A CONTEXTUAL MODEL OF CONCURRENT-CHAINS CHOICE

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An extension of the generalized matching law incorporating context effects on terminal-link sensitivity is proposed as a quantitative model of behavior under concurrent chains. The contextual choice model makes many of the same qualitative predictions as the delay-reduction hypothesis, and assumes that the crucial contextual variable in concurrent chains is the ratio of average times spent, per reinforcement, in the terminal and initial links; this ratio controls differential effectiveness of terminal-link stimuli as conditioned reinforcers. Ninety-two concurrent-chains data sets from 19 published studies were fitted to the model. Averaged across all studies, the model accounted for 90% of the variance in pigeons' relative initial-link responding. The model therefore demonstrates that a matching law analysis of concurrent chains—the assumption that relative initial-link responding equals relative terminal-link value—remains quantitatively viable. Because the model reduces to the generalized matching law when terminal-link duration is zero, it provides a quantitative integration of concurrent schedules and concurrent chains.

Key words: concurrent chains, generalized matching law, delay-reduction hypothesis, conditioned reinforcement, context of reinforcement, pigeons, key peck

Choice in the concurrent-chains procedure, as operationalized by relative initial-link responding, has proven highly resistant to a single, comprehensive quantitative analysis. In a recent review, Davison (1987) fitted three competing concurrent-chains models, those of Davison and Temple (1973), Killeen (1982), and Squires and Fantino (1971), to archival data from 10 studies. He reported that none of the models provided an adequate quantitative description of the data across the studies. Davison (1987) concluded that model building in concurrent chains was premature, and that research should proceed through a careful parametric analysis of circumscribed aspects of the concurrent-chains procedure. Other researchers have noted the difficulties that the “complex contingencies of the concurrent-chains procedure” create for a quantitative model (Mazur, 1984, p. 436; see also Houston, 1991).

In spite of these difficulties, the concurrent-chains paradigm has inspired some of the most popular theories of choice. These theories, such as the delay-reduction hypothesis (Squires & Fantino, 1971), incentive theory (Killeen, 1982), and melioration (Vaughan, 1985), embody to some extent what Davison (1987) termed the “lore” of concurrent chains: (a) As initial links increase relative to terminal links, preference becomes more sensitive to initial-link and less sensitive to terminal-link differences; and (b) as initial links decrease relative to terminal links, preference becomes less sensitive to initial-link and more sensitive to terminal-link differences.

A new model is presented here that provides an adequate quantitative description of a wide range of concurrent-chains data. The model is called the contextual choice model (CCM) because it generalizes the generalized matching law (Baum, 1974b) through the incorporation of context effects. CCM demonstrates that a matching law analysis of concurrent chains, that is, the assumption that relative initial-link responding equals relative terminal-link value, remains quantitatively viable.

It is important to clarify exactly what is meant by *context*. When preference for an alternative is determined by factors other than attributes of the alternative itself, we say that preference is affected by context. There are several ways in which context may influence preference, however. Grace (1993) showed that

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Fig. 1. Schematic diagram of a typical concurrent-chains procedure. In the initial links, both keys are illuminated white, and responding is occasionally reinforced according to *conc VI VI* schedules by entry into one of two mutually exclusive terminal links, during which one key is illuminated red or green and the other is extinguished. Responding during terminal links is reinforced by access to grain, after which the initial links are reinstated. Relative responding in the initial links is taken as a measure of preference for left (red) versus right (green) terminal-link schedules.

if sensitivity exponents in a generalized matching model were affected by similarities between attributes of the choice alternatives, violations of weak stochastic transitivity (i.e., a is preferred to b and b to c , but c is preferred to a) could result. Context effects of this nature are often studied in human preference (e.g., Tversky, 1969). Another way in which context can influence preference occurs when parameters of the choice procedure, common to both alternatives, affect preference separately from a direct comparison of the alternatives. For example, increasing the period during which a pigeon chooses between the same pair of alternatives results in less extreme preference (Fantino, 1969). This kind of context effect has been widely reported in concurrent chains, and is the focus of this paper. More precisely, context in concurrent chains refers to relative initial- and terminal-link lengths. Before de-

scribing the new model (CCM), I will review previous applications of the matching and generalized matching laws to concurrent chains.

CONCURRENT CHAINS AND THE MATCHING LAW

The concurrent-chains procedure was introduced by Autor (1960, 1969) and Herrnstein (1964b) to determine whether the matching law could describe allocation of behavior maintained by conditioned rather than immediate primary reinforcement. A typical arrangement is diagrammed in Figure 1. Responding on concurrent variable-interval variable-interval (*conc VI VI*) schedules (initial links) is reinforced by access to one of two mutually exclusive schedules (terminal links) that terminate in primary reinforcement. After a reinforcer has been delivered in a terminal link, the initial links are reinstated. Relative initial-link responding is assumed to be a measure of preference for terminal-link schedules, or a measure of the value of the terminal-link stimuli as conditioned reinforcers. Autor and Herrnstein found that relative initial-link responding matched relative terminal-link reinforcement rates, and concluded that (a) the value of a terminal-link stimulus as a conditioned reinforcer was proportional to the rate of reinforcement in the presence of the stimulus, and (b) the matching law described *conc VI VI* performance maintained by conditioned reinforcement.

The generality of both conclusions was soon questioned, however. Fantino (1969) showed that absolute initial-link duration strongly affected preference, and proposed the delay-reduction hypothesis—the value of a terminal-link stimulus as a conditioned reinforcer is determined by the reduction in time to primary reinforcement, relative to the onset of the initial links, signaled by the stimulus—as a model for concurrent chains. In concurrent schedules, frequent empirical deviations from strict matching resulted in the generalized matching law, which introduced two parameters: a bias term, b , that reflected a constant proportionality in behavior due to factors such as unequal response key force requirements, and an exponent, a , that indicated the degree of undermatching or overmatching (Baum, 1974b):

$$\frac{B_L}{B_R} = b \left(\frac{\mu_R}{\mu_L} \right)^a \quad (1)$$

In Equation 1, B_L and B_R are the responses emitted, and μ_L and μ_R are the average intervals between reinforcers on the left and right schedules, respectively.¹ The generalized matching law has proven to be a reliable quantitative description of behavior allocation in concurrent schedules (Baum, 1979; Davison & McCarthy, 1988; Wearden & Burgess, 1982).

There has been no consensus on a quantitative description of concurrent chains, however. Williams (1988) suggested that the concurrent-chains procedure was so complex that it might be impossible for a single model to describe it adequately. Instead, concurrent chains has inspired a variety of competing theoretical explanations, such as delay reduction (Squires & Fantino, 1971), incentive theory (Killeen, 1982), and melioration (Vaughan, 1985). Although these models may bear some relationship to the matching law (e.g., the Squires and Fantino model reduces to strict matching when terminal-link durations are zero), it would be desirable for reasons of parsimony for a concurrent-chains model to reduce to the generalized matching law in the limit as terminal-link duration approached zero. Because *conc VI VI* can be viewed as a special case of concurrent chains when terminal-link delays are zero, such a model would provide a quantitative integration of *conc VI VI* and concurrent chains (Davison, 1983, 1987).

One further generalization of the matching law is relevant for concurrent chains. According to Baum and Rachlin (1969), response allocation in *conc VI VI* matched the value of the alternative schedules. Furthermore, different independent variables (such as rate, amount, and immediacy of reinforcement) could be combined to produce a single intervening variable of reinforcer value. If we allow an arbitrary number of independent variables to be combined multiplicatively to determine

reinforcement value, the most general form of the matching law becomes (Baum, 1974b):

$$\frac{B_L}{B_R} = b \prod_{i=1}^n \left(\frac{x_{iL}}{x_{iR}} \right)^{a_i} = \frac{V_L}{V_R}. \quad (2)$$

According to Equation 2, the value of a schedule is a multiplicative concatenation of its appropriately scaled attributes. Killeen (1972) has noted that the matching law is really a statement about how value is to be scaled: Given that relative behavior corresponds to relative value, we need only specify how independent variables differentially affect value. Of course, this is the assumption made by Autor (1969) and Herrnstein (1964b): Relative initial-link responding in concurrent chains equals the relative value of the terminal-link stimuli as conditioned reinforcers. The failure to date of matching models of concurrent chains means the validity of this assumption is uncertain, and that other important questions remain unanswered: How is the conditioned reinforcement value of a terminal-link stimulus to be defined? How is the interaction between initial and terminal links to be represented? To address these questions, we need to consider in detail research that has applied the generalized matching law to concurrent chains.

