RECOGNITION BY THE PIGEON OF STIMULI VARYING IN TWO DIMENSIONS¹

DONALD S. BLOUGH

BROWN UNIVERSITY

Pigeons served in four experiments, each of which involved about 44,000 discrete 1.2-sec trials under steady-state conditions. The first experiment scaled a short segment of the visual wavelength continuum; this dimension was then combined in a conditional discrimination with each of three others; time after reinforcement, tone frequency, and line tilt. In the two-stimulus experiments, the birds' responses were reinforced in the presence of only one stimulus combination: "582 nm" together with "2 min after reinforcement", "3990 Hz", or "vertical line". Many other stimulus combinations also appeared equally often and went without reinforcement. The wavelength stimuli conformed to an equal-interval scale, and per cent response was generally linear with wavelength, when scaled on cumulative normal coordinates. The components of the compound stimulus were found to interact in a multiplicative fashion; when one component differed greatly from its reinforcement value, changes in the other component had relatively little effect. For the "time"-"wavelength" compound, this interaction appeared to be modified by the effects of set or attention. Certain response latency data are reported, and other combination rules are discussed.

The stimuli that ordinarily control behavior are difficult to define, for they are apt to be physically complex and extended in space and time (Gibson, 1960; Garner, 1970). It is more difficult still to determine how an organism performs the integration that is necessary to recognize such complex stimuli. To make stimulus integration a manageable object of study (while, of course, greatly restricting its scope) one may define a stimulus by two classes of events that can be independently manipulated. The effect upon response of changing either stimulus component alone may then be recorded, together with the effect of simultaneous changes in both components. If the effects of single component changes are related to the effects of two-component changes by a combination rule, the form of this rule should reflect the manner in which the subject puts together information from the two components.

Students of animal behavior have been little concerned, until recently, with the definition and integration of stimuli. Even now, the

literature on stimulus control is heavy with references to selective rather than integrative mechanisms (attention, masking, overshadowing, blocking—*cf.*, Sutherland and Mackintosh, 1971); relatively few papers examine the joint control of behavior by components or dimensions of stimuli. When such control has been studied (*e.g.*, Butter, 1963; Wolf, 1963; Miller and Ackley, 1970), some sort of summation or improved control by two-dimensional stimulus change, as contrasted to one-dimensional change, is usually reported. In a few cases, subjects have shown a response rate to stimulus combinations intermediate between those to separate components. In these cases, there is reason to believe that the responses initially trained to the two components fall into different classes (even though these classes may be recorded in the same way). This is rather obvious when one combines a negative conditioned stimulus with a positive conditioned stimulus (*e.g.*, Reberg and Black, 1969), perhaps not so obvious when a stimulus associated with a low response rate is combined with a stimulus associated with a higher rate (Weiss, 1967). In the latter case, the measured switch-closing response probably suffers competition from a second response class of "waiting behaviors". The present research was con-

¹This research was supported in part by USPHS Grant MH-02456. Reprints may be obtained from the author, Department of Psychology, Brown University, Providence, Rhode Island 02912.

cerned, so far as possible, with a single class of responses; an exception, short-latency responses, will be considered in due course.

Combination Rules and Stimulus Scales

Most animal experiments using compound stimuli have not been designed to test quantitative models of the recognition process. One notable exception is the work of Chase and Heineman (1972); these authors fit their data with equations based upon statistical decision theory and an attention process. In multidimensional work with human subjects, combination rules commonly arise from another sort of model, which relates response to the psychological distance between stimuli (Torgerson, 1958; Shepard, 1965; Garner, 1970). Where such a spatial conception is used, the distinction is often drawn between the city-block rule and the Euclidean rule. According to the city-block rule, all pairs of values from two stimulus dimensions that sum to a constant will yield a constant response. The Euclidean rule follows the familiar formula for a straight-line distance between two points. In this case, pairs of stimulus values yield a constant response only if the squares of these values sum to a constant. It has been hypothesized that distinct or separable stimulus dimensions follow the city-block rule, while unanalyzable or "integral" dimensions follow the Euclidean rule. I shall return to these various models and rules later on.

A combination rule can provide insight into the recognition process only to the extent that it uniquely describes the data, for if many substantially different rules fit the data, many hypotheses about the combination process will remain tenable. A rule will remain ambiguous, however, if it is not clear how numbers are to be assigned to the variables that are incorporated in the rule. The ambiguity that can result where scales are lacking is exemplified by the alternative interpretations of Butter's (1963) two-dimensional generalization data. Butter suggested that a multiplicative rule described his data, for the effect of a simultaneous change along his two dimensions could be predicted by multiplying the effects of changing each dimension alone. However, a multiplicative rule is hard to handle with a spatial model. Cross (1965) therefore suggested that a logarithmic transformation be applied

to Butter's data; where stimulus effects had previously multiplied, they added under the logarithmic transformation, and thus they were made to conform to the city-block rule. Since no independent scales of stimuli or response were available, there is no convincing way to distinguish between the multiplicative and the additive rules for Butter's data—or, for that matter, to exclude other rules, based on other transformations. This is not to say that the data could be fit by any rule at all (*cf.*, Luce and Tukey, 1964; Torgerson, 1958; Anderson, 1970), but the distinction between a multiplicative rule, which implies stimulus interaction, and an additive rule, which does not, is of very considerable interest.

