TRANSITIVITY AS A PROPERTY OF CHOICE¹DOUGLAS J. NAVARICK² AND EDMUND FANTINO

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Pigeons' pecks in the presence of two concurrently available initial-link stimuli occasionally produced one of two stimuli associated with mutually exclusive terminal links. Pecks during either terminal link produced food according to aperiodic (variable-interval and variable-ratio) or periodic (fixed-interval and fixed-ratio) schedules of reinforcement. Aperiodic and periodic schedules to which the pigeons were indifferent, in the sense that these schedules maintained equal responding in the initial links, often yielded different preferences in separate choice tests with a third schedule. Conversely, aperiodic and periodic schedules that were equally preferred to a third schedule often failed to generate indifference. These intransitivities imply that (1) aperiodic and periodic schedules are not functionally equivalent in their effects upon choice, and (2) efforts to find a simple method for transforming aperiodic schedules into their periodic equivalents will fail.

Transitive relations take the following general form: if $A \geq B \geq C$, then $A \geq C$. Three levels of transitivity have been defined—"weak", "moderate", and "strong" (Coombs, 1964). Where $P(a,b)$ represents the proportion of choices for A over B, and $P(a,b)$ and $P(b,c) \geq 0.5$, an outcome, $P(a,c)$, satisfies

Weak transitivity

$$\text{if } P(a,c) \geq 0.5 \quad (1)$$

Moderate transitivity

$$\text{if } P(a,c) \geq \text{minimum } [P(a,b), P(b,c)] \quad (2)$$

Strong transitivity

$$\text{if } P(a,c) \geq \text{maximum } [P(a,b), P(b,c)] \quad (3)$$

For example, suppose that $P(a,b) = 0.6$ and $P(b,c) = 0.8$. Strong transitivity would be satisfied if $P(a,c) = 0.9$, moderate transitivity if $P(a,c) = 0.7$, and weak transitivity if $P(a,c) = 0.55$. A strengthened definition of strong stochastic transitivity by Tversky and Russo (1969) distinguished as a separate case the occurrence of indifference between two stimuli. For this situation, the two stimuli to which the subject is indifferent must be chosen the same

proportion of the time over the third stimulus. That is,

$$P(a,b) = 0.5 \text{ implies } P(a,c) = P(b,c) \quad (4)$$

For example, if $P(a,b) = 0.50$, and $P(b,c) = 0.8$, then implication (4)—which the authors will refer to as "functional equivalence"—would be satisfied if $P(a,c) = 0.8$. The relation between functional equivalence and Tversky and Russo's strong stochastic transitivity is discussed at the end of this paper.

The present experiment investigated the conditions under which the choice behavior of pigeons exhibits functional equivalence. One paradigm addressed the question: if indifference holds between Schedules A and B, will A and B be equally preferred to Schedule C? A second paradigm asked: if Schedules A and B are equally preferred to Schedule C, will indifference hold between A and B?

The question of transitivity directly relates to the assumption of several recent experiments that aperiodic schedules (such as variable-interval—VI—or variable-ratio—VR) possess periodic (e.g., fixed-interval—FI—or fixed-ratio—FR) "equivalents" (Herrnstein, 1964; Fantino, 1967; Killeen, 1968; Davison, 1969; Duncan and Fantino, 1970). An aperiodic schedule is preferred to a periodic schedule with the same mean interreinforcement interval (Herrnstein, 1964) or the same mean response requirement (Fantino, 1967). It has been assumed that were the appropriate

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method found for averaging the components of aperiodic schedules, then the effects of all basic schedules upon choice could be treated within a single conceptual framework. Thus, Killeen (1968, pp. 263-264) stated "... whenever an organism is indifferent between different schedules of reinforcement, appropriate measures of reinforcement frequency for these schedules will be equal." It will be shown that this kind of approach can succeed only to the extent that choices for schedules of reinforcement exhibit transitivity in the present sense of functional equivalence. Unfortunately, the present data indicate that functional equivalence is not a general property of choice behavior, at least in the choice procedure that was utilized in all the experiments cited above.

The procedure employed in this and the previous experiments was the concurrent-chains procedure (Autor, 1960). In a chained schedule of reinforcement (Ferster and Skinner, 1957), the reinforcer is presented after two or more schedules are completed in succession. Each schedule is correlated with an exteroceptive stimulus, so that the consequence of completing a schedule (other than the one terminating in reinforcement) is production of the stimulus for the next schedule. In concurrent-chains, the organism responds on two keys illuminated by the color associated with the initial links of two chained schedules. One chain is identified with the left key, the other chain with the right key. A peck during the initial links occasionally produces the stimulus for the terminal link on the key pecked, and renders the alternate key dark and inoperative. Further responding on the lighted key produces food according to the schedule of reinforcement in that link. A choice proportion for the terminal-link schedules is calculated by: (1) counting the total number of pecks during the *initial links*; (2) dividing this sum into the number of initial-link pecks on one key. The choice proportion in concurrent chains is equivalent to the "relative rate of responding", since the durations of the two initial-link stimuli are always equal.

In a typical transitivity test, one terminal-link schedule might be a VI schedule. The value of the FI schedule in the other terminal link would be manipulated until each terminal link maintained a comparable amount of responding in the initial links (choice propor-

tion = 0.50). Will the resultant FI schedule (Schedule B) be functionally equivalent to the VI schedule (Schedule A)? This would be tested by comparing each schedule to a third schedule (Schedule C), for example, a different FI schedule. Choice proportions would first be determined when Schedules C and A were in the terminal links, and then when Schedules C and B were in the terminal links. If Schedules A and B were indeed functionally equivalent, they should yield approximately the same choice proportion when so compared with Schedule C.