CONCURRENT CHAINS AND GENERALIZED MATCHING

Davison and colleagues have attempted a generalized matching analysis of concurrent chains (Alsop & Davison, 1988; Davison, 1983, 1987; Davison & McCarthy, 1988; Wardlaw & Davison, 1974). Noting that differential terminal-link delays to reinforcement could correspond to differential reinforcement magnitude in *conc VI VI* schedules, Davison (1983) suggested that the following version of the concatenated generalized matching law might apply for at least some concurrent-chains performances:

$$\frac{B_L}{B_R} = b \left(\frac{\mu_{1R}}{\mu_{1L}} \right)^{a_1} \left(\frac{\mu_{2R}}{\mu_{2L}} \right)^{a_2}, \quad (3)$$

where b is a bias term, μ_{1L} and μ_{1R} are the mean initial-link intervals between conditioned reinforcers, μ_{2L} and μ_{2R} are the mean terminal-link delays to primary reinforcement, and a_1 and a_2 are the sensitivities, respectively, to initial-link conditioned reinforcement and

¹ Although it has been more common to write Equation 1 in terms of rate of reinforcement rather than time between reinforcers, I will express this and all subsequent equations in terms of time—average interval between reinforcers for initial links and average delay to reinforcement for terminal links. This is a matter of notation for concurrent schedules, because the average interval between reinforcers is the reciprocal of rate. For concurrent chains, terminal-link delay and rate of reinforcement can be manipulated separately, and delay seems to be the more potent controlling variable (Davison, 1988; Davison & Smith, 1986; Mazur, Snyderman, & Coe, 1985).

terminal-link primary reinforcement rates. Davison (1983) argued that because *conc VI* VI can be viewed as a special case of concurrent chains when terminal-link delay to reinforcement is zero, a model of concurrent chains should reduce to the generalized matching law in the limit as terminal-link delay approaches zero. However, Davison (1987) noted that Equation 3 could not account for the effect of absolute initial-link length on preference (Fantino, 1969), and suggested that a_2 , the sensitivity to terminal-link reinforcement, might be a decreasing function of the smaller initial-link duration instead of a fixed parameter. In that case, one might question the legitimacy of a generalized matching law without parameter invariance (Davison & McCarthy, 1988). In discussing these difficulties, Davison concluded, "such considerations seriously compromise the generalized matching approach to concurrent chains" (1987, p. 228). Interaction between terminal-link sensitivity and relative initial-link to terminal-link length has also been reported by Duncan and Fantino (1970), MacEwen (1972), and Williams and Fantino (1978).

Although such context effects on terminal-link sensitivity (a_2) remain unexplained by a straightforward application of the generalized matching law to concurrent chains, it may be that a_1 , the sensitivity to initial-link reinforcement, is unaffected by context and hence is adequately represented by Equation 3. Using a 3-s initial-link changeover delay (COD) and fixed-duration blackout as the terminal links, Davison (1983) found that for a wide range of conditions, a_1 was independent of changes in either relative or absolute terminal-link delays. Davison (1983) concluded that the use of an initial-link COD may have eliminated troublesome interactions and made a generalized matching analysis more feasible. However, in a replication and extension of Davison's (1983) study, Alsop and Davison (1988) reported that a_1 increased significantly in a nonparametric trend test as a function of shorter initial-link duration. Alsop and Davison (1988), who used VI terminal links, concluded that concurrent-chains performance, even with a COD, was considerably more complex than Davison (1983) originally thought, and that a_1 might be a function of both initial-link and terminal-link durations.

Davison noted that constancy of a_1 was "quite

at variance" with the "lore" of concurrent chains: Initial-link sensitivity should increase as initial-link length increases (1987, p. 236). He presented a reanalysis of group mean data from Fantino and Davison (1983) showing a monotonic increase in a_1 as the average initial-link duration increased (Davison, 1987, Figure 11.2). Terminal links were constant and equal (VI 20 s) across the conditions analyzed. This result can be questioned on two grounds. First, the increase may be an artifact of using group mean data. Second, instead of the average initial-link schedule value that Davison (1987) used for the x axis, the reciprocal of the sum of reciprocals of the initial-link lengths may be more appropriate, because it represents the programmed average time spent, per terminal-link entry, in the initial links. The importance of this measure is implied by the delay-reduction hypothesis (Fantino, 1969), because it is the average time from the onset of the initial links to conditioned reinforcement. Also, this measure is much more sensitive to changes in the shorter initial link: The average time spent in the initial links is never greater than the shorter initial link. The importance of the shorter initial link has been noted by Davison (1983), who suggested that it may affect a_2 , and by Alsop and Davison (1988), who showed that grouping conditions according to the shorter initial link produced data that were well described by a generalized matching model of concurrent chains (Equation 3).

I have reanalyzed the Fantino and Davison (1983) individual subject data (in log-ratio form) for the conditions analyzed by Davison (1987), and plotted the initial-link sensitivities against the average time spent in the initial links. The results are shown in Figure 2. Although there is individual variation in initial-link sensitivity (which may have resulted from only two data points being used to obtain each estimate), Figure 2 indicates that, across subjects, there is no consistent monotonic trend. Bird 34, for example, appears to have an increasing trend and Bird 36 a decreasing one.

In the three largest concurrent-chains data sets, therefore, there is evidence for a monotonic, increasing trend in a_1 as the initial links increased in one study (Alsop & Davison, 1988), but not in two others (Davison, 1983; Fantino & Davison, 1983). Because of the simplification it affords, I will make the assumption that a_1 is constant and unaffected by con-

text. It should be emphasized that this assumption is provisional and future data may require its revision.

Although the status of a_1 may be unclear, researchers have more consistently reported that a_2 , the sensitivity to terminal-link reinforcement, is affected by procedural context. Specifically, a_2 appears to decrease as initial links increase relative to terminal links and to increase as initial links decrease relative to terminal links. Williams and Fantino (1978) found that, with constant initial links, preference between two fixed-interval (FI) schedules in constant ratio (2:1) became more extreme as the absolute terminal-link duration increased. Also using constant initial links, MacEwen (1972) reported that preference between two FI schedules in constant ratio (4:1) became more extreme as absolute terminal-link duration increased, but that preference seemed to reach an asymptote; that is, preference was a negatively accelerated, monotonic increasing function of terminal-link duration. Fantino and Davison (1983) arranged three sets of equal initial-link conditions (VI 15 s, VI 60 s, and VI 180 s) and found that preference for VI terminal links became less extreme as absolute initial-link length increased, which can be interpreted as a decrease in a_2 . Alsop and Davison (1988), however, using a constant pair of terminal links (VI 6 s and VI 12 s), found no systematic change in a_2 as a function of the shorter initial link, but noted that their terminal links may have been too short to observe the expected decrease in a_2 .

The finding that preference between two terminal links becomes less extreme as absolute initial-link length increases was first reported by Fantino (1969), and led to the delay-reduction hypothesis, probably the most popular theory of conditioned reinforcement: The strength of a stimulus as a conditioned reinforcer depends on the reduction in delay to reinforcement signaled by the stimulus, relative to the overall context of delay to reinforcement. As Davison (1987) noted, this result creates fundamental problems for a parameter-invariant generalized matching analysis of concurrent chains. A generalized matching analysis may still be possible, however, if we can specify exactly how terminal-link sensitivity changes as a function of procedural context. If terminal-link sensitivities can be shown to be a function of initial or

Fig. 2. Sensitivity to initial-link reinforcement (a_1 in Equation 3) plotted as a function of the average time spent, per reinforcement, in the initial links. Data are from Fantino and Davison (1983). Four pairs of initial links (VI 15 s and VI 45 s; VI 15 s and VI 180 s; VI 30 s and VI 180 s; VI 60 s and VI 180 s) with equal terminal links (VI 20 s) were arranged. Data were analyzed in log-ratio form. Absence of a consistent monotonic trend across subjects suggests there was no consistent interaction between initial-link sensitivity and relative terminal- to initial-link duration.

terminal-link parameters, the generalized matching law would gain rather than lose support (Davison, 1987). With this in mind, it may be helpful to examine archival data.