To avoid the sort of ambiguity just described, the one stimulus dimension (a short segment of the wavelength continuum) that appeared in all of the present experiments was independently scaled (Experiment 1). The scaling method was suggested by Shepard (1965). A series of closely spaced, overlapping discrimination functions was collected along the wavelength continuum. The method next dictates that the continuum be transformed, stretching here, squeezing there, until the gradients all have the same shape. The result is an equal-interval scale over that portion of the dimension for which the gradients overlap. In the present case, it happened that squeezing and stretching were unnecessary, for the gradients were all of approximately the same shape on the untransformed wavelength continuum. Thus, a unit of "hue" change results from a unit wavelength change.

The other scales used in the experiment are, in effect, pinned to this scale of hue. For example, a unit change in "pitch" results from any change in sound frequency that yields the same response change as does a unit change in wavelength. The response scale has no independent definition, but the data indicate that a cumulative normal transformation of response probability grows as an approximately linear function of wavelength. This makes it convenient to present most of the data in a familiar form, with response on a cumulative normal ordinate and stimulus wavelength on the abscissa.

Experiments 2, 3, and 4 combined a second variable with the scaled wavelength stimulus. Three different sorts of variable were used, because the integration process might well de-

pend upon the nature of the input. "Time" was a variable in Experiment 2; since time is measured from the moment of reinforcement, it was conceptualized as a variable that (like "payoff") might bias a decision process controlling wavelength recognition. In Experiments 3A and 3B, a tone was combined with wavelength; this is the sort of separable stimulus dimension that might be expected to add its effect to the wavelength variable in some simple way—perhaps following the "city-block" rule. In Experiment 4, line tilt was combined with wavelength. Since the "form" of a stimulus seems relatively inseparable from its hue (Garner, 1970), one might guess that the combination rule here would be different than for tone, perhaps following the Euclidean rule.

In the Discussion section, a variety of possible combination processes and their associated rules are considered. Most attention is focussed on signal-detection models, because these suggest plausible processes underlying the data, and because there is growing interest in applying this sort of formulation to animal data. However, the data appear, to conform most closely to a multiplicative rule that seems incompatible with either simple statistical decision models or with simple spatial models. Because of the prior scaling of wavelength, this rule cannot be arbitrarily transformed to an additive rule, nor can it be equated with Butter's multiplicative rule, which implies different stimulus scales. The essential conclusion is that the stimuli interact, rather than exerting their effects independently.

The experiments were conducted in the order reported, except that Experiment 1 was conducted between Experiments 3 and 4, and Experiment 4 followed almost a year later, with some other work on the same subjects intervening (Blough, 1969b).

EXPERIMENT 1: SCALING THE WAVELENGTH CONTINUUM

METHOD

Subjects

Four male White Carneaux pigeons served; two, No. 476 and No. 556, worked in all the experiments; a third, No. 33, died and was replaced by No. 156 for Experiment 4. All birds

had extensive experience in experiments concerned with reinforcement schedules and with stimulus control (Blough, 1966, 1969b). The birds were maintained at approximately 80% of their free-feeding weights by supplementary feeding, if necessary, after each experimental session. Except as noted below, they were run daily for approximately 2 hr, and received most of their food (mixed grain) in the experimental sessions.

Apparatus

The birds worked simultaneously in three Lehigh Valley pigeon chambers. The chambers were illuminated indirectly by a standard houselight. The boxes were equipped with three keys, of which only the center key was functional; this key operated on application of about 10 g (0.10 N) force. A loudspeaker supplied white noise to each box; a ventilating fan added to this masking noise. A Bausch and Lomb 250-mm grating monochromator provided visual stimuli. Light from a ribbon filament lamp operated at 17 amp ac passed through entrance and exit slits set to provide a half-width dispersion of 6.6 nm. The beam passed through a set of shutters and projected an image of the grating on the ends of three fiber-optics light guides of 0.125 in. (0.32 cm) diameter by 3 ft length (91.4 cm). The terminal ends of the light guides were mounted approximately 0.25 in. (0.64 cm) behind the translucent plastic response keys in the pigeon chambers. They produced a stimulus patch consisting of a fuzzy spot 0.375 in. (0.95 cm) in diameter, centered on the circular response key. The luminance of the spot was approximately 7 millilamberts at 582 nm. The spot appeared on the key only during brief trials, as described below.

A small digital computer (the LINC, Clark and Molnar, 1964) controlled stepping motors that adjusted the visual stimulus wavelengths, relays, and shutters that turned the stimuli on and off, and the solenoid controlling food presentation. The computer also recorded responses and response latencies.

Procedure

Birds 476 and 556 served in Experiment 1. They worked daily in a session consisting of 2499 test trials without reinforcement, among which approximately 78 reinforcement trials were randomly interspersed. The test trials

were arranged in 51 successive series of 49 stimuli each. In each series, each of seven wavelengths appeared on the response key. These wavelengths appeared in random order, except that each stimulus appeared exactly seven times in each series. No pecks were reinforced on any of these test trials. Reinforcement trials were scheduled with a probability of $1/32$, independently of the test series; they were randomly distributed among the test trials. Thus, a bird might get several reinforcement trials in a row, or none for long intervals. However, if the random process did not generate a reinforcement in 128 trials, a reinforcement was scheduled on the one hundred and twenty-ninth trial. The same stimulus wavelength appeared on all reinforcement trials; this wavelength was the same as the highest wavelength used in the test series. If the bird pecked at this stimulus on a reinforcement trial, it received about 2-sec access to grain; reinforcement duration was adjusted for each bird to compensate for differences in eating speed, such that the birds consumed approximately their daily food ration during a session.

Trials lasted 1.2 sec; the stimulus light came on, and remained on until the bird pecked or until the end of the trial. The brevity of the trials arises from unpublished data showing that, in this situation, few responses occur with latency exceeding 1 sec. The intertrial interval varied randomly between 1.20 and 1.45 sec, except that it was 3.5 sec long after reinforcement trials. Pecks during the intertrial interval were rare; if they occurred, they delayed the start of the next trial by 0.5 sec.