METHOD

Subjects

Eleven male adult White Carnaux pigeons were maintained at approximately 80% of free-feeding weights. All were experimentally naive at the start of the experiment.

Apparatus

Each of two experimental chambers (Ferster and Skinner, 1957) contained a solenoid-operated grain hopper centrally located between two circular translucent response keys, mounted 3 in. (7.62 cm) apart and 9 in. (22.9 cm) above the floor. The reinforcer consisted of 3.25-sec access to mixed grain. Stimulus projectors (Industrial Electronic Engineers, Inc., #1820) were used to transilluminate the keys with white, blue, green, red, yellow, or amber light. Two 6-w houselights provided general illumination except during reinforcement. Each peck on a lighted key produced auditory feedback by operating a 110-v ac relay adjacent to the chamber. A minimum force of 12 to 15 g (0.118 to 0.147 N) was required to operate the keys. White noise masked extraneous sounds. Standard electromechanical scheduling equipment was located in an adjacent room.

Procedure

Concurrent-chains schedule. Figure 1 presents a diagram of the concurrent-chains procedure. Equal VI schedules arranged access to the terminal links. When the terminal link on one side became available, the VI timer associated with that side stopped operating while the alternate timer continued to operate. Entry into a terminal link occasioned three events: (1) the key color changed on the side pecked; (2) illumination was removed from the other

Fig. 1. Diagram of the concurrent-chains procedure. Figure 1a illustrates the order of events when responses were reinforced on the left key, Figure 1b the order of events when responses were reinforced on the right. Initial links were associated with white light, the two terminal links with different colors (typical colors shown). Responding in the terminal links produced food according to VI, VR, FI, or FR schedules.

key, which became inoperative and provided no feedback clicks; (3) the VI timer associated with the other key stopped operating. Further responding on the lighted key produced food according to the schedule in that terminal link. If both timers had scheduled access to their respective terminal links at the time one terminal link was entered, entry into the other terminal link remained available upon return to the initial links. The initial links were reinstated after one reinforcement.

A pair of terminal-link schedules was presented for choice until the daily choice proportions satisfied the following stability criterion. After 18 days (and every day thereafter until stability was reached), the choice proportions of the last nine days were divided into three successive blocks of three sessions. Performance was considered stable when two conditions were met: (1) the means of these blocks did not differ from each other by more than 0.05; (2) the means showed neither an upward nor a downward trend, *i.e.*, neither $\bar{X}_1 < \bar{X}_2 < \bar{X}_3$ nor $\bar{X}_1 > \bar{X}_2 > \bar{X}_3$. The reported choice proportions are the means of the final nine sessions.

Transitivity paradigms. Two procedures

were used to test for transitivity. Procedure 1 addressed the question: if indifference holds between two schedules, will these schedules be equally preferred to a third schedule? Procedure 2 addressed the question: if two schedules are equally preferred to a third schedule, will indifference hold between the first two schedules? For ease of communication, the letters A, B, and C are used to designate the significance of the schedules as follows. Schedule A represents the aperiodic schedule for which a "periodic equivalent" was sought; Schedule B represents the periodic equivalent; and Schedule C represents the third schedule used to test the equivalence of A and B.

In Procedure 1, Schedules A and B were equated to each other and then presented for choice with Schedule C. If transitivity holds, the preference for Schedule A *vs* Schedule C should equal the preference for Schedule B *vs* Schedule C. Table 1 illustrates the order of conditions for Procedure 1 with VI and FI schedules (typical values shown). Choice proportions in this and subsequent tables represent preferences for Schedule A wherever possible. In steps that exclude Schedule A, the choice proportions for Schedule B are indicated. Thus, in the choices A *vs* B and A *vs* C, preferences are for A. In the choice B *vs* C, preferences are for B. The letter designations appear in parentheses beside the schedules and choice proportions.

In Step 1 of Procedure 1, Schedules A (*e.g.*, VI 23-sec) and B (FI X-sec) were equated to each other by adjusting the value of X of Schedule B until the stable preference for A was about 0.50. The acceptable range of values was arbitrarily designated as 0.45-0.55. Steps 2 and 3 constituted the transitivity test, in which Schedules A and B were each presented for choice with Schedule C. Schedules A and B were presented on the same keys on which they were initially equated in Step 1. Transitivity requires that the choice proportion for

Table 1
Order of Conditions for Procedure 1

Step	Schedule		Choice proportion
	Left	Right	
1	VI 23 (A)	FI X (B)	0.50 (A)
2	VI 23 (A)	FI 20 (C)	M (A)#
3	FI 20 (C)	FI X (B)	M (B)#

A in the A *vs* C choice (represented by the value M) equal the preference for B in the B *vs* C choice (also represented by M). If transitivity was obtained (*i.e.*, choice proportions not differing by more than 0.05), the generality of this equivalence was tested by presenting a new schedule as Schedule C. In cases of intransitivity, the original point in Step 1 was replicated. For ease of inspecting the tables of results, those choice proportions that represent a transitivity test or a test of the generality of transitivity appear with the symbol “#”. The closer these choice proportions are to one another, the closer would be the approximation to equivalence. The difference between the values represents the *deviation from transitivity*.