Fantino and Davison (1983) reported a large number of conditions for which, with different pairs of constant initial links, terminal links whose durations summed to 60 s

Fig. 3. Sensitivity to terminal-link reinforcement (a_2 in Equation 1) plotted as a function of the initial-link length. Data are three sets of conditions—equal VI 15 s, VI 60 s, and VI 180 s initial links—from Fantino and Davison (1983). For each initial-link length, nine pairs of terminal links that summed to 60 s were arranged. For all 6 subjects, a_2 is a monotonic decreasing, negatively accelerated function. Data were analyzed in choice-proportion form.

were scheduled. Because Fantino and Davison's data also have at least nine conditions per pair of initial-link schedules, a reliable estimate of a_2 can be obtained. Figure 3 shows a reanalysis of their individual subject data. For each subject, a_2 (the y axis in Figure 3) was estimated for three groups of initial-link conditions: equal VI 15 s, VI 60 s, and VI 180 s (the x axis in Figure 3). Figure 3 demonstrates that, for all 6 subjects, a_2 , the sensitivity to terminal-link reinforcement rate, was a decreasing monotonic, negatively accelerated function of initial-link length. Although this rules out a parameter-invariant generalized matching model of concurrent chains, such as Equation 3 (Davison, 1983), it implies that a model only moderately more complex, one that captures the functionally invariant effects of initial-link and terminal-link context on terminal-link sensitivity, may be adequate. Based on the studies reviewed here, terminal-link sensitivity is a function of the ratio of average terminal-link to initial-link duration: As terminal links increase relative to initial links, a_2 increases (MacEwen, 1972; Williams & Fantino, 1978); as terminal links decrease relative to initial links, a_2 decreases (Fantino, 1969; Fantino & Davison, 1983).

To recapitulate: Davison's (1983) application of the generalized matching law to concurrent chains (Equation 3) assumed that initial-link and terminal-link determinants of preference were additive and independent, with invariant sensitivity exponents. Whereas conflicting results exist regarding invariance of a_1

(initial-link sensitivity), a_2 (terminal-link sensitivity) is not invariant, but is a function of the ratio of average terminal-link to initial-link durations. So, in what follows we will continue to assume that a_1 is invariant and that initial-link and terminal-link determinants of preference are additive, as in Equation 3; but we will now assume that the interaction of initial and terminal links—the extent to which independence is violated—affects terminal-link sensitivity.

THE CONTEXTUAL CHOICE MODEL

Terminal-link sensitivity is a function of the ratio of the average terminal-link to initial-link durations. A natural way to represent this quantitatively, as an extension of Equation 3, is to raise the terminal-link term to an additional exponent: the (generalized) ratio of the average terminal-link and initial-link durations. Because the new exponent should also affect any other terminal-link variables, such as reinforcement magnitude, we have the following general equation:

$$\frac{B_L}{B_R} = b \left(\frac{\mu_{1R}}{\mu_{1L}} \right)^{a_1} \left[\left(\frac{\mu_{2R}}{\mu_{2L}} \right)^{a_2} \left(\frac{x_{iL}}{x_{iR}} \right)^{a_i} \right]^{\left(\frac{Tt}{Ti} \right)^k}, \quad (4)$$

where $\left(\frac{Tt}{Ti} \right)^k$ is the additional exponent, k is a scaling parameter, and the ratio $\left(\frac{x_{iL}}{x_{iR}} \right)$ represents a possible additional variable, such as reinforcement magnitude. Tt is the average terminal-link duration, including any post-reinforcer blackouts, and Ti is the average initial-link duration, per reinforcement. The scaling parameter, k , represents how sensitive the terminal-link sensitivities (a_2 and a_i) are to changes in relative initial-link and terminal-link duration, and may be necessary because the different terminal-link stimulus conditions that have been employed in concurrent chains (e.g., terminal links as delays in blackout vs. signaled terminal links) might be unequally affected by temporal context.

Equation 4 is the most general form of the contextual choice model (CCM). Terminal-link sensitivities are an increasing function of terminal-link duration and a decreasing function of initial-link duration, as suggested by Davison and McCarthy (1988, p. 214). Be-

cause of the laws of exponents, the effective terminal-link sensitivity to delay becomes $a_2 \left(\frac{Tt}{Ti} \right)^k$. Equation 4 can therefore be regarded

as a kind of "recursive" generalized matching law: Sensitivity exponents are themselves a generalized ratio of the procedural context.

It would be preferable (for reasons of parsimony) if we could assume the effect of context to be consistent across experiments; in other words, to set $k = 1$. As will be discussed below, with this restriction Equation 4 is still able to predict nearly all data sets accurately, and changes in the value of k appear to be related to terminal-link stimulus conditions. When $k = 1$ and delay to reinforcement is the only attribute varied, Equation 4 becomes

$$\frac{B_L}{B_R} = b \left(\frac{\mu_{1R}}{\mu_{1L}} \right)^{a_1} \left(\frac{\mu_{2R}}{\mu_{2L}} \right)^{a_2 \left(\frac{Tt}{Ti} \right)} \quad (5)$$

Equation 5 predicts that terminal-link sensitivities will approach zero (indifference) as initial-link length increases relative to terminal-link length, as required by the delay-reduction hypothesis (Fantino, 1969, 1977). Also, Equation 5 predicts that terminal-link sensitivity will become more extreme as terminal-link length increases relative to initial-link length, the empirical result reported by MacEwen (1972) and Williams and Fantino (1978).

In addition, Equation 5 has the advantage of reducing to the generalized matching law in the limit as terminal-link length approaches zero, because when $Tt = 0$ the terminal-link ratio equals one. Thus it meets one of Davison's (1987) criteria for a candidate model for concurrent chains. If Equation 5 can provide successful quantitative predictions for concurrent chains, it may serve to integrate concurrent-schedule and concurrent-chains research. Before discussing the fits of CCM to the data, it is worth considering some of the theoretical assumptions of the model with respect to conditioned reinforcement and preference scaling.

Theoretical Assumptions

According to CCM, relative responding in the initial links is controlled by relative rate of conditioned reinforcement, as described by the generalized matching law, and terminal-link value can be concatenated multiplicatively (Baum & Rachlin, 1969; Davison, 1983). CCM retains the traditional assumption (Kel-

leher & Gollub, 1962; Nevin, 1973) that the value of a terminal-link stimulus as a conditioned reinforcer is determined by the average delay to reinforcement (i.e., rate) in its presence (but see below). The effectiveness of differences in conditioned reinforcement value depends on context, however: When relatively more time is spent in the presence of terminal-link discriminative stimuli, differences in conditioned reinforcement value exert more control over behavior allocation; when relatively less time is spent in the presence of terminal-link discriminative stimuli, differences in conditioned reinforcement value exert less control over behavior allocation. This parallels a result in delayed-reward discrimination learning, where the effectiveness of immediate conditioned reinforcement increases as the absolute delay to primary reinforcement increases (Winter & Perkins, 1982).

There is an important distinction in the way in which conditioned reinforcement value is defined in CCM and in the delay-reduction hypothesis. The delay-reduction hypothesis states that values of stimuli as conditioned reinforcers are determined by context (Fantino, 1969, 1977). CCM states that values of stimuli are determined independently of context, but that differential effectiveness of the stimuli is context dependent. The separation of value and effectiveness makes sense for concurrent chains, because context affects variables other than terminal-link delay in a similar way. For example, several studies have shown that as absolute delay increases, preference becomes more sensitive to differences in magnitude of reinforcement (Ito, 1985; Ito & Asaki, 1982; Navarick & Fantino, 1976; White & Pipe, 1987). If relative terminal-link and initial-link durations are functionally equivalent in their effect on sensitivity to different terminal-link variables such as delay and magnitude of reinforcement, then the assumption of Equation 4—that terminal-link variables combine to determine relative value before being rescaled by context—is supported. Therefore, CCM assumes that conditioned reinforcement value and effectiveness of differences in conditioned reinforcement value are separately determined.