The birds were experienced when this experiment began; they had achieved asymptotic performance on a discrimination that included 582 nm as the positive stimulus and, as negative stimuli, six nonreinforcement test wavelengths spaced below 582 nm in 1-nm steps. For the present study, the birds were run, for four days, on each of six different, overlapping series of wavelengths. They started with 584 nm as the positive stimulus ($S+$); four days later the $S+$ became 583 nm and each test stimulus shifted down 1 nm; four days later there was a similar shift to a series with $S+ = 582$ nm, and so on down to $S+ = 579$ nm. The lowest series, with $S+ = 579$ nm, was given for six sessions, and then successively higher series were given for four days each until a final four sessions where

the series had begun, at $S+ = 584$ nm. The effect of this procedure was to yield 12 sets of heavily overlapping gradients, such that effects of stimulus spacing would be reflected simultaneously in each of several curves.

The wavelengths used here and in the other experiments were chosen because the output of the monochromator and the pigeon's photopic luminosity function are both relatively flat over this range (Blough, 1957). Thus, no correction was necessary to yield stimuli of approximately equal luminance. Also, generalization gradients suggest that this region is reasonably free of rapid changes in the appearance of these stimuli to pigeons (Blough, 1961).

RESULTS AND DISCUSSION

Figure 1 shows the results of Experiment 1. Each point represents the mean of six sessions with a common $S+$, three sessions from the first half of the experiment, when $S+$ was being reduced every four days, and three from the second half, when $S+$ was being increased. Though there appeared to be no learning effect over the four days at each wavelength, the first of these days was omitted from the data. For each bird, a mean psychometric function was determined by superimposing all of the data as though the same $S+$ had been used for all sessions. The curves in Figure 1 represent this mean performance.

The curves in Figure 1 are identical, and are spaced at 1-nm intervals regardless of the data points. Any systematic deviation of points around these curves can thus be attributed either to changes over time from one group of sessions to another, or to deviations of the wavelength scale from equal-interval spacing. Inspection reveals some systematic deviations, but these are attributable to sessions, rather than to wavelength scale factors. For example, in a few cases, most of the data points fall either above or below the corresponding curve, but in no case do the points associated with a particular wavelength all appear to be too high or too low. Though, no doubt, a slight rescaling of the abscissa could improve the fit of the points, this was not attempted.

The data thus indicate that, over the range of wavelengths used in this and in the following experiments (575 to 582 nm), the wavelength continuum approximates an equal-

Fig. 1. Results of Experiment 1, on the scaling of wavelength. Each point represents the mean of six sessions conducted with a single S+. The curves are not fit separately for each S+; instead, a single curve was used for each bird, and this curve is displaced one nanometer for each data set across the graph. Since the points do not depart systematically from this common curve, equal wavelength steps produce approximately equal behavioral effects over this stimulus range (see text).

interval scale for the pigeon. This is not unexpected, for the range of stimuli here is very small relative to the range over which the human sees changing hues. It also is in accord with recent data indicating that wavelength discriminability of the pigeon is relatively constant in this region of the spectrum (Wright, 1971). Though expected, the result tends to exclude from consideration in the following pages any interpretation that calls for unequal spacing of the wavelength stimuli or implies that some pairs of wavelengths "look more similar" than others equally spaced.

The sets of curves for the two birds in Figure 1 differ somewhat in shape. This difference suggests a correction, based on response latency, that is applied throughout this report. Earlier studies (e.g., Blough, 1969a) have shown that responses following very quickly upon each other ("short IRT responses") are poorly controlled by stimulus changes; the same was true of short latency responses in the present experiments. Bird 556 gave many responses of short latency, Bird 476 few. When responses shorter than 0.2 sec are removed from the data of Bird 556, its curves look somewhat more similar to those of Bird 476. Figure 2 shows this, together with

Fig. 2. Data from Figure 1 (one bird at one S+) showing that responses of short latency (here, those shorter than 0.2 sec) are poorly controlled by stimulus wavelength. The per cent response for responses longer than 0.2 sec has been corrected for opportunities to respond that were missed because of the short-latency responses. The ordinate is correct only for the longer-latency responses; the curve for short latencies was multiplied by a constant to match the two at the S+.

the fact that the short latency responses, taken by themselves, are poorly controlled by the wavelength variable. Unfortunately, the up-

per tail of the uncontrolled "short latency" distribution overlaps with the lower tail of the controlled "long latency" distribution; cutting at 0.2 sec is a compromise that appears to eliminate most of the uncontrolled responses. Response percentages for the remaining responses are calculated "per opportunity"; this conditional probability eliminates from consideration those trials that were, in effect, missed because of overly fast responses. Except in Experiment 2, Bird 476 gave very few short-latency responses, so the agreement of the data from this bird with those of the other subjects serves as a partial check on the validity of the latency correction. The latency correction may be considered a partial adjustment for "attention", a matter that is given further consideration below.

EXPERIMENT 2: WAVELENGTH AND TIME VARIABLE

The theory of signal detection provides a model for predicting behavior as a function of two variables, these variables commonly being called "signal" and "bias". In this experiment, the "signal" varies with the wavelength of light on the pigeon's response key. "Bias" is changed by scheduling reinforcement at fixed times, so that response probability rises as the time for reinforcement approaches.

METHOD

Subjects

Birds 476, 556, and 33, described above, served.

Apparatus

The apparatus was the same as that used in Experiment 1.