If one accepts certain assumptions about how key bias operates, it can be shown Procedure 1 would not control for key bias in the sense that if a key bias were present, a deviation from transitivity might be expected on that basis alone. Data from Killeen (1968) provide a rough estimate of the extent to which key bias may have operated in the present experiment, since he employed similar schedule values. In Killeen's experiment, VI and FI schedules were presented in the terminal links of concurrent chains, and estimates were made of the FI schedules that would yield indifference with respect to the VI schedules. For VI 23-sec, this FI was 8.4 sec, while for VI 54-sec, the FI was 9.6 sec. In Procedure 1 of the present study, the same VI schedules reported by Killeen (1968) were arranged in both the initial and terminal links. If a substantial key bias existed in either of the two studies, the FI schedules required for indifference points in Step 1 might be expected to deviate substantially from Killeen's (unless both studies had equivalent key biases in the same direction). In three of the four cases involved, Killeen's values were closely approximated. When VI 23-sec was Schedule A, the values required for indifference were FI 7-sec and FI 10-sec (Birds 1 and 2). When VI 54-sec was Schedule A, FI 4-sec and FI 10-sec were required for indifference (Birds 5 and 6). The FI 4-sec falls about 5.5 sec short of Killeen's value (9.6). Thus, for three of four birds, the assumption of no substantial key bias appears to be a reasonable one in the sense that the indifference points obtained in the present experiment were equivalent to those obtained by Killeen. More-

over, there are additional suggestions lowering the plausibility of attributing intransitivity to key bias for two of these three birds: (1) For Bird 1, Schedules A and B were functionally equivalent with respect to one Schedule C but not with respect to another; (2) the intransitivity displayed by Bird 2 could be deduced from previous experiments (this deduction is developed in the Discussion section).

In Procedure 2, Schedules A and B were first equated with respect to Schedule C in the sense that each was preferred to Schedule C by the same amount. Schedules A and B were then presented for choice with one another. If transitivity held, the preference for A should have equalled the preference for B, a choice proportion for A of 0.50. Table 2 presents the order of conditions for this procedure with the same schedules used in Table 1. In Step 1, Schedule A was presented for choice with Schedule C and the choice proportion for A was determined, represented here by M. In Step 2, Schedule A was removed and replaced with Schedule B (FI X). The value of X in Schedule B was then adjusted until a stable preference for Schedule B was obtained approximately equal to M (difference not exceeding 0.05), the preference for A in the A *vs* C choice. Steps 3 and 4 constituted the transitivity test. Schedules A and B were presented for choice with each other, with the sides reversed in different steps. If transitivity held, the average choice proportion for Schedule A should have been about 0.50. Acceptable values ranged from 0.45-0.55. As with Procedure 1, if transitivity was obtained, a test of generality was conducted by presenting Schedule A and B for choice with a new third schedule (for example, Conditions 5 and 6 for Bird 7 in Table 3). If intransitivity was obtained, the original points in Steps 1 and 2 were replicated (for example, Conditions 5 and 6 for Bird 9 in Table 3). A deviation from transitivity in Procedure 2 is the difference between the average

Table 2
Order of Conditions for Procedure 2

Step	Schedule		Choice proportion
	Left	Right	
1	VI 23 (A)	FI 20 (C)	M (A)
2	FI X (B)	FI 20 (C)	M (B)
3	VI 23 (A)	FI X (B)	0.50 (A)#
4	FI X (B)	VI 23 (A)	0.50 (A)#

choice proportion for Schedule A in Steps 3 and 4, and the value, 0.50.

Procedure 2 controls for key bias in the sense that a conclusive demonstration of transitivity is possible even if a key bias exists (providing that any such bias is roughly constant across Steps 1 and 2, and across Steps 3 and 4; key bias in Steps 1 and 2 can differ from the key bias in Steps 3 and 4).

Treatment of "error". In Procedure 1, the indifference point in Step 1 was replicated if the deviation from transitivity exceeded 0.05. The purpose of this replication was to determine whether the deviation from transitivity obtained in Steps 2 and 3 fell within the limits of replication error. If the deviation from transitivity exceeds replication error, Schedules A and B are considered non-equivalent or intransitive. If the deviation is within the limits of error, the test is considered inconclusive. For example, assume that the choice proportion for Schedule A in Step 1 was 0.52 and the deviation from transitivity in Steps 2 and 3 was 0.15. In the replication of Step 1, a choice proportion for Schedule A of 0.47 is obtained. The replication error in this case would be $(0.52 - 0.47) = 0.05$. Because this estimate of error may itself be subject to error, we shall require that the deviation from transitivity exceed the estimate of error by at least 0.05 (in this example, 0.10). Since the deviation from transitivity in the present example was 0.15, we would conclude that choices for Schedules A and B were intransitive.

Similarly, in Procedure 2, Steps 1 and 2 were replicated if the deviation from transitivity exceeded 0.05. The purpose of this replication was to ascertain whether Schedules A and B were still equally preferred to Schedule C. To make such a determination, we compared the difference between the choice proportions for Schedules A and B before the transitivity test with the difference between the choice proportions for A and B after the test. The difference between these differences represents replication error. For example, Bird 9 in the present study (Table 3) gave choice proportions for A and B before the transitivity test of 0.86 and 0.87, respectively (in Conditions 1 and 2). A deviation from transitivity of 0.14 was then obtained (Conditions 3 and 4). In the replication of Steps 1 and 2, the choice proportions for A and B were 0.95 and 0.96, respectively (Conditions 5 and 6). The estimate of error in this case

would be $[(0.86 - 0.87) - (0.95 - 0.96)] = 0$, suggesting that intransitivity obtained in Conditions 3 and 4 was not due to replication error. Moreover, in a second transitivity test with Bird 9, the deviation was 0.13, virtually identical to the original deviation (Conditions 7 and 8). For Procedure 2, replication error based on changes in absolute values will be indicated as well as changes based on differences. For example, Table 3 shows that Bird 8 gave a choice proportion for A over C in Condition 1 of 0.70 and a choice proportion in Condition 5 of 0.86, a change of 0.16. In Condition 2 the choice proportion for B over C was 0.71, while in Condition 6 it was 0.65, a change of 0.06. The average change in absolute values would be $(0.16 + 0.06)/2 = 0.11$.