CCM represents the effect of temporal context through changes in terminal-link sensitivity. Baum (1974b) suggested that changes in generalized matching sensitivity could be related to discriminability. Miller, Saunders, and Bourland (1980) supported this interpre-

tation by demonstrating that when discriminative stimuli for *conc* VI VI schedules were made more similar (in terms of line orientation), the sensitivity exponent decreased substantially. Because decreases in sensitivity have been observed in concurrent chains without systematic manipulation of physical stimulus differences, it is unclear why spending relatively more (or less) time in the presence of terminal-link stimuli would affect discriminability of the value of the stimuli. An analysis of reinforcement as *situation transition* (Baum, 1973, 1974a) may predict this result, however: As a pigeon spends relatively more time in the terminal-link situation, it is exposed to more information about the situation; hence, terminal-link cues should be more effective as discriminative stimuli.

As noted above, Equation 5 assumes that the value of a terminal-link stimulus as a conditioned reinforcer depends on the rate (i.e., arithmetic-mean delay) of primary reinforcement in its presence. Clearly, the preference for variable over fixed schedules that has been repeatedly demonstrated (Davison, 1969, 1972; Herrnstein, 1964a; Hursh & Fantino, 1973; Killeen, 1968; Mazur, 1984) contradicts this assumption. However, the present analysis is restricted to experiments that arranged either two variable (VI VI) or two fixed (FI or fixed-delay) schedules. When delay-of-reinforcement distributions do not differ by a scalar multiple (e.g., FI vs. VI), it will be necessary to modify Equation 5.²

When terminal links are uncued (i.e., when the stimuli during the terminal links are the same), it is unclear whether temporal context will have the same effect on initial-link allocation as when the stimuli are different. Temporal context may not have the same overall effect on preference in cued and uncued procedures, because only in the cued case is the relative time spent in the presence of exteroceptive terminal-link discriminative stimuli a function of context. Although some studies that arranged uncued terminal links have found that terminal-link sensitivity increases with absolute duration (Williams & Fantino, 1978; and their reanalysis of Chung & Herrnstein,

1967), the magnitude of the context effect may be different.

Another way to understand the generalization represented by CCM is to consider the matching law, generalized matching law, and CCM as pure scaling procedures. The original or strict matching law (Herrnstein, 1961) states that measures of behavior and reinforcement will be equal ("match") in a choice situation, and that behavior and reinforcement can be measured on interval scales with equal interval sizes and a common zero point. The generalized matching law states that measures of behavior and reinforcement will match in a choice situation, and that behavior and reinforcement can be measured on interval scales with unequal interval sizes (sensitivity exponent) and different zero points (bias term). CCM states that measures of behavior and reinforcement will match in a choice situation, that behavior and reinforcement can be measured on interval scales with different zero points (bias term) and with interval sizes that may depend systematically on procedural context, and that systematic effects of procedural context on interval size can be represented through a combination of independent scaling operations. Therefore, CCM generalizes the generalized matching law by specifying how context—relative terminal-link to initial-link duration—systematically rescales preference.

Empirical Evaluation of CCM

To assess the adequacy of CCM as a quantitative model of concurrent chains, Equation 5 was fitted to every archival concurrent-chains data set meeting the following criteria: (a) at least four data points available per subject; (b) use of time-based terminal-link schedules whose delay distributions differed by a scalar multiple (i.e., FI FI or VI VI); (c) equal terminal-link reinforcer magnitudes; and (d) obtained preference measures with a range of at least 12.5%. The data sets included independent and nonindependent (Stubbs & Pliskoff, 1969) initial links, changeover (and no changeover) delays in the initial links, response-dependent and response-independent terminal-link schedules, FI versus FI and VI versus VI terminal links, uncued delays in blackout terminal links (Chung & Herrnstein, 1967; Davison, 1983; Gentry & Marr, 1980), and constant duration terminal links (Davison, 1988; Omino & Ito, 1993). Tandem schedules were

² Grace, R. C. (1993, May). *Models for choice between fixed and variable delays*. Paper presented at the annual meeting of the Society for the Quantitative Analysis of Behavior, Chicago.

not included (Leung & Winton, 1985; Schneider, 1972), and neither were probabilistically reinforced terminal links (Spetch & Dunn, 1987). Average initial-link duration was calculated as the reciprocal of the sum of the reciprocals of the initial-link intervals between reinforcers. Average terminal-link duration was calculated as a weighted average in studies in which terminal-link entries were reported; otherwise, it was calculated as a simple average of terminal-link schedule values. Zero-second initial-link and terminal-link delays were taken as 1 s (Davison, 1983). In total, 92 data sets from 19 published studies were fitted to Equation 5. Relative choice proportions rather than log ratios were used, as is usual with concurrent chains (Davison, 1987; Fantino & Davison, 1983). Parameter values were determined through a nonlinear optimization procedure (Microsoft Excel 4.0 Solver; Microsoft, Inc., 1992). To assess the model, obtained choice proportions were related to predicted choice proportions that were calculated using fitted values of b , a_1 , and a_2 . The context scaling parameter, k , was fitted for nine data sets, as will be discussed below.

Table 1 compares the percentage of variance accounted for by predictions from CCM (Equations 4 and 5) with the percentage of variance accounted for by the three models fitted to the data from 10 studies in the review by Davison (1987). It is clear that CCM was more successful in this regard than were the other models. It should be noted, in fairness, that CCM should be expected to account for more variance because it contains more free parameters.

Table 2 lists the average estimated parameter values, percentage of variance accounted for, and obtained versus predicted regression statistics for CCM in each of the 19 studies. (A table of individual-subject parameter estimates for all 92 data sets is available from the author.) Column 1 in Table 2 lists the number of data points per subject for each study. Columns 2 through 5 list average estimated parameter values. Column 6 gives the average proportion of variance accounted for by the model. Columns 7 through 11 list average parameters of regressions performed on the obtained versus predicted data. Accurate prediction is indicated by a slope (column 7) near or equal to one, an intercept (column 8) near or equal to zero, and small standard errors of

Table 1

A comparison of concurrent-chains models: percentage of variance accounted for (VAC) when fitted to 10 archival data sets. Fits for Davison and Temple (1973; "D&T"), Killeen (1982; "K") and Squires and Fantino (1971; "S&F") models are those reported in the review by Davison (1987). These fits were to group mean data. VAC reported for CCM (Equation 5) are averaged across individual-subject data sets, except for Davison (1976) and Wardlaw and Davison (1974), where only group means were reported.

Data set	D&T	K	S&F	CCM
Fantino (1969)	7	2	95	99
Duncan and Fantino (1970)	76	90	64	98
Squires and Fantino (1971)	53	56	56	86
MacEwen (1972)	0	0	0	92
Davison and Temple (1973)	95	98	87	92
Wardlaw and Davison (1974)	91	85	70	94
Davison (1976)	78	65	31	98
Gentry and Marr (1980)	0	0	0	78
Fantino and Davison (1983)	77	65	82	92
Davison (1983)	74	47	59	82
Average VAC	55	51	54	91

slope (column 9), intercept (column 10), and prediction (column 11).

Averaged across all studies, CCM accounted for 90% of the variance, significantly more than the roughly 55% accounted for by the other models. Although accounting for a large percentage of variance is desirable, it is also important, as Davison (1987) noted, that regressions performed on the obtained versus predicted data points indicate little systematic deviation. Davison suggested two regression criteria for an adequate quantitative fit to a data set: The slope and intercept estimates should differ from 1.0 and 0.0 (perfect prediction) by less than two standard deviations, and the standard error of prediction (SE_y) should be 0.10 or less. All but 3 of the 92 fits to the individual-subject data sets met these criteria.