Procedure

The procedure was much like that of Experiment 1, with the time factor added. The wavelength of light on the response key varied from trial to trial as before, with reinforcement associated only with 582 nm. However, reinforcement became available on every forty-ninth trial, and at no other time except at the beginning of each session, when two trials involved reinforcement. Reinforcement lasted for 4 sec, and the intertrial interval was extended to 10 sec after each reinforcement. Since each trial lasted, including the

intertrial interval, about 2.45 sec, the total interval between reinforcements was approximately 128 sec. This could be extended if a pigeon pecked during intertrial intervals, but the incidence of such pecking was low. For purposes of data analysis, each of these intervals of about 2 min was divided into blocks of eight trials each. During each successive block, the wavelength projected on the response key assumed each of eight values in random order. These included 582 nm and seven other values spaced in 1-nm steps down to 575 nm.

A session was composed of 52 intervals of about 2 min each, as just described, each interval including six successive blocks of eight stimulus wavelengths, and ending with one presentation of the reinforcement wavelength, 582 nm. To allow for possible warm-up effects, the first two intervals were excluded from the analysis of data that is reported below.

Birds worked in this experiment for 34 sessions, of which the last 17 provide the data reported below. Over these 17 sessions, each wavelength appeared in each time interval on 850 occasions. However, on about 1% of these occasions, the bird had missed the last reinforcement by failure to peck at the S+. The data from any series following such a missed reinforcement were excluded from analysis. Response percentages were computed for the remaining trials: 838 trials at each time-wavelength combination for Birds 476 and 556; 843 for Bird 33.

RESULTS AND DISCUSSION

Figure 3 shows the data of Bird 476 in isometric projection. The effect of each variable acting alone can be seen on the back "walls" of this diagram, with wavelength changes at the time S+ to the left, and time changes at the wavelength S+ to the right. These marginal curves show that each dimension alone controlled almost the entire range of response. The figure also shows that the combined effect of the variables was always greater than the effect of either variable acting alone. Figure 4 shows the same data in two dimensions, together with the data from a second bird. (Data from the third bird are omitted here, though they are much like those of Bird 33. The third bird was probably able to see the stimulus dimly between some trials, due to shutter failure.)

Fig. 3. Results of Bird 476 in Experiment 2. The reinforcement wavelength-time combination is in the "back corner" of this isometric projection; that is, the reinforcement wavelength is 582 nm, and the reinforcement time is at the end of the "120-sec" interval.

Experiment 2 was originally conceived as an attempt to bias the detection of wavelength changes by altering the pigeon's tendency to peck as a function of time. Signal-detection theory provides a quantitative prediction of the outcome, if it is assumed that time since reinforcement shifts the subject's criterion

across a distribution of signal (wavelength) effects that is normally distributed on a decision axis (*cf.* Green and Swets, 1966). According to this model, the pigeon's task is to detect departures from the reinforcement wavelength. Thus, the various test wavelengths are "signals" of varying strength, depending upon their distance from S+ (*cf.* Blough, 1967). Detecting such a signal means not responding, so a "false alarm" is a trial on which the S+ is present and yet the bird does not respond, while a "correct detection" is lack of response to any other wavelength. Figure 5 shows ROC curves for Bird 476 plotted according to this scheme, with the axes scaled according to a cumulative normal transformation.

The simplest detection model, assuming normal distributions of equal variance associated with all stimulus wavelengths, predicts that the curves of Figure 5 will be straight lines parallel to the main diagonal. Clearly, they are neither straight nor parallel. With regard to their convergence, they look much like the data from the other experiments reported, and this matter is considered below. The curvature of the data could be accounted for if "time" acted not only on the subject's criterion, but also controlled attention to the visual stimulus, to some extent (*cf.* Heine-

Fig. 4. Complete data of two birds from Experiment 2. Data of the third bird look much like that of Bird 33, but are omitted because of shutter failure, leading to excessive overly fast responses.

Fig. 5. ROC curves for Experiment 2, plotted on the assumption that time from reinforcement changes the criterion that controls the bird's decision on the wavelength dimension. As in subsequent figures, points representing less than 1% response are omitted; this accounts for the blank space to the upper right. Each line represents responding to a single wavelength; points along the lines correspond to different times.

mann and Chase, 1970). This would mean that on some trials, especially early in the time interval, the bird did not respond no matter what wavelength appeared. (Such inattention could sometimes be observed; shortly after reinforcement, the birds might stand for several trials with their backs to the key, or pecking the floor.)

Latency records support the notion that time affected the pigeon's readiness to respond. The birds responded much more rapidly, on the average, when the time for reinforcement was near, regardless of stimulus wavelength. This is illustrated in Figure 6, where two time-wavelength combinations are compared. Each of the combinations gave about the same number of responses (see legend), but the latency distributions are markedly different. Though some latency effects are noted in other experiments below, this experiment produced a large latency shift, and also produced the only marked change in the form of the latency distribution.

The failure of the data from Experiment 2 to fit a simple detection model, and the probable complexity of the underlying processes, suggest that simpler situations ought to be studied. "Time" is, after all, a difficult vari-

able to cope with. It cannot be manipulated as such, and the basis of so-called "temporal discrimination" is unknown. Even if the process incorporates an internal stimulus generator with an output that varies through time, the experimenter has no way to turn off or reset this "clock". For this reason, I turn now to two situations that involve controlled exteroceptive stimuli. The results with these seem more clear-cut.

EXPERIMENT 3: WAVELENGTH AND TONE FREQUENCY VARIABLE

Experiments 3 and 4 combined the wavelength dimension with each of two other stimulus dimensions, tone frequency, and line tilt. As suggested above, these two combinations might produce different results; for example, separate dimensions roughly corresponding to "hue" and "pitch" for the human might exert their effects in an additive ("city-block") way. "Hue" and the form variable "tilt" seem less separable, so these might combine in a different way. Experiments 3A and 3B combined visual wavelength with auditory frequency, differing only in the ranges of the visual and auditory stimuli used.