Replication of Step 1 in Procedure 1, and of the difference between Steps 1 and 2 in Procedure 2, served as a control for possible "hysteresis" effects during the selection of Schedule B, that is, for the possible influence of preceding schedule values upon the value of Schedule B. Replication of these values would suggest that any such influence was not substantial and that Schedules A and B had been genuinely equated. In addition, the value of Schedule B was usually adjusted several times up and down before the final value was selected.

Intransitivity findings are qualified in two ways. In the first place, considerable shifts in key bias sometimes occurred over successive tests with the same bird. Such shifts may potentially account for obtained intransitivities. In the second place, we have assumed that differences between choice proportions ≤ 0.05 represent the influence of incidental factors. All of the subsequent analysis proceeds on this assumption. Should one instead view all differences between choice proportions, however small, as indicative of differences in schedule values, an alternative analysis is possible (cf. Tversky and Russo's (1969) analysis of substitutability). This analysis takes into account the direction of intransitivities. In Procedure 2, for example, if $P(a,c)$ and $P(b,c)$ were 0.82 and 0.80, respectively, and $P(a,b)$ were 0.92, functional equivalence but not substitutability would be violated. If $P(a,b)$ were 0.08, both would be violated. If differences within the 0.05 limit represent incidental factors, as we assume, then substitutability should be satisfied in about 50% of the cases, since the chance that $P(a,c) > P(b,c) \approx 0.50$. Of 11 intransitivi-

Table 3
Functional Equivalence Tests for VR versus FR Schedules

(i)	(ii)	Bird Condition	Schedules		(iv)	(v)	Choice Proportion	Outcome		Error		(ix)	(x)	(xi)		(xiii) Initial link response rates combined (R/min)	(xiv) Sessions	
			Left	Right				Transit.3*	Dev.**	Diffs.**	Abs.			Conds.	Sec/reinf			
															Left			Right
7	1	FR 5 (C)	VR 15 (A)	0.39 (A)									1.96	8.65	43.4	18		
	2	FR 5 (C)	FR 15 (B)	0.38 (B)									1.85	4.68	51.1	42, 19 ¹		
	3	FR 15 (B)	VR 15 (A)	0.40 (A)#									4.08	3.92	57.6	20		
	4	VR 15 (A)	FR 15 (B)	0.54 (A)#	YES		0.03	0.15	8-7;11-12				4.38	3.75	58.0	22		
	5	FR 40 (C)	VR 15 (A)	0.62 (A)#									8.80	2.76	58.5	30		
	6	FR 40 (C)	FR 15 (B)	0.59 (B)#	YES		0.03	0.15	8-7;11-12				8.31	2.87	42.8	18		
	7	FR 50 (C)	FR 15 (B)	0.58 (B)#									12.18	3.06	49.9	21		
	8	FR 50 (C)	VR 15 (A)	0.73 (A)#	NO		0.15	0.15	8-7;11-12				10.56	2.64	45.7	29		
	9	FR 15 (B)	VR 15 (A)	0.55 (A)#									2.88	2.64	55.1	24		
	10	VR 15 (A)	FR 15 (B)	0.52 (A)#	YES		0.04	0.15	8-7;11-12				3.06	3.54	48.4	31		
	11	FR 50 (C)	VR 15 (A)	0.89 (A)#									23.52	2.88	49.3	34		
	12	FR 50 (C)	FR 15 (B)	0.72 (B)#	NO		0.17	0.15	8-7;11-12				13.38	3.36	51.5	27		
8	1	FR 35 (C)	VR 15 (A)	0.70 (A)									13.25	5.75	46.8	18		
	2	FR 35 (C)	FR 18 (B)	0.71 (B)									10.00	5.37	50.0	44, 20		
	3	FR 18 (B)	VR 15 (A)	0.58 (A)#									4.77	5.58	59.9	21		
	4	VR 15 (A)	FR 18 (B)	0.57 (A)#	MAYBE		0.08	0.11	1-2;5-6				2.76	5.04	57.8	18		
	5	FR 35 (C)	VR 15 (A)	0.86 (A)									14.46	5.28	49.1	20		
	6	FR 35 (C)	FR 18 (B)	0.65 (B)									9.18	5.88	45.1	22		
	7	FR 35 (C)	FR 14 (B)	0.87 (B)									14.16	6.54	42.3	59, 18		
	8	FR 14 (B)	VR 15 (A)	0.45 (A)#									3.84	5.34	57.6	19		
	9	VR 15 (A)	FR 14 (B)	0.63 (A)#	YES		0.04	0.11	1-2;5-6				2.88	5.40	59.9	20		
	10	FR 45 (C)	VR 15 (A)	0.86 (A)#									14.14	5.40	50.5	19		
	11	FR 45 (C)	FR 14 (B)	0.93 (B)#	MAYBE		0.07	0.11	1-2;5-6				16.92	5.58	46.4	21		
	12	VR 45 (A)	FR 45 (C)	0.86 (A)									15.90	13.02	41.4	24		
9	1	FR 32 (B)	FR 45 (C)	0.87 (B)									7.98	15.00	61.4	53, 20		
	2	VR 45 (A)	FR 32 (B)	0.69 (A)#									11.45	7.56	72.1	22		
	3	FR 32 (B)	VR 45 (A)	0.58 (A)#	NO		0.14	0.09	1-2;5-6				9.00	15.16	71.4	19		
	4	FR 45 (A)	FR 45 (C)	0.95 (A)									13.68	11.64	94.1	19		
	5	VR 45 (A)	FR 45 (C)	0.96 (B)									8.76	14.04	62.6	21		
	6	FR 32 (B)	FR 32 (B)	0.80 (A)#									12.30	6.66	84.3	22		
	7	VR 45 (A)	VR 45 (A)	0.46 (A)#	NO		0.13	0.02	5-6;9-10				9.30	15.36	57.9	22		
	8	FR 32 (B)	FR 45 (C)	0.94 (A)									12.24	14.22	91.7	18		
	9	VR 45 (A)	FR 45 (C)	0.98 (B)									8.34	15.24	59.1	19		
	10	FR 32 (B)	FR 45 (C)	0.76 (A)#									14.52	6.36	62.6	20		
	11	VR 45 (A)	FR 32 (B)	0.41 (A)#	NO		0.09	0.02	5-6;9-10				9.24	16.38	52.1	27		
	12	FR 32 (B)	VR 45 (A)															

ties found in this study, six cases satisfied substitutability, close to the 50% to be expected on the basis of chance.

