, 607-616.
- Feller, W. An introduction to probability theory and its applications. 2nd edition. New York: John Wiley and Sons, 1957.
- McGill, W. J. Stochastic latency mechanisms in R. D. Luce, R. R. Bush, and E. Galanter (eds.) *Handbook of mathematical psychology: Vol. I*. New York: John Wiley and Sons, 1963, pp. 309-360.
- Millenson, J. R. Random interval schedules of reinforcement. *J. exp. Anal. Behav.*, 1963, 6, 437-443.
- Mueller, C. G. Theoretical relationships among some measures of conditioning. *Proced. Nat. Acad. Sci.*, 1950, 36, 123-130.
- Schoenfeld, W. N., Cumming, W. W., Snapper, A. G., and Haas, P. Some electronic control units for operant behavior studies: II. A random ratio generator. *J. exp. Anal. Behav.*, 1960, 3, 107-108.