METHOD

Subjects

Birds 476, 556, and 33, described above, served.

Apparatus

The apparatus was the same as that used in Experiment 1, with the addition of auditory stimuli. Except during stimulus presentations the loudspeakers in each box maintained white noise, as in Experiment 1, at about 90 dB, as measured with a GR1558-AP noise analyzer set at "all band". During stimulus presentations, this noise was replaced briefly by tones between 3000 and 4000 Hz supplied by an HP-200 CD oscillator. These sine wave signals were amplified and supplied to the small chamber loudspeakers. The tones were distorted to some extent by the amplifier and speaker system, and marked standing waves developed in the boxes when the tones were presented. These were attenuated somewhat by lining three walls of the pigeon chamber with carpeting, but the intensity of the tones varied from place to place in the box, depen-

Fig. 6. Representative latency data from Experiment 2. The open circles show responses in the presence of the reinforcement time, together with "visual S_4 ", 578 nm, which was four steps from the visual S_+ . The dots show responses to the reinforcement wavelength at "time S_3 ", which was three steps from the time S_+ (about 60 sec after the previous reinforcement). The two combinations of time and wavelength shown here each produced almost the same response (circles—35.9%; dots—38.9%) but quite different latency distributions. The shift exemplified here was apparent in data from all birds.

dent upon frequency, and it varied between approximately 89 and 95 dB at the pigeon's head level near the response key. A stepping motor controlled by the computer changed the tone frequency from trial to trial.

Procedure

Experiment 3A. The procedure differed from that of Experiment 2 in the following ways. Two stimulus variables were now correlated with reinforcement. One of these was the wavelength of light projected on the pigeon's key, as in Experiments 1 and 2. The other was the frequency of a tone in the pigeon's box, which had the physical characteristics already described. These tones and lights occurred simultaneously and for the same duration; thus, their presentation might be considered a "compound" stimulus, varying in two dimensions. Reinforcement was available only on the first trial in each session and on subsequent trials with a constant probability of 1/32. On such reinforcement trials, the visual stimulus wavelength was always 582 nm and the auditory stimulus frequency was always 3990 Hz. Nonreinforcement trials, 2499 in number, made up the remainder of the session. The stimuli to ap-

pear on any of these nonreinforcement trials were chosen from among seven tones and seven lights, the tones including the reinforcement value (3990 Hz) and six other values spaced in approximately equal steps down to 3185 Hz, and the light including 582 nm and six other values spaced at 0.5 nm intervals down to 579 nm. The combination of tone and light to appear on any trial was chosen randomly, except that no combination reappeared until the entire series of 49 had appeared. As in Experiment 1, the S_+ , that is, the combination 582 nm and 3990 Hz, was one of the 49 nonreinforcement combinations. The random appearances of this combination on reinforcement trials were in addition to the test presentations. The series of 49 nonreinforcement combinations was repeated 51 times in each session. Only the last 50 of these series are included in the data analysis; responses on all reinforcement trials are also excluded.

Birds were exposed to this schedule for 54 sessions, of which the last 17 provided the data analyzed below.

Experiment 3B. The range of visual stimuli employed in Experiment 3A was inadequate to yield complete coverage of the response range. The procedure in Experiment 3B was the same as that for Experiment 3A, except that the visual stimuli varied from 382 to 376 nm in 1-nm steps, and the auditory stimuli varied from 3990 to 3370 Hz in approximately 90-Hz steps.

The birds were run on this procedure for 24 sessions, of which the last 17 days provide the data analyzed below.

RESULTS

Since reinforcement was correlated with only one of 49 stimulus combinations, 582 nm (visual) and 3990 Hz (auditory), response probability should drop with departures from these values. Figure 7 shows an example of this response pattern. Responding dropped monotonically with wavelength, as in Experiments 1 and 2. The auditory dimension was not so well behaved; though per cent response changed uniformly with frequency in Experiment 3A, by the time the birds had stabilized on Experiment 3B some irregularities had appeared in the frequency functions. This irregularity was more marked in the data of

Fig. 7. Sample results from Experiment 3B. As in Figure 3, the reinforcement combination is in the "back corner". The nonmonotonicity of response to frequency changes (see curve on right "wall") can perhaps be ascribed to the discrimination of intensity (standing waves) as well as frequency (see text).

Bird 33, and less in the data of Bird 556, than that shown by Bird 476 in Figure 7. It seems probable that the birds began to discriminate

apparent loudness, to some extent, based on the strong standing waves in the boxes, which shifted with frequency.

Figures 8 and 9 show all the data from Experiment 3. The equal-interval wavelength scale suggested by Experiment 1 falls along the abscissa; tone frequency, which was not scaled, is the parameter, with each curve corresponding to one frequency; response, on the ordinate, has been scaled according to a cumulative normal transformation. Points representing response percentages of 1% or less (based on fewer than nine responses in 17 sessions) are omitted. These figures, and the similar ones for Experiment 4, also include a correction for attention that straightens the uppermost function at its high end. This correction is based on the assumption that, on a small percentage of trials, birds failed to respond due to non-stimulus causes. To correct for this, all response percentages were multiplied by a constant equal to the reciprocal of the assumed percentage of effective trials. The amount of the correction was such as to make the uppermost curve as straight as possible; this meant, on the average, that

Fig. 8. Complete data from Experiment 3A, but with responses shorter than 0.2 sec latency omitted. The data of Bird 556 are corrected for attention, assuming nonresponse on 2% of the trials; the remaining data are not corrected for attention.