Tests with VI and VR schedules. Six birds were employed in tests of equivalence for VI *vs* FI schedules. The VI schedules in both the initial and terminal links were the same as those reported by Killeen (1968). The initial links contained VI 56-sec schedules; Schedule A was either VI 23-sec or VI 54-sec.

Five birds were employed in tests of equivalence for VR *vs* FR schedules. The initial links contained VI 1-min schedules, the intervals of which were determined according to Segal's (1964) method. Schedule A was VR 15, VR 45, or VR 90, with ratios also selected by Segal's method (1964).

Sessions were terminated after 50 reinforcements, except in the case of Bird 9, where there were 40 reinforcements per session following the first condition to prevent satiation.

RESULTS

Transitivity became less likely as the aperiodic schedules increased in size. With ratio schedules, four tests of six with VR 15 that permitted definitive judgements by our criteria (*i.e.*, excluding tests resulting in "Maybe" judgements in Tables 3 and 4), indicated transitivity, while zero tests of five with VR 45 or VR 90 did so. With interval schedules, three tests of four that involved VI 23-sec (but excluded VI 54-sec), and that permitted definitive judgements, indicated transitivity, while zero tests of three involving VI 54-sec did so. The ratio and interval data are now considered separately.

Ratio schedules. Table 3 presents data for tests with ratio schedules. The terminal-link schedules are listed in columns (iii) and (iv) in their order of presentation. All data in this and Table 4 are based on performances during the nine-day period in which the daily choice proportions satisfied the stability criterion defined in the Procedure section. Values in the two tables were calculated in the same manner. We are primarily interested in the choice proportions shown in column (v). The choice proportion is obtained by dividing the number of initial-link responses on one key by the total number of initial-link responses. Letter designations appear beside both the schedules and the choice proportions. The deviation from

transitivity (column vii) and the estimate of error based on differences (column viii) are absolute values. The estimate of error based on changes in the absolute values of choice proportions (column ix) is calculated from the same conditions (column x) as the estimate of error based on differences. For example, suppose that column (x) listed the conditions 1-2; 5-6. The estimate of error based on differences would be obtained with the formula $[(1-2) - (5-6)]$, where the condition numbers are replaced with their associated choice proportions. The estimate based on absolute values would be obtained with the formula $(|1-5| + |2-6|)/2$. Judgements as to whether a test indicated transitivity are based on error estimates derived from differences and are entered in column (vi). A "Yes" judgement is entered if the deviation from transitivity does not exceed 0.05. A judgement of "Maybe" is entered if the deviation from transitivity exceeds 0.05 but does not exceed the estimate of error by at least 0.05. A "No" judgement is entered if the deviation from transitivity exceeds 0.05 and exceeds error by at least 0.05.

In addition to these data, Table 3 presents the overall response rates in the initial links (mean of nine sessions; the response rate in each initial link can be derived from this figure and the choice proportion) in column (xiii). Since the rate of reinforcement on ratio schedules partially depends on the rate of responding, the obtained interreinforcement intervals are presented in columns (xi) and (xii). The reciprocals of the intervals are the rates of reinforcement. The relative rate of reinforcement can be obtained by dividing the rate of reinforcement on one key by the sum of the rates of reinforcement on both keys.

Bird 7 preferred VR 15 and FR 15 equally to FR 5 (Conditions 1 and 2). VR 15 and FR 15 also produced indifference in the transitivity test (Conditions 3 and 4). When presented for choice with FR 40 in a test of the generality of this equivalence, VR 15 and FR 15 continued to yield equal preferences over FR 40 (about 0.60—Conditions 5 and 6). Functional equivalence failed to generalize to FR 50 as Schedule C, with VR 15 yielding a higher preference than FR 15 in each of two tests (Conditions 8 and 7, 11 and 12). The transitivity obtained in the first test with VR 15 and FR 15 (Conditions 3 and 4) was recovered in a second test (Conditions 9 and 10) after equiva-

lence failed to generalize to FR 50 (Conditions 8 and 7). Thus, functional equivalence depended on the size of the common schedule, holding in the lower range of values (FR 5, FR 40), but breaking down at higher values (FR 50). Bird 8 preferred VR 15 and FR 14 equally to FR 35 (Conditions 5 and 7); VR 15 and FR 14 also generated indifference (Conditions 8 and 9). Results for Birds 7 and 8 are thus consistent in showing functional equivalence with VR 15 as Schedule A, although for Bird 7, non-equivalence resulted when Schedule C was made sufficiently large.

In contrast to these results, transitivity was not obtained in any of the tests involving VR 45 or VR 90. Deviations from transitivity were consistently in the direction of a greater preference for the VR schedules. Bird 9 preferred VR 45 and FR 32 equally to FR 45 in Conditions 1 and 2. In the transitivity test (Conditions 3 and 4), VR 45 was preferred to FR 32 by a substantial amount—an average choice proportion for VR 45 of 0.64. Intransitivity was observed with the same schedules in two additional tests. Indeed, in the third test, Bird 9 gave a somewhat greater choice proportion for FR 32 over FR 45 than it did for VR 45 over FR 45; yet VR 45 was preferred to FR 32. Results for Bird 10 were consistent with those for Bird 9. Although VR 45 and FR 50 were equally preferred to FR 15 (Conditions 1 and 2), VR 45 was strongly preferred to FR 50 (Conditions 3 and 4)—an average choice proportion for VR 45 of 0.69.