Davison (1987) added an additional criterion, that the slope be accurately estimated (defined as a standard deviation of one tenth or less of the parameter estimate). The fits were less successful in this regard, with 39 data sets not meeting this criterion. However, this criterion may be affected by the number of data points. Of the the 60 sets with 12 or fewer data points, 37 had slope standard deviations greater than one tenth the slope estimate, whereas for the 32 sets with more than 12 data points, only 2 had slope standard de-

Table 2

Summary of estimated parameters, percentage of variance accounted for, and obtained versus predicted regression statistics when CCM (Equation 5) was fitted to each data set. All estimated parameters and regression statistics are averaged across individual data sets. n = number of data points in set. b , a_1 , a_2 , and k are parameters estimated for CCM. A dash indicates that the parameter was not estimated for each data set in the study. m = slope, b' = intercept. SDm = standard deviation of slope, SDb' = standard deviation of intercept, and SEy = standard error of prediction, are statistics for regression performed on the obtained versus predicted data points. See text for further explanation.

	n	b	a_1	a_2	k	VAC	m	b'	SDm	SDb'	SEy
Alsop and Davison (1988)	26	1.68	0.82	0.19	—	91	0.99	0	0.06	0.04	0.08
Chung and Herrnstein (1967)	10	1.41	—	2.85	0.84	91	1.00	0	0.11	0.06	0.06
Duncan and Fantino (1970)	6	1.03	—	5.40	—	98	1.00	0.01	0.07	0.05	0.07
Dunn and Fantino (1982)	6	1.08	—	1.81	—	100	1.00	0	0.03	0.02	0.03
Davison and Temple (1973)	26	1.02	1.94	2.05	—	92	1.01	−0.01	0.08	0.04	0.08
Davison (1976)	20	1.59	0.55	3.03	—	98	1.01	0	0.03	0.02	0.04
Davison (1983)	61	1.12	0.68	1.18	0.42	82	0.99	0.01	0.06	0.03	0.10
Davison (1988)	24	1.12	—	0.48	0.86	92	0.94	0.03	0.06	0.04	0.09
Fantino (1969)	4	0.93	—	1.24	—	99	1.00	−0.01	0.06	0.03	0.04
Fantino and Davison (1983)	56	0.92	0.19	1.11	—	92	0.95	0.03	0.04	0.02	0.08
Fantino and Royalty (1987)	7	1.49	—	0.86	—	81	1.00	0.01	0.22	0.16	0.08
Gentry and Marr (1980)	10	1.08	—	0.93	0.57	78	0.96	0.02	0.18	0.10	0.08
Killeen (1970)	4	0.83	—	4.35	—	99	0.96	0.01	0.05	0.03	0.04
MacEwen (1972)	8	1.60	—	3.27	—	92	0.94	0.05	0.11	0.09	0.02
Omino and Ito (1993)	5	1.03	—	0.94	—	95	1.00	0	0.14	0.07	0.05
Preston and Fantino (1991)	9	2.22	0.77	0.20	—	75	0.99	0.01	0.25	0.18	0.07
Squires and Fantino (1971)	9	0.88	0.45	—	—	86	1.01	0	0.20	0.10	0.07
Wardlaw and Davison (1974)	20	1.36	—	3.91	—	94	0.97	0.02	0.06	0.03	0.07
Williams and Fantino (1978)	12	1.43	—	3.82	—	91	0.99	0.01	0.10	0.06	0.09
Average VAC across all studies						90.7					

viations greater than one tenth the slope estimate. This was significant on a chi-square, $\chi^2 = 15.12$, $p < .001$. Hence, requiring the standard deviation of the slope estimate to be less than one tenth the slope estimate may be inappropriate for smaller data sets.

It was noted previously that the effect of temporal context may depend on stimulus conditions—cued versus uncued terminal links. To investigate this hypothesis, the following procedure was used for fitting k , the context scaling parameter. First, Equation 4 (with $k = 1$) was fitted to each data set. Of the 92 fits, 81 satisfied Davison’s (1987) regression criteria (except that of requiring the slope standard deviation to be less than one tenth the slope). For the remaining 11 data sets, a value of k was estimated. If the fit now met the regression criteria, or a significantly greater proportion of variance was accounted for, then k was set at the estimated value; otherwise k was set equal to one.

With this procedure, estimated values of k were used for 9 of the 92 data sets: two in

Gentry and Marr (1980), five in Davison (1983), and one each in Chung and Herrnstein (1967) and Davison (1988). For these data sets, the estimate of k was always less than one, and in two cases a value of zero was obtained, indicating no effect of context. These studies, however—with the exception of Davison (1988), who noted (p. 226) that Bird 33’s performances were anomalous—were the only 3 out of 19 analyzed to arrange exclusively uncued terminal links. The effect of context on cued and uncued terminal-link sensitivity may not be equivalent because, for uncued terminal links, relative time spent in the presence of exteroceptive terminal-link discriminative stimuli does not depend on context. The fact that the three studies that used uncued terminal links exclusively required different values of k from the cued terminal-links studies is evidence that the effect of temporal context in concurrent chains is mediated by terminal-link stimulus conditions. It is unclear, however, why every data set in these studies did not require values of k less than one.

As an example of how context effects on terminal-link sensitivity are represented in CCM, Figure 4 shows a reanalysis of data for Bird 31 from Fantino and Davison (1983). For each of three pairs of equal initial links (VI 15 s, VI 60 s, and VI 180 s), nine pairs of terminal links whose duration summed to 60 s were arranged. First, Equation 3 (i.e., no effect of temporal context) was fitted individually to each group of nine conditions. The three values estimated for a_2 are plotted as points in Figure 4. Then, Equation 5 (including context) was fitted to all 27 conditions as a single group, and a value of a_2 was estimated (with $k = 1$). The solid line in Figure 4 shows

the effective terminal-link sensitivity, $a_2 \left(\frac{T_i}{T_i} \right)$,

produced by the estimated value of a_2 for Bird 31, as initial-link duration increased. Figure 4 illustrates how CCM (Equation 5) has replaced the parameter invariance of the original generalized matching law with a functional invariance: Terminal-link sensitivity is no longer constant, but becomes a specified function of procedural context.

Nevin (1984) rightly noted that parameter invariance is a desirable, although not easily attainable, property of a quantitative model. Determination of parameter invariance poses a special difficulty for CCM, however. The problem is that because the curve-fitting procedure is nonlinear, in some cases multiple sets of parameter values can be found that minimize error variance. How, then, do we choose between two parameter sets for a data fit, one that minimizes error and another whose values are more consistent with other data sets? This issue has rarely arisen in the quantitative analysis of behavior because models have usually employed linear regression, which gives a unique solution (although the difficulties posed by estimating parameters for Herrnstein's, 1970, hyperbola have been much discussed; cf. Davison & McCarthy, 1988). An attainable initial goal, however, is identification of a quantitative model that captures the functional invariances of concurrent-chains behavior; parameter invariance across studies must be addressed after functional invariance has been adequately modeled. Its success in predicting results across a wide range of studies is strong evidence for CCM (Equation 5) modeling the functional invariances of concurrent chains.

Fig. 4. This figure illustrates how context effects on terminal-link sensitivity (a_2) are represented in CCM (Equation 5). Data points are values of a_2 for Bird 31 for three groups of equal initial-link conditions from Fantino and Davison (1983), estimated for a parameter-invariant generalized matching model (Equation 3). The solid line shows the "effective a_2 " produced when CCM (Equation 5) was fit to the same data. The parameter invariance (fixed a_2) of the original generalized matching law is replaced by a functional invariance (a_2 is a monotonic decreasing, negatively accelerated function of context) in CCM.

Another difficulty in estimating parameters for CCM arose in some studies in which not all independent variables represented in the model were manipulated. In this case the parameters of the model were not independent in their effect on the dependent variable. For example, Alsop and Davison (1988) arranged a constant pair of terminal links, VI 6 s (left) and VI 12 s (right). Sensitivity to terminal-link reinforcement (a_2) and bias (b) were not independent, because increasing a_2 could be compensated for by a decrease in b . In some data sets, this resulted in combinations of extreme parameter values accounting for asymptotically smaller increments of variance (e.g., a b of nearly zero and an a_2 of more than 50). However, in these instances, virtually the same percentage of variance was accounted for by parameter values more in accord with the other data sets, and these values were chosen.

The fits exhibit more parameter variability than is usually obtained with the generalized matching law (Baum, 1979), although not more than obtained when fitting Herrnstein's hyperbola (de Villiers & Herrnstein, 1976). In particular, terminal-link sensitivity (a_2) was the most variable parameter, ranging from 0.19 to 5.40 across the studies, with an average of 2.09. Fitted values of a_2 appear to be a function of the type of terminal-link schedules arranged, however. In the 12 studies that ar-

ranged fixed (e.g., FI vs. FI) terminal links, average a_2 was 2.68; in the seven studies that arranged variable (e.g., VI vs. VI) terminal links, average a_2 was 0.90. This confirms the finding that preference between fixed delays to reinforcement frequently results in overmatching (Killeen, 1970; Williams & Fantino, 1978), whereas preference between variable delays results in undermatching (Fantino & Davison, 1983). Similarly, Duncan and Fantino (1970) argued that terminal-link sensitivity was a function of the shortest delay interval comprising either schedule. Although a popular theory of temporal discrimination (scalar expectancy theory; Gibbon, Church, Fairhurst, & Kacelnik, 1988) is able to explain this result (through postulating representations in memory), it is unclear in terms of CCM why sensitivity to relative terminal-link delay to reinforcement should vary as a function of the distributions of delays comprising the terminal links.