Fig. 9. Complete data for Experiment 3B, as in Figure 8. Here, attention corrections were, for the birds (from left to right), 4, 2, and 1%.

attention was assumed on 98.6% of the trials. (The correction for each bird is given in the figure captions.) The correction had little effect on the pattern of response below 90%.

EXPERIMENT 4: WAVELENGTH AND LINE TILT VARIABLE

METHOD

Subjects

Birds 476, 556, and 156, described above, served.

Apparatus

The apparatus was the same as that used in Experiment 1, except in the following respects. Three slits of variable angular displacement were placed at an image of the monochromator grating. Each slit, in turn, was imaged on the end of a coherent fiber-optics light guide. This guide terminated about 0.25 in. (0.63 cm) behind a glass response key in the pigeon box. The pigeon thus viewed, on a dark background, a stimulus line of approximately 3.5 nm by 1.0 nm, of variable tilt. Since the dimensions of the line approached the grain of the rather coarse transmitting fibers (10 mil), the line image was distorted at some angles of tilt; it is possible that discrimination could to some extent have been based on this dis-

tortion as well as the angular position of the line. Because of light losses in this system, the monochromator slits were opened to a half-width dispersion of 13.2 nm (2-mm slits), with a resulting luminance at 582 nm of about 17 millilamberts.

Procedure

The procedure was identical to that of Experiment 3, except that variations in line tilt replaced variations in auditory frequency. The S+ was a vertical, 582-nm line. This was also one of the 49 test stimuli. The other test stimuli included all combinations of seven wavelengths, spaced at 1-nm intervals, and seven line tilts, spaced at $5\frac{1}{3}^\circ$ steps over a range of 32° . (For two birds the line tilted to the right; for one bird to the left.) White noise was on continuously during experimental sessions.

The birds were exposed to the basic procedure for 43 sessions. Over the first eight sessions, the density of reinforcement was gradually reduced from probability 1/8 to 1/32. The last 17 sessions provided the data reported here.

RESULTS

The combination surfaces generated by wavelength-tilt combinations looked very much like those exemplified in Figure 7 for

Fig. 10. Complete data for Experiment 4, as in Figures 8 and 9. Here, attention corrections were, for the birds from left to right, 1, 2, and 2%.

Experiment 3. Except that the tilt dimension did not exhibit the irregularity noted above in some of the auditory data, the joint effect of these two visual dimensions seemed much like that of the auditory and visual dimension. Figure 10 shows the complete data; again, points of less than 1% are omitted, and there is a small correction for attention.

DISCUSSION

The data of Experiments 3 and 4 display a high degree of regularity, which excludes certain rules that have been suggested for stimulus combination situations. I shall briefly describe some of these, and then suggest a rule that seems to fit the data quite well.

1. *Two changes are better than one: interference and dominance excluded.* All of the two-variable experiments agree on one point: a stimulus altered from S+ in two dimensions evokes less response than a stimulus altered to the same degree in either dimension alone. This fact excludes several possible combination rules. First, the stimulus changes do not somehow interfere with each other, such that a change in two dimensions yields more responses than a change in either alone. Second, two "dominance" rules are inapplicable. Were responses controlled only by the stimulus that was the most similar to the S+, data points

should be high when either stimulus was at S+. Were responses controlled only by the stimulus that was most different from the S+, data points should fall no lower for a stimulus combination than for one of the components alone (cf. Cross, 1965). Neither outcome is seen here.

2. *Additive and multiplicative combinations of untransformed probabilities.* If, as just stated, stimuli do not act alone, what rule summarizes their interaction? Perhaps the simplest rule that has been proposed, sometimes called the "discrimination hypothesis" (Guttman, 1956; Johnson, 1970), asserts that a change in two stimuli dimensions yields a change in per cent response equalling the sum of the response changes produced by either stimulus change alone. A glance at any of the data sets quickly dispels this notion, which generates very low predictions, including many "below zero" points for stimuli that actually yield many responses.

The multiplicative rule deserves more serious consideration. Some previous data support it (e.g., Butter, 1963) and it arises from a plausible decision rule. The rule states that if a change in stimulus component A reduces response to R_A per cent of its value at S+, and a change in stimulus component B reduces response to R_B per cent of its value at S+, the

two changes together will yield $R_A \times R_B$ per cent of the response at $S+$.

Independent decisions about the two stimulus components can yield this rule, if response is controlled by a simple statistical decision process. In fact, the multiplicative rule defines events as statistically independent in probability theory (Feller, 1950, p. 86). Thus, if event A is "the decision that the auditory stimulus is $S+$ " and event B is "the decision that the visual stimulus is $S+$ ", the multiplicative rule follows if a response occurs only when these decisions are both made, and if the decisions are independent.

An attention model may yield the multiplicative rule by providing for the independent sequential analysis of two stimuli. For example, a pigeon might attend first to a tone stimulus. If the tone was judged $S+$, then the bird would look at a visual stimulus. It would peck if the visual stimulus now appeared to be $S+$. Such conditional control or "control of attention" to one dimension by another has been suggested by several authors (*e.g.*, Heinemann and Chase, 1970). It has the characteristics of a response chain, a chain that could be seen if overt observing responses were required. For example, "if auditory $S+$, peck key A; if key A pecked, visual stimulus appears; if visual $S+$, peck key B". Pecks on key B should follow the multiplicative rule. As I mentioned earlier, the difference between the data of Experiment 2 and those of Experiments 3 and 4 may be due to a process of this sort. However, the assumption of sequential processing is not necessary to a multiplicative model. The result follows, for example, from a two-dimensional signal-detection model, providing that the underlying distributions of sensory events arising from each stimulus are assumed to be independent, and that the criteria employed on each dimension are also independent. (See discussion below and Figure 14B; see also Heinemann and Chase, 1970.)

