TECHNICAL NOTE

A NOTE ON HERRNSTEIN'S EQUATION¹

In 1970, Herrnstein described a clear and concise mathematical treatment of the relation between reinforcement frequency and response rate for single variable-interval schedules. Herrnstein's equation is expressed as follows:

$$P = kR/(R + R_0) \quad (1)$$

where P is the response rate, k is the maximum rate of responding as a function of reinforcer frequency, R is the reinforcement rate, and R_0 is a constant representing the overall rate of reinforcement for other behaviors.

This note describes a way of converting Herrnstein's equation into a simple linear function that allows the two constants, R_0 and k , to be easily determined. This technique provides a quick assessment of the manner in which an independent variable can alter response rate.

According to the Herrnstein equation, a change in the response rate under single variable-interval schedules that is assignable to some variable other than reinforcer frequency could be the result of that variable's effect on R_0 , on k , or on both. For example, suppose one observes that the administration of a drug to an animal on a variable-interval schedule of food reinforcement reduces its rate of responding. This depression in response rate might be attributed to a number of factors: the drug might be an anorexic, a neuromuscular blocking agent, or it may increase the reinforcement value of grooming behavior. The first two factors imply that the reduction occurs through an alteration in k whereas the last factor implicates R_0 , assuming Herrnstein's interpretation of R_0 is correct. It would appear to be useful, therefore, to develop a simple technique for rapidly determining the values of these two constants.

If equation (1) is expressed in terms of reciprocals, a linear equation relating $1/P$ to $1/R$ results.

$$1/P = 1/k + (R_0/k)(1/R) \quad (2)$$

The slope of the linear function is R_0/k , and the Y-intercept is $1/k$. The X-intercept is obtained by setting $1/P = 0$ giving a value of $-1/R_0$ (cf. Figure 1). The linear function intercepts the X-axis at a negative reciprocal of reinforcement rate. This value has no empirical or theoretical connotation other than as a mathematical term useful in obtaining R_0 .

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In order to illustrate the utility of this technique, values of R_0 and k were determined using data derived from Catania and Reynolds (1968, Figure 1, Animal 281). Reciprocals were taken of the X and Y values plotted in their graph and then the best-fitting straight line was obtained by the method of least squares. Figure 1 presents the data graphed in their original form and also on a double-reciprocal plot. Using the present technique, values determined for R_0 and k were 5.4 and 66.6, respectively. Herrnstein's

Fig. 1. In the upper panel, the reciprocal of response rate is plotted as a function of the reciprocal of reinforcement rate. The linear function was fitted by the method of least squares. The slope of the function is R_0/k , and the X-intercept and Y-intercept are $-1/R_0$ and $1/k$, respectively. The lower panel presents the same data untransformed. The hyperbolic curve was derived using the values of R_0 and k determined from the double-reciprocal linear function.

(1970) determinations for the same data were 5 and 68, respectively. It may be noted parenthetically that this method of plotting tends to give a great deal of weight to low rates of reinforcement in determining the slope of the function. Other methods of plotting have been evaluated by Dowd and Riggs (1965).

If the hypothetical drug referred to previously reduced response rate by altering k but not R_0 , the X-intercept of the drug and control groups would be identical but the slopes of the two functions as well as the Y-intercepts would differ. If, on the other hand, the drug affected R_0 without affecting k , the slopes and X-intercepts would differ but not the Y-intercepts. The latter result would imply that the reduction in response rate produced by the drug could be overcome by shifting the animal to a denser food reinforcement schedule.

It is of some interest that the Herrnstein equation describes a function that is also observed in other branches of science and has been analyzed in a similar manner. In enzymology, for example, the substrate curve, which relates the rate of an enzymatic reaction to the concentration of its substrate, takes a similar form and has been analyzed by the double-reciprocal plot (Lehninger, 1970). The relevant equation is:

$$r = v_{\max}[S]/([S] + K_m) \quad (3)$$

where r is the rate of the enzymatic reaction, $v_{\max}$ is the maximum reaction velocity, $[S]$ is the substrate concentration, and K_m is the Michaelis-Menten dissociation constant, a term that reflects the relative rates of dissociation and association of the enzyme-substrate complex. Without implying any causal re-

lation between equations (1) and (3), it is interesting to note the analogies that exist in the quantities expressed by the two equations, *i.e.*, rate of reaction *vs* rate of responding and substrate concentration *vs* rate (density) of reinforcement. The analogy can be extended to K_m and R_0 if one views both constants as reflecting opposing processes, the opposing processes for K_m being as described above and those of R_0 being responding for reinforcers other than those selected by the experimenter and responding for the scheduled reinforcer. Although the underlying factors governing chemical reaction rates and response rates are, undoubtedly, physically different, it is of some interest that the same mathematical model has been applied to such widely different "behaviors".

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