To summarize: The good fits in Tables 1 and 2 indicate that CCM provides an adequate quantitative description of a wide range of concurrent-chains data. That CCM accurately predicts results across a wide range of experiments, with different independent variable manipulations and procedural variations, is strong evidence for its representing the essential determinants of preference in the concurrent-chains procedure. The model's aggregate performance, in terms of percentage of variance accounted for, is on a par with the generalized matching law description of *conc* VI VI (Baum, 1979). That CCM reduces to the generalized matching law when terminal-link delays are zero permits *conc* VI VI and concurrent chains to be described by a single integrative quantitative model.

CCM and Delay-Reduction Theory

One motivation for CCM was to represent the "lore" of concurrent chains quantitatively within the generalized matching framework. Much of this lore stems from the intuitively plausible predictions of the delay-reduction hypothesis: Preference for an alternative is determined by the reduction in delay to reinforcement signaled by that alternative, in terms of the overall context of delay of reinforcement (Fantino, 1969, 1977, 1981; Squires & Fantino, 1971; see also Fantino & Davison, 1983). The delay-reduction hypothesis was the first

attempt to incorporate the effect of procedural context on preference in concurrent chains. Delay reduction's intuitive plausibility is reflected by its being the most popular and widely cited theory of conditioned reinforcement (see Fantino, Preston, & Dunn, 1993, for a review), and by its extension to other areas such as foraging (Abarca & Fantino, 1982), self-control (Navarick & Fantino, 1976), observing responses (Case & Fantino, 1981), the serial-position effect in short-term memory (Wixted, 1989), and three-alternative choice (Fantino & Dunn, 1983). I will consider the relationship between CCM and the delay-reduction hypothesis with respect to three issues: (a) choice as a function of the length of the initial link preceding the longer terminal link (Preston & Fantino, 1991), (b) operant simulations of foraging (Fantino & Abarca, 1985), and (c) effect of postreinforcer blackouts. In each case, CCM performs as well or better than the delay-reduction model.

Choice as a bitonic function. CCM represents the qualitative predictions of delay reduction through the double sensitivity exponent, which enables preference to become less sensitive to terminal-link differences as initial links increase relative to terminal links, and more sensitive to terminal-link differences as initial links decrease relative to terminal links. In a recent article, Preston and Fantino (1991) compared the predictions made by three models—delay reduction (Squires & Fantino, 1971), melioration (Vaughan, 1985), and incentive theory (Killeen, 1982)—of preference as a function of the length of the initial link leading to the longer of the two terminal links. Consistent with the predictions of delay reduction, but not melioration or incentive theory as currently formulated, Preston and Fantino reported that pigeons' preference for the shorter terminal link was a bitonic (not monotonic) function of the initial link leading to the longer terminal link. After Preston and Fantino's Figure 4 (1991, p. 161), Figure 5 shows preference for the shorter (VI 10 s) terminal link as a function of the initial link preceding the longer (VI 20 s) terminal link, as predicted by CCM (Equation 5). (The initial link preceding the shorter terminal link was VI 60 s. Representative parameter values were chosen: $b = 1$, $a_1 = 0.80$, $a_2 = 1$.) Figure 5 shows that preference for the shorter terminal link predicted by CCM is also a bitonic function of the length of the

initial link preceding the longer terminal link, consistent with the qualitative prediction of the delay-reduction hypothesis (Preston & Fantino, 1991).

Operant simulations of foraging. The delay-reduction hypothesis has been applied to operant simulations of foraging using a procedure similar to concurrent chains (Abarca & Fantino, 1982; Fantino & Abarca, 1985). In the *successive-encounters procedure*, introduced by Lea (1979), a search phase (similar to initial links) terminates in a prey encounter (or choice phase) in which the subject can either accept or reject the "prey," which is typically a short or long delay to reinforcement (similar to terminal links). Abarca and Fantino (1982) and Lea (1979) found that as the choice phase increased in length, the probability of accepting the long delay increased. However, probability increased gradually, not in a step function as predicted by both delay-reduction and optimal foraging theory (Fantino & Abarca, 1985).

When adapted to the successive-encounters procedure, CCM predicts a gradual increase in probability of accepting the long delay. To demonstrate this, the model will be applied to group mean data from Experiment 1 of Abarca and Fantino (1982). Assume that the relative probability of accepting the long delay equals the relative value of the long-delay choice stimulus as a conditioned reinforcer. Relative value is calculated as a power function of the ratio of handling times (i.e., terminal-link delays), and temporal context (average handling time divided by search time) modulates sensitivity as in Equation 5. We have the following equation for the relative value of the long and short delays:

$$\frac{V_L}{V_S} = \frac{p(L)}{p(S)} = \left(\frac{H_S}{H_L} \right)^{a_2 \left(\frac{Th}{Ts} \right)}, \quad (6)$$

where V_L and V_S are the values, $p(L)$ and $p(S)$ are the probabilities of accepting, and H_L and H_S are the handling times, respectively, for the long- and short-delay choices. Th is the average time spent in the handling phase (terminal links), and Ts the average time spent in the search phase (initial links). Note that $p(S) = 1$, because the short delay was accepted 100% of the time (Abarca & Fantino, 1982, p. 120). Equation 6, containing one parameter (a_2),

Fig. 5. Preference for shorter terminal link (VI 10 s) plotted as a function of the initial link preceding the longer terminal link (VI 20 s), predicted by CCM (Equation 5). Parameter values were set as follows: $b = 1$, $a_1 = 0.80$, $a_2 = 1$. The initial link preceding the VI 10-s terminal link was VI 60 s. The figure demonstrates that preference for the VI 10 s was a bitonic, not monotonic, function of the initial link preceding the longer terminal link, as reported by Preston and Fantino (1991). It indicates that CCM makes a qualitative prediction similar to the delay-reduction hypothesis when the initial link leading to the longer terminal link is varied.

was fitted to the group mean data from four search-phase conditions from Abarca and Fantino (1982). The results are shown in Figure 6.

With $a_2 = 0.29$, the model accounted for 92% of the variance in Abarca and Fantino's (1982) data. The smooth curve in Figure 6 shows the predictions of CCM as adapted to the successive-encounters procedure, and dem-

Fig. 6. Probability of accepting the long delay to reinforcement as a function of increasing choice phase length, as predicted by CCM adapted to an operant simulation of foraging, the successive encounters procedure (see Equation 6). Data are averages of 4 birds from Abarca and Fantino (1982). Unlike delay-reduction and optimal foraging theory, CCM predicts a gradual increase in the probability of accepting the less valued prey item as search time increases.

onstrates that, unlike delay-reduction and optimal foraging theory, CCM predicts a gradual increase in the probability of accepting the less valuable item as search time increases. CCM's success in predicting Abarca and Fantino's (1982) data indicates that the effect of temporal context in the successive-encounters procedure—the ratio of average handling to search time—may be similar to the effect of context in concurrent chains. In that case, CCM could prove to be a useful quantitative model for operant foraging simulations.

Postreinforcer blackouts. There is another instance in which CCM and the delay-reduction hypothesis make different predictions. If total terminal-link duration is kept constant while two delays to reinforcement in constant ratio are varied, CCM predicts no increase in preference for the shorter delay as the absolute delay duration increases, whereas delay reduction predicts that preference for the shorter delay will increase as absolute delay duration increases. Postreinforcer blackouts have no effect, according to delay reduction, because the critical measure is the reduction in delay signaled by the terminal-link stimulus relative to the average delay from the onset of the initial links. Because differential blackouts are associated with terminal-link stimuli, their value is included in T_t . If total terminal-link (and initial-link) duration is constant across conditions, CCM predicts that there will be no context effect.