Unfortunately, the simple multiplicative rule does not describe the present data, for the predicted points are all too high (Figure 11). The data of Experiment 2 are not fitted well, but they are fitted somewhat better than those of Experiments 3 and 4. This supports my earlier suggestion that, to some extent, time controlled attention to wavelength in Experiment 2.

The additive and multiplicative rules are

Fig. 11. Response predicted by the multiplicative rule in three experiments. Responses to single-stimulus (one-dimensional) stimulus change, as a per cent of response to $S+$, are multiplied to predict the combined effect of the two changes together. Note that the prediction for Experiment 2 is more adequate than that for the other experiments, which did not differ. This suggests an important "set" or "attention" factor in Experiment 2 where time affects response to wavelength (see text).

but simple instances of possible rules for predicting responses to compound stimuli from untransformed marginal response probabilities. A number of models of two-cue learning suggest other rules, though they cannot be directly applied here because they are designed for redundant cue situations (*cf.* Sutherland and Mackintosh, 1971, Chapter 5). On a common-sense basis, the failure of the multiplicative rule is not surprising, for a subject seeking maximum payoff (the most food for the least work) would probably not make independent decisions about the two dimensions. The statistical decision approach can take this into account, but before considering this approach, it is necessary to consider the extent to which attention and other extraneous factors, though not major factors in the data, might alter the results.

3. *Corrections for inhomogeneity: attention and other matters.* Whenever one wishes to summarize data by a mathematical function, inhomogeneity poses a potential threat. The data values will represent some composite, usually a sum or average, across trials, days, and perhaps subjects. If some of the data arise

from different processes than others, or processes with different parameters, it may happen that the data will be fitted by equations that do not represent unitary processes, or formulae that have misleading parameters. Estes (1956) summarized the sorts of trouble that averaging could get us into, and the kinds of functions where it poses much, or little, problem. Though his arguments (and those of others) are most familiar as applied to averaging across subjects, he points out that they apply equally to repeated measures on the same subject. In general, they apply to any source of inhomogeneity.

I have argued elsewhere for caution in the interpretation of response rate, because of potential inhomogeneity in this measure (Blough, 1965). As noted above, the inhomogeneity of the rate measure may apparently be increased by removing from it "short IRT" responses; similarly, the homogeneity of the present data is, hopefully, increased by eliminating short latency responses. Short latency trials may be considered a subset of those trials on which the birds were "inattentive". (By definition, a subject is not attending to a stimulus on a trial if that stimulus has no effect on the subject's behavior on that trial.) Inattentive trials might also be identified by observing behavior, if some record could be kept of this: the behavior of a bird with its head in the food magazine cannot be controlled by a stimulus on the response key. However, apart from latency, there seems no practical way directly to identify "inattentive" trials; thus, a statistical correction, similar to the "correction for guessing" in psychophysics, becomes necessary.

A sufficient condition for "perfect attention" to a stimulus is that, over many trials, one value of the stimulus always evokes response while another never does. In the present experiments, both dimensions in several cases nearly meet this criterion, for on trials where only one of the two dimensions departed from S+, response rate often dropped from near 100% to near 0% (*cf.* Bird 476, Figures 9 and 10). Where these extremes are not reached, some trials remain on which one or both of the stimuli may have been ineffective. If inattentive on such trials, the bird might have responded anyway, or it might have failed to respond. In this paper, I assume that if the bird responded on an inattentive trial, it

always did so with a latency below the 0.2-sec cutoff. I justify this assumption only on the ground that an additional statistical correction for such trials seems to add little clarity to the data.

This leaves for statistical correction only those inattentive trials on which the bird failed to respond. (If one watched a bird long enough, one would see such trials, on which the bird was pecking at the floor or flapping its wings upon stimulus presentation.) A correction for these is incorporated in Figures 10 through 19; it is the same as that used by Heinemann and his co-workers (*e.g.*, Heinemann *et al.*, 1969) for such trials. This correction attempts to raise the upper asymptote of the psychometric function to 100%, by multiplying all data points by a constant that makes up for the trials effectively missed through inattention. To estimate the actual asymptote, the upper curve of each of the families in Figures 8, 9, and 10 was plotted on a cumulative normal ordinate, and the correction aimed to straighten the upper end of this curve. A correction was not necessary in some cases; overall it averaged 1.4%.

The correction just mentioned assumes lack of control by either of the two dimensions that were varied in these experiments. Much more difficult is the estimation of trials on which one dimension is attended while the other is not. The statistical problems are complex; perhaps more important, such a correction must assume some controlling process in the absence of input from one dimension. In lieu of correction for attention to single dimensions, computer simulations were run with various estimates for attention parameters and an additive rule, described in the next section, that derives from signal-detection theory. Though some departures from linearity in the data might be accounted for on the basis of such effects (for example, those to which the eye is drawn by dotted lines for Bird 476, Figure 10), there seems little reason to believe that such effects were a major factor in the present data.

A word is in order about other sources of inhomogeneity. It is plausible to suppose that the birds might discriminate more sharply on some days than on others; thus, cumulative normal functions of different variance might describe their data from day to day. In extreme cases, summing such data will lead to distorted

functions (cumulative normal being an instance of Estes "Class C" function). The average function remains undistorted, in such a case, if the cumulative normal transformation is made before the data are combined across days. This was done for a large sample of the data, but the shift in data points was very small, in most cases not plottable. Similarly, the multiplicative prediction was carried out within days, and the results averaged, with insignificant change in predicted points.