The foregoing tests indicated that higher valued VR schedules and FR schedules are not functionally equivalent in tests involving other FR schedules. Bird 11 was employed to determine whether VR and FR schedules exhibit equivalence with respect to FI schedules. The results were consistent with the previous data; although VR 90 and FR 28 were equally preferred to FI 30-sec (Conditions 1 and 2), VR 90 was strongly preferred to FR 28 (Conditions 3 and 4)—the average choice proportion for VR 90 exceeding 0.90.

In summary, equivalence was demonstrable with VR 15 as Schedule A (the schedule for which a "periodic equivalent" was sought), but was not demonstrable with VR 45 or VR 90 as Schedule A. A typical pattern of intransitivity emerged. Although the VR and FR schedules were about equally preferred to another FR schedule or an FI schedule, the VR

schedule was preferred to its "periodic equivalent" in the transitivity test. Transitivity also appeared to be dependent upon the size of Schedule C. VR 15 and FR 15 were equivalent for Bird 7 with respect to FR 5 and FR 40, but did not exhibit equivalence with respect to FR 50.

Interval schedules. Table 4 presents the results of tests involving interval schedules. Obtained interreinforcement intervals are omitted because they were virtually identical to the scheduled intervals. Response rates in the terminal links are means of the response rates in each of the sessions during the nine-day stability period. In all other respects, Table 4 is analogous to Table 3.

As with ratio schedules, transitivity became less likely as the aperiodic schedules increased in size. In most cases, VI 23-sec and its empirically-determined "periodic equivalent" exhibited functional equivalence with respect to other FI schedules, but did not exhibit equivalence with respect to VI 54-sec. Moreover, the VI 54-sec schedule and its periodic equivalent failed to show functional equivalence. These trends are clearly illustrated with Birds 1 and 6.

Bird 1 was indifferent between VI 23-sec and FI 7-sec (Conditions 5 and 10). These two schedules exhibited functional equivalence with respect to FI 15-sec (Conditions 6 and 7). However, when VI 54-sec was substituted for FI 15-sec (as "Schedule C"), VI 23-sec and FI 7-sec were no longer equivalent. Instead, VI 23-sec produced a greater choice proportion (over VI 54-sec) than did FI 7-sec (Conditions 8 and 9). Bird 6, tested with VI 54-sec as Schedule A, gave results consistent with those of Bird 1. Although VI 54-sec and FI 10-sec were equally preferred to one another (Condition 1), VI 54-sec was more strongly preferred to FI 20-sec than was FI 10-sec (Conditions 2 and 3). Thus, VI 54-sec gave intransitive results whether utilized as Schedule C or as Schedule A.

DISCUSSION

The present results show that schedule preferences are frequently intransitive. With Procedure 1, aperiodic and periodic schedules were found to which the bird was indifferent; the preferences for these schedules over a third schedule were often unequal. With Procedure

Table 4
Functional Equivalence Tests for VI versus FI Schedules

(i)	(ii)	Bird	Condition	(iii)		(iv)	(v)	(vi)		(vii)	(viii)	(ix)	(x)	Response rates (R/min)		(xiiv)			
				Left	Right			Transit.?	Dev.**					Error	Terminal Links		Initial Combined	Sessions	
															Left				Right
PROCEDURE 1																			
1	1		VI 23 (A)	FI 5 (B)	0.48 (A)										70.2	125.7	34.6	46, 19 ^a	
	2		FI 10 (C)	FI 5 (B)	0.60 (B)#										80.9	136.2	46.2	18	
	3		VI 23 (A)	FI 10 (C)	0.54 (A)#	MAYBE	0.06								72.8	69.0	28.9	19	
	4		VI 23 (A)	FI 5 (B)	0.43 (A)										66.5	90.0	56.7	20	
	5		VI 23 (A)	FI 7 (B)	0.53 (A)										55.4	50.9	51.5	20, 18	
	6		FI 15 (C)	FI 7 (B)	0.66 (B)#										55.6	64.0	45.1	20	
	7		VI 23 (A)	FI 15 (C)	0.66 (A)#	YES	0.00								46.0	43.6	49.1	22	
	8		VI 54 (C)	FI 7 (B)	0.55 (B)#										45.1	70.1	40.5	25	
	9		VI 23 (A)	VI 54 (C)	0.67 (A)#	NO	0.12								42.5	38.8	53.5	20	
	10		VI 23 (A)	FI 7 (B)	0.52 (A)										42.4	64.5	46.3	18	
2	1		FI 10 (B)	VI 23 (A)	0.47 (A)										82.9	80.3	53.2	92, 27	
	2		FI 20 (C)	VI 23 (A)	0.60 (A)#										42.8	79.6	59.0	18	
	3		FI 10 (B)	FI 20 (C)	0.79 (B)#	NO	0.19								101.3	22.9	54.1	32	
	4		FI 10 (B)	VI 23 (A)	0.44 (A)										80.4	62.0	60.4	22	
5	1		VI 54 (A)	FI 4 (B)	0.49 (A)										68.2	248.9	72.2	39, 18	
	2		VI 23 (C)	FI 4 (B)	0.41 (B)#										78.5	251.9	69.2	25	
	3		VI 54 (A)	VI 23 (C)	0.56 (A)#										69.8	43.4	56.5	18	
	4		VI 54 (A)	FI 4 (B)	0.57 (A)	NO	0.15								71.3	277.0	66.1	19	
	5		FI 20 (L)	FI 10 (R)	0.45 (L)										83.5	238.0	57.9	18	
	6		VI 54 (A)	FI 4 (B)	0.48 (A)										67.7	251.3	68.2	20	
	7		VI 54 (A)	FI 23 (C)	0.54 (A)#										65.0	52.8	61.0	18	
	8		VI 23 (C)	FI 4 (B)	0.45 (B)#	MAYBE	0.09								77.6	248.7	57.6	23	
6	1		FI 10 (B)	VI 54 (A)	0.50 (A)										104.0	63.2	20.1	29, 19	
	2		FI 20 (C)	VI 54 (A)	0.97 (A)#										58.8	84.8	23.9	32	
	3		FI 10 (B)	FI 20 (C)	0.87 (B)#	NO	0.10								169.4	87.2	40.1	31	
	4		FI 10 (B)	VI 54 (A)	0.45 (A)										145.8	101.6	42.5	27	