Omino and Ito (1993) reported data that are relevant to this question. For one group of pigeons, total terminal-link duration was 30 s, and delays of 4 s versus 8 s and 8 s versus 16 s were arranged, for both FI and fixed-delay (FD) schedules. For a second group, terminal-link duration was 40 s, and delays of 8 s versus 16 s and 16 s versus 32 s, also for both FI and FD schedules, were arranged. Pigeons showed no systematic increase in preference for the shorter delay as absolute delay increased, for either FI or FD schedules. For the first group (30 s terminal-link duration), obtained preferences for the shorter delay averaged across 3 birds were, for FD 4 s versus 8 s: 64%; for FD 8 s versus 16 s: 62%; for FI 4 s versus 8 s: 64%; and for FI 8 s versus FI 16 s: 64%. For the second group (40 s terminal-link duration), obtained preferences for the shorter delay averaged across 3 birds were, for FD 8 s versus 16 s: 60%; for FD 16 s versus

32 s: 58%; for FI 8 s versus 16 s: 76%; and for FI 16 s versus 32 s: 75%. Thus, preferences were approximately equal for delays in constant ratio as absolute delay duration increased with total terminal-link duration controlled, as predicted by CCM but not by the delay-reduction hypothesis. However, this result may have been influenced by the terminal-link stimulus conditions arranged by Omino and Ito (1993). For the FD conditions, terminal links were nondifferentially signaled delays in blackout, whereas for the FI conditions, the stimulus for a terminal link was that the other key was extinguished (not a change in color on the terminal-link key).

In summary, even though there are some areas in which the models make different predictions, CCM can account for the same qualitative effects as the delay-reduction hypothesis does, with the additional advantage of accurate quantitative prediction across a wide range of schedules and procedural variations.

For the Future

CCM is based on the assumption that the ratio of the average terminal-link to initial-link durations, T_t/T_i , is the crucial contextual variable in concurrent chains. It must be stressed that other formulations might be as successful empirically—in fact, any monotonic decreasing, negatively accelerated function that would fit the data of Figure 3 might work as well as Equation 5. As this article was being prepared, many alternative models were fitted to the same data, and some were just as successful. Equation 5 was chosen on the basis of parsimony; however, if more comprehensive parametric data are collected, a more complicated equation may be necessary. For this analysis, the dependent variable is no longer preference, but rather the terminal-link sensitivities themselves. A multiple-component concurrent-chains procedure (e.g., Cerutti & Catania, 1986; Safar, 1982; White & Pipe, 1987) may be appropriate for data collection of this type, if the experiments are to be completed within a manageable time frame.

According to CCM, the ratio T_t/T_i is a higher order independent variable, and as such its control over behavior needs to be explicitly tested. This research might proceed in at least two directions. First, parametric experiments on the order of those of Fantino and Davison (1983), with conditions tailored to examine

different terminal-link to initial-link duration ratios, could be conducted. Alternatively, given that the ratio T_t/T_i determines context effects on preference, experiments could test predictions that follow from this assumption. For example, preference should be unchanged when both initial and terminal links increase but their ratio is held constant. Also, manipulating average initial-link and terminal-link durations should be functionally equivalent: Increasing T_t by a factor of two should have the same effect as decreasing T_i by a factor of two, and vice versa. Experiments of this sort might indicate whether T_t/T_i determines context effects in concurrent chains, or whether a more complex relation is needed.

CCM makes a specific recommendation about how concurrent chains can be optimized as a preference scaling procedure. Average initial-link and terminal-link durations should be kept constant across conditions. In that case, the relation between terminal-link independent variables and initial-link responding should be uncontaminated by context effects. Although some (e.g., Gentry & Marr, 1980) have attempted to increase control in concurrent chains by arranging postreinforcer blackouts to equalize terminal-link reinforcement rates, it may be more effective to control average terminal-link and initial-link durations across conditions.

Because CCM is based on the matching law (Equation 2), it permits natural incorporation of additional terminal-link variables such as magnitude of reinforcement. Although studies that have varied magnitude of reinforcement were not considered here, the effect of context on sensitivity to magnitude appears to be equivalent to that on sensitivity to delay (Ito, 1985; Navarick & Fantino, 1976; White & Pipe, 1987). Another variable that has been manipulated in concurrent chains is probability of reinforcement (Spetch & Dunn, 1987); it may be possible to incorporate this as well.

Concurrent chains is an extremely rich experimental procedure—it permits the manipulation of many different reinforcement and stimulus parameters. A generalization of the generalized matching law incorporating context effects on terminal-link sensitivity was proposed. The empirical success of the contextual choice model (CCM) shows that the matching law remains a viable framework for the quantitative analysis of concurrent chains.

Because the model reduces to the generalized matching law when terminal-link duration is zero, it serves to integrate behavior allocation in concurrent schedules and concurrent chains. Future research should explore the generality of the model's assumptions, and the extent to which additional variables, such as magnitude and probability of reinforcement, can be incorporated while maintaining accurate quantitative prediction.

REFERENCES

- Abarca, N., & Fantino, E. (1982). Choice and foraging. *Journal of the Experimental Analysis of Behavior*, 38, 117–123.
- Alsop, B., & Davison, M. (1988). Concurrent-chain performance: Effects of absolute and relative terminal-link entry frequency. *Journal of the Experimental Analysis of Behavior*, 49, 351–365.
- Autor, S. M. (1960). *The strength of conditioned reinforcers as a function of frequency and probability of reinforcement*. Unpublished doctoral dissertation, Harvard University.
- Autor, S. M. (1969). The strength of conditioned reinforcers as a function of frequency and probability of reinforcement. In D. P. Hendry (Ed.), *Conditioned reinforcement* (pp. 127–162). Homewood, IL: Dorsey Press.
- Baum, W. M. (1973). The correlation-based law of effect. *Journal of the Experimental Analysis of Behavior*, 20, 137–153.
- Baum, W. M. (1974a). Chained concurrent schedules: Reinforcement as situation transition. *Journal of the Experimental Analysis of Behavior*, 22, 91–101.
- Baum, W. M. (1974b). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 22, 231–242.
- Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior*, 32, 269–281.
- Baum, W. M., & Rachlin, H. C. (1969). Choice as time allocation. *Journal of the Experimental Analysis of Behavior*, 12, 861–874.
- Case, D. A., & Fantino, E. (1981). The delay-reduction hypothesis of conditioned reinforcement and punishment: Observing behavior. *Journal of the Experimental Analysis of Behavior*, 35, 93–108.
- Cerutti, D. T., & Catania, A. C. (1986). Rapid determinations of preference in multiple concurrent-chain schedules. *Journal of the Experimental Analysis of Behavior*, 46, 211–218.
- Chung, S. H., & Herrnstein, R. J. (1967). Choice and delay of reinforcement. *Journal of the Experimental Analysis of Behavior*, 10, 67–74.
- Davison, M. C. (1969). Preference for mixed-interval versus fixed-interval schedules. *Journal of the Experimental Analysis of Behavior*, 12, 247–252.
- Davison, M. C. (1972). Preference for mixed-interval versus fixed-interval schedules: Number of component intervals. *Journal of the Experimental Analysis of Behavior*, 17, 169–176.