4. *Combinations of normalized data: signal-detection theory.* The results of Experiment 1 imply that equal wavelength steps should have about equal behavioral effects over the range of wavelengths used in this research. In Experiments 3 and 4, a cumulative normal transformation of response probability seems to grow in an almost linear fashion with wavelength. This linearity suggests, in turn, that combination rules governing these data may find their simplest expression in terms of the normalized measure.

The signal-detection approach readily accepts such a normal transformation of response probabilities, for, though it deftly bypasses the scaling problem, it typically assumes normal distributions of sensory events along a decision axis (Green and Swets, 1966). When the criterion (or distribution of sensory events) shifts in equal steps along this axis, the resulting response probabilities conform to a cumulative normal function. If this model applies to the present data, equal wavelength steps elicit approximately equal shifts in the distribution of events along the decision axis.

Classic detection theory incorporates an additive law for the combination of variables that control response, in the sense that changes in response probability are determined by the algebraic sum of shifts, along the decision axis, of distributions of sensory events and of a criterion. When the variance of sensory and criterion distributions remains constant, the additive process is reflected by "receiver-operating-characteristic" (ROC) curves that are parallel to the main diagonal of the standard ROC graph with normalized coordinates. As I indicated above, the data of Experiment 2 deviate in systematic ways from this picture. Sample data from Experiment 4 are plotted in this form in Figures 12 and 13, with each dimension considered in one figure as "signal" and in the other as "bias". There

Fig. 12. ROC curves for a bird in Experiment 4, plotted on the assumption that wavelength affects the criterion applied to judgements of tilt.

is no technical reason for choosing one form over the other, and the figures emphasize the symmetry of the situation.

The data in Figures 12 and 13 are fitted remarkably well by a series of straight lines. Also, because normalized response probability is almost a linear function of wavelength, the reader will find that if he rotates Figure 13 through 180° he will almost duplicate the

Fig. 13. Same data as Figure 12, plotted on the assumption that tilt affects the criterion applied to the judgement of wavelength.

leftmost family of curves in Figure 11, which are based on the same data. Reversing the process with the other data sets in Figures 8, 9, and 10, the reader may confirm that all yield a picture similar in its general outlines to that in Figure 12. The ROC curves, then, are well approximated by straight lines, but they are obviously convergent, and hence do not support the additive model of simple detection theory. Interestingly, such convergence is frequently also found in human psychophysical data; the detection formulation can accommodate such data either by abandoning the assumption that events on the decision axis are normally distributed, or by assuming that the variance of the distributions is itself a function of signal or bias. Before elaborating on this matter, it is necessary to consider a more comprehensive two-dimensional detection scheme that Chase and Heinemann (1972) have applied to data similar to the present results.

The expanded detection scheme adds a degree of freedom by treating sensory events distributed over a two-dimensional decision surface. Panel A of Figure 14 shows how the S+ stimulus in the present studies may be assumed to generate a bivariate normal distribution of sensory events on such a surface. The two axes represent the directions in which this distribution is shifted by changes in each of the two stimulus dimensions. A sample test distribution is shown, such as might result from trials upon which an auditory stimulus and a visual stimulus simultaneously departed from S+.

A criterion line across the surfaces in Figure 14 divides stimulus effects that evoke response from those that do not. If the point that represents the momentary effect of the two stimuli on a trial falls on one side of this line, a response will occur; if on the other, no response will occur. Many combination rules can be generated by choosing different func-

Fig. 14. Hypothetical distribution of sensory events over a decision surface defined by two orthogonal stimulus dimensions. Here, for example, changing wavelength away from S+ causes sensory events to become more probable toward the right foreground; changing tone frequency causes them to become more probable toward the left foreground. Height on the vertical axis represents the probability of each combination of sensory events, given a particular combination of stimuli. Thus, with each stimulus combination is associated one bivariate distribution, or "mountain". Criteria slice the mountains in various ways, as described in the text. The mountain for only a single one of the 49 test stimuli is shown in panels B, C, and D.

tions for this criterion line. For example, the multiplicative rule, resting as it does upon independent decisions about the two dimensions, arises when a separate criterion line runs parallel to each axis, as shown in Figure 14B. A response follows only for sensory effects that fall within both these lines, as indicated by the shaded area in the Figure; this is equivalent to saying "yes" about both stimulus dimensions. Figure 14B shows that stimulus changes, either of which alone yield 50% response (suggested by dotted distribution curves on the respective axes), yield 25% response (one quarter of the stimulus "mountain") when they are combined. This, of course, exemplifies the multiplicative rule.

A straight criterion line (Figure 14C) was derived by Chase and Heinemann (1972) for their situation. Here, a large departure from S+ on one axis can be compensated by a small difference on the other axis to yield a point still within the criterion line. Hence, the same stimulus changes that yielded 25% response in Figure 14B combine here to yield much less, the actual percentage depending on the variance of the distribution. (If the S+ stimulus yields 98% response, the test stimulus yields 2%.)

What criterion might apply to the present situation? Boneau and Cole (1967) derived the criterion for pigeons' responding reinforced on part of a single stimulus dimension, and their derivation may be generalized to the present case. The outcome is equivalent to the customary "likelihood ratio" solution. Boneau and Cole replace the all-knowing "ideal observer" of detection theory with a pigeon that keeps track of the relative frequency with which any combination of sensory events is associated with reinforcement. The frequency of food is relatively high in the presence of stimulus combinations near the center of the S+ distribution. But it is unlikely that food will accompany a stimulus effect far out on both axes, because such