PROCEDURE 2

3	1	VI 23 (A)	FI 23 (C)	0.88 (A)				121.6	79.2	51.7	22
	2	FI 11 (B)	FI 23 (C)	0.88 (B)				166.3	107.7	51.2	48, 19
	3	VI 23 (A)	FI 11 (B)	0.58 (A)#				138.1	243.6	57.2	18
	4	FI 11 (B)	VI 23 (A)	0.50 (A)#	YES	0.04	0.09	230.7	107.9	63.3	18
	5	VI 23 (A)	FI 5 (C)	0.54 (A)#				119.6	247.2	53.8	18
	6	FI 11 (B)	FI 5 (C)	0.43 (B)#	MAYBE	0.11	0.09	190.0	244.7	51.9	24
	7	VI 23 (A)	FI 5 (C)	0.45 (A)#	YES	0.02	0.09	99.4	230.8	58.1	25
4	1	FI 2.7 (C)	VI 23 (A)	0.29 (A)				174.3	105.8	93.6	18
	2	FI 2.7 (C)	FI 9 (B)	0.29 (B)				173.8	111.2	90.8	5, 18
	3	FI 9 (B)	VI 23 (A)	0.46 (A)#				94.0	98.4	71.9	19
	4	VI 23 (A)	FI 9 (B)	0.66 (A)#	MAYBE	0.06	0.03	78.8	104.2	50.7	32
	5	FI 2.7 (C)	VI 23 (A)	0.24 (A)				185.3	88.5	77.6	21
	6	FI 2.7 (C)	FI 9 (B)	0.27 (B)				182.5	105.3	80.7	19

*Criteria for judgements are as follows: *Yes*—deviation from transitivity is 0.05 or less, regardless, of error; *No*—deviation from transitivity exceeds 0.05 and exceeds error based on differences by at least 0.05; *Maybe*—deviation from transitivity exceeds 0.05 but does not exceed error based on differences by at least 0.05.

**Absolute values †The first of two numbers in a pair indicates the number of sessions before selection of Schedule B in this step. #Values involved in a transitivity test.

2, aperiodic and periodic schedules were found that were equally preferred to a third schedule; the bird often failed to show indifference between these two schedules. Such intransitivities suggest that aperiodic schedules and their empirically-determined "periodic equivalents" do not necessarily function in the same way in tests involving other schedules.

That transitivity in the present sense of functional equivalence cannot be a general property of choice follows from recent attempts to find a rule by which aperiodic schedules may be transformed into their periodic equivalents (*e.g.*, Fantino, 1967; Killeen, 1968, Davison, 1969, 1972; Duncan and Fantino, 1970). Killeen (1968) presented VI and FI schedules for choice in the terminal links of concurrent chains. When the interreinforcement intervals were transformed, choice proportions were found to "match" the relative rate of reinforcement provided by a schedule. The appropriate transformation involved an harmonic mean of the intervals comprising the VI schedule, or, in the following formula, an exponent $r = -1$:

$$\frac{P_1}{P_1 + P_2} = \frac{x^r}{x^r + \frac{1}{N} \sum_{i=1}^N y_i^r} \quad (5)$$

where P_1 and P_2 represent the number of pecks during the initial links on the keys associated with the FI and VI schedules, respectively; x represents the value of the terminal-link FI schedule; y_i is the value of the i^{th} interreinforcement interval in the terminal-link VI schedule; and N is the number of intervals in the VI schedule.

The harmonic transformation does not yield matching when other schedules are presented for choice. Davison (1969) found that for choice between mixed-interval and FI schedules an exponent of -3 in equation (5) was necessary. Duncan and Fantino (1970) presented periodic schedules for choice (two FIs or two FRs), and found that matching required exponents substantially lower than -1 (*e.g.*, up to -6). In other words, the pigeons in these experiments responded more for the schedule providing the higher rate of reinforcement than would be expected with an exponent of -1 . On the basis of their own and previous data, Duncan and Fantino suggested that the exponent required to produce matching may decrease with the size of the smallest

interreinforcement interval of the terminal-link schedules.

This rough rule would predict intransitivity (non-equivalence) if the size of the smallest interreinforcement interval were not constant across the set of comparisons comprising the transitivity test, *i.e.*, A vs B , B vs C , and A vs C . For example, Bird 2 received a VI 23-sec schedule in which the smallest interreinforcement interval was 2.7 sec. An FI 10-sec schedule yielded indifference with respect to VI 23-sec and was thus selected as the periodic equivalent. On the basis of both Killeen's (1968) and Duncan and Fantino's (1970) analysis, the exponent for this choice—FI 10-sec vs VI 23-sec—should be about -1 . Now, FI 20-sec was chosen as the common schedule to test for functional equivalence of VI 23-sec and FI 10-sec. When comparing FI 20-sec with VI 23-sec, the smallest interreinforcement interval is still 2.7 sec and an exponent of about -1 would continue to be appropriate. In the other comparison, however—FI 20-sec vs FI 10-sec—the smallest interval is now 10 sec and an exponent of about -3.5 would be appropriate according to Duncan and Fantino's general rule. Thus, the choice proportions in the two comparisons should be different, resulting in intransitivity. The present results support this prediction. Conversely, the empirical fact of intransitivity across aperiodic and periodic schedules argues against the possibility of discovering a single appropriate transform for determining periodic equivalents. Different choice proportions in a transitivity test would imply different exponents. Thus, intransitivity helps to explain the absence of a single appropriate transform: aperiodic and periodic schedules are not functionally equivalent.