- Davison, M. C. (1976). Preference for fixed-interval schedules: Effects of unequal initial links. *Journal of the Experimental Analysis of Behavior*, 25, 371-376.
- Davison, M. (1983). Bias and sensitivity to reinforcement in a concurrent-chain schedule. *Journal of the Experimental Analysis of Behavior*, 40, 15-34.
- Davison, M. C. (1987). The analysis of concurrent-chain performance. In M. L. Commons, J. E. Mazur, J. A. Nevin, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 5. The effect of delay and of intervening events on reinforcement value* (pp. 225-241). Hillsdale, NJ: Erlbaum.
- Davison, M. C. (1988). Delay of reinforcers in a concurrent-chain schedule: An extension of the hyperbolic-decay model. *Journal of the Experimental Analysis of Behavior*, 50, 219-236.
- Davison, M., & McCarthy, D. (1988). *The matching law: A research review*. Hillsdale, NJ: Erlbaum.
- Davison, M., & Smith, C. (1986). Some aspects of preference between immediate and delayed periods of reinforcement. *Journal of Experimental Psychology: Animal Behavior Processes*, 12, 291-300.
- Davison, M. C., & Temple, W. (1973). Preference for fixed-interval schedules: An alternative model. *Journal of the Experimental Analysis of Behavior*, 20, 393-403.
- de Villiers, P. A., & Herrnstein, R. J. (1976). Toward a law of response strength. *Psychological Bulletin*, 83, 1131-1153.
- Duncan, B., & Fantino, E. (1970). Choice for periodic schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 14, 73-86.
- Fantino, E. (1969). Choice and rate of reinforcement. *Journal of the Experimental Analysis of Behavior*, 12, 723-730.
- Fantino, E. (1977). Conditioned reinforcement: Choice and information. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 313-339). Englewood Cliffs, NJ: Prentice-Hall.
- Fantino, E. (1981). Contiguity, response strength, and the delay-reduction hypothesis. In P. Harzem & M. D. Zeiler (Eds.), *Advances in analysis of behaviour: Vol. 2. Predictability, correlation, and contiguity* (pp. 169-201). Chichester, England: Wiley.
- Fantino, E., & Abarca, N. (1985). Choice, optimal foraging, and the delay-reduction hypothesis. *Behavioral and Brain Sciences*, 8, 315-330.
- Fantino, E., & Davison, M. (1983). Choice: Some quantitative relations. *Journal of the Experimental Analysis of Behavior*, 40, 1-13.
- Fantino, E., & Dunn, R. (1983). The delay-reduction hypothesis: Extension to three-alternative choice. *Journal of Experimental Psychology: Animal Behavior Processes*, 9, 132-146.
- Fantino, E., Preston, R. A., & Dunn, R. (1993). Delay reduction: Current status. *Journal of the Experimental Analysis of Behavior*, 60, 159-169.
- Fantino, E., & Royalty, P. (1987). A molecular analysis of choice on concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior*, 48, 145-159.
- Gentry, G. D., & Marr, M. J. (1980). Choice and reinforcement delay. *Journal of the Experimental Analysis of Behavior*, 33, 27-37.
- Gibbon, J., Church, R. M., Fairhurst, S., & Kacelnik, A. (1988). Scalar expectancy theory and choice between delayed rewards. *Psychological Review*, 95, 102-114.
- Grace, R. C. (1993). Violations of transitivity: Implications for a theory of contextual choice. *Journal of the Experimental Analysis of Behavior*, 60, 185-201.
- Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior*, 4, 267-272.
- Herrnstein, R. J. (1964a). Aperiodicity as a factor in choice. *Journal of the Experimental Analysis of Behavior*, 7, 179-182.
- Herrnstein, R. J. (1964b). Secondary reinforcement and rate of primary reinforcement. *Journal of the Experimental Analysis of Behavior*, 7, 27-36.
- Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior*, 13, 243-266.
- Houston, A. (1991). Violations of stochastic transitivity on concurrent chains: Implications for theories of choice. *Journal of the Experimental Analysis of Behavior*, 55, 323-335.
- Hursh, S. R., & Fantino, E. (1973). Relative delay of reinforcement and choice. *Journal of the Experimental Analysis of Behavior*, 19, 437-450.
- Ito, M. (1985). Choice and amount of reinforcement in rats. *Learning and Motivation*, 16, 95-108.
- Ito, M., & Asaki, K. (1982). Choice behavior of rats in a concurrent-chains schedule: Amount and delay of reinforcement. *Journal of the Experimental Analysis of Behavior*, 37, 383-392.
- Kelleher, R. T., & Gollub, L. R. (1962). A review of positive conditioned reinforcement. *Journal of the Experimental Analysis of Behavior*, 5, 543-597.
- Killeen, P. (1968). On the measurement of reinforcement frequency in the study of preference. *Journal of the Experimental Analysis of Behavior*, 11, 263-269.
- Killeen, P. (1970). Preference for fixed-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 14, 127-131.
- Killeen, P. (1972). The matching law. *Journal of the Experimental Analysis of Behavior*, 17, 489-495.
- Killeen, P. (1982). Incentive theory: II. Models for choice. *Journal of the Experimental Analysis of Behavior*, 38, 217-232.
- Lea, S. E. G. (1979). Foraging and reinforcement schedules in the pigeon: Optimal and non-optimal aspects of choice. *Animal Behaviour*, 27, 875-886.
- Leung, J.-P., & Winton, A. S. W. (1985). Preference for unsegmented interreinforcement intervals in concurrent chains. *Journal of the Experimental Analysis of Behavior*, 44, 89-101.
- MacEwen, D. (1972). The effects of terminal-link fixed-interval and variable-interval schedules on responding under concurrent-chained schedules. *Journal of the Experimental Analysis of Behavior*, 18, 253-261.
- Mazur, J. E. (1984). Tests of an equivalence rule for fixed and variable reinforcer delays. *Journal of Experimental Psychology: Animal Behavior Processes*, 10, 426-436.
- Mazur, J. E., Snyderman, M., & Coe, D. (1985). Influences of delay and rate of reinforcement on discrete-trial choice. *Journal of Experimental Psychology: Animal Behavior Processes*, 11, 565-575.
- Microsoft, Inc. (1992). *Microsoft Excel 4.0 spreadsheet with business graphics and database*. Redmond, WA: Author.
- Miller, J. T., Saunders, S. S., & Bourland, G. (1980). The role of stimulus disparity in concurrently available reinforcement schedules. *Animal Learning & Behavior*, 8, 635-641.

- Navarick, D. J., & Fantino, E. (1976). Self-control and general models of choice. *Journal of Experimental Psychology: Animal Behavior Processes*, 2, 75-87.
- Nevin, J. A. (1973). Conditioned reinforcement. In J. A. Nevin & G. S. Reynolds (Eds.), *The study of behavior: Learning, motivation, emotion, and instinct* (pp. 154-198). Glenview, IL: Scott, Foresman.
- Nevin, J. A. (1984). Quantitative analysis. *Journal of the Experimental Analysis of Behavior*, 42, 421-434.
- Omino, T., & Ito, M. (1993). Choice and delay of reinforcement: Effects of terminal-link stimulus and response conditions. *Journal of the Experimental Analysis of Behavior*, 59, 361-371.
- Preston, R. A., & Fantino, E. (1991). Conditioned reinforcement value and choice. *Journal of the Experimental Analysis of Behavior*, 55, 155-175.
- Safar, T. (1982). *Tests of transitivity and functional equivalence in concurrent-chains choice behavior*. Unpublished doctoral dissertation, University of California, San Diego.
- Schneider, J. W. (1972). Choice between two-component chained and tandem schedules. *Journal of the Experimental Analysis of Behavior*, 18, 45-60.
- Spetch, M. L., & Dunn, R. (1987). Choice between reliable and unreliable outcomes: Mixed percentage-reinforcement in concurrent chains. *Journal of the Experimental Analysis of Behavior*, 47, 57-72.
- Squires, N., & Fantino, E. (1971). A model for choice in simple concurrent and concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior*, 15, 27-38.
- Stubbs, D. A., & Pliskoff, S. S. (1969). Concurrent responding with fixed relative rate of reinforcement. *Journal of the Experimental Analysis of Behavior*, 12, 887-895.
- Tversky, A. (1969). Intransitivity of preferences. *Psychological Review*, 76, 31-48.
- Vaughan, W., Jr. (1985). Choice: A local analysis. *Journal of the Experimental Analysis of Behavior*, 43, 383-405.
- Wardlaw, G. R., & Davison, M. C. (1974). Preference for fixed-interval schedules: Effects of initial-link length. *Journal of the Experimental Analysis of Behavior*, 21, 331-340.
- Wearden, J. H., & Burgess, I. S. (1982). Matching since Baum (1979). *Journal of the Experimental Analysis of Behavior*, 38, 339-448.
- White, K. G., & Pipe, M. E. (1987). Sensitivity to reinforcer duration in a self-control procedure. *Journal of the Experimental Analysis of Behavior*, 48, 235-249.
- Williams, B. A. (1988). Reinforcement, choice, and response strength. In R. C. Atkinson, R. J. Herrnstein, G. Lindzey, & R. D. Luce (Eds.), *Stevens' handbook of experimental psychology: Vol. 2. Learning and cognition* (2nd ed., pp. 167-244). New York: Wiley.
- Williams, B. A., & Fantino, E. (1978). Effects on choice of reinforcement delay and conditioned reinforcement. *Journal of the Experimental Analysis of Behavior*, 29, 77-86.
- Winter, J., & Perkins, C. C. (1982). Immediate reinforcement in delayed reward learning in pigeons. *Journal of the Experimental Analysis of Behavior*, 38, 169-179.
- Wixted, J. T. (1989). Nonhuman short-term memory: A quantitative reanalysis of selected findings. *Journal of the Experimental Analysis of Behavior*, 52, 409-426.

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