Not all cases of non-equivalence in the present study are predictable from Duncan and Fantino's (1970) rule-of-thumb for the appropriate exponent. Additional intransitivities occurred in other cases as well. For example, the data of Birds 1 and 7 show that aperiodic and periodic schedules were sometimes functionally equivalent with respect to one schedule but not another, despite the absence of appreciable change in the value of the smallest interval. These intransitivities were produced with a pair of schedules that had previously shown equivalence within the same organism. For example, Bird 7 preferred VR 15 and FR 15 equally to FR 5 and, in the transitivity test,

preferred VR 15 and FR 15 equally to each other. The bird also preferred VR 15 and FR 15 equally to FR 40, but did not prefer these equally to FR 50. The smallest intervals with FR 40 as the common schedule were the same as those with FR 50 as the common schedule. The smallest interreinforcement interval was determined by the VR 15 or FR 15 schedule, whichever one was presented with the common schedule. Since the size of the smallest interval did not vary with the common schedule, the size of the common schedule itself is implicated as a factor in functional equivalence. Therefore, while intransitivity was often found when the size of the smallest interreinforcement interval varied across comparisons, intransitivity was sometimes obtained even when the sizes of the smallest intervals were comparable (e.g., Bird 1). A case of non-equivalence could probably be virtually guaranteed by sufficiently increasing the difference among the smallest interreinforcement intervals across comparisons. This prediction is supported by the very large deviation from transitivity shown by Bird 11, the only organism subjected to a difference substantially above 10 sec (we refer here to the mean interreinforcement interval on the FR 28 schedule, which was about 24 sec, as compared with the interval associated with the shortest ratio on the VR 90 schedule—three responses, or about 3 sec).

Functional Equivalence and Strong Transitivity

Tversky and Russo's (1969) definition of strong stochastic transitivity consists of two relations: the implication we have labelled functional equivalence (4), and the following one:

$$\begin{aligned} P(a,b) > 0.50, P(b,c) > 0.50 &\text{ imply} \\ P(a,c) > \text{maximum } [P(a,b), P(b,c)] &\quad (6) \end{aligned}$$

Tversky and Russo do not make explicit the distinction between these two relations. It can be shown that these implications bear different logical relationships to the mathematical property known as the "product rule" (Luce and Suppes, 1965). Any set of values that satisfies functional equivalence also satisfies the product rule. However, a set of values that satisfies implication (6) may violate this rule. This distinction is an important one from the standpoint of experimental tests of transitivity. If one could draw the same logical conclusions from implication (6) as from functional equivalence (4), a test of (6) would be preferable,

since an inequality is easier to establish experimentally than an equality. Since the relations have different properties, however, a test of one is not equivalent to a test of the other.

In summary, the present experiment showed the following: (1) transitivity in the very strong sense of functional equivalence is not a general property of choice, at least in the concurrent-chains procedure; (2) non-equivalence is likely when the size of the smallest interreinforcement interval varies markedly across comparisons; (3) intransitivity argues against a single appropriate method for transforming aperiodic schedules into their "periodic equivalents". Each of these findings is consistent with—indeed may be deduced from—earlier work.

REFERENCES

- Autor, S. M. *The strength of conditioned reinforcers as a function of frequency and probability of reinforcement*. Unpublished doctoral dissertation. Harvard University, 1960.
- Coombs, C. H. *A theory of data*. New York: John Wiley & Sons, 1964.
- Davison, M. C. Preference for mixed-interval versus fixed-interval schedules. *Journal of the Experimental Analysis of Behavior*, 1969, 12, 247-252.
- Davison, M. C. Preference for mixed-interval versus fixed-interval schedules: number of component intervals. *Journal of the Experimental Analysis of Behavior*, 1972, 17, 169-176.
- Duncan, B. and Fantino, E. Choice for periodic schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1970, 14, 73-86.
- Fantino, E. Preference for mixed- versus fixed-ratio schedules. *Journal of the Experimental Analysis of Behavior*, 1967, 10, 35-43.
- Ferster, C. B. and Skinner, B. F. *Schedules of reinforcement*. New York: Appleton-Century-Crofts, 1957.
- Herrnstein, R. J. Aperiodicity as a factor in choice. *Journal of the Experimental Analysis of Behavior*, 1964, 7, 179-182.
- Killeen, P. On the measurement of reinforcement frequency in the study of preference. *Journal of the Experimental Analysis of Behavior*, 1968, 11, 263-269.
- Luce, R. D. and Suppes, P. Preference, utility, and subjective probability. In R. D. Luce, R. R. Bush, and E. Galanter (Eds.), *Handbook of mathematical psychology*. Vol. 3. New York: Wiley, 1965. Pp. 249-410.
- Segal, E. F. A rapid procedure for generating random reinforcement intervals on VI and VR tapes. *Journal of the Experimental Analysis of Behavior*, 1964, 7, 20.
- Tversky, A. and Russo, J. E. Substitutability and similarity in binary choices. *Journal of Mathematical Psychology*, 1969, 6, 1-12.

Received 11 August 1971.

(Final acceptance 19 July 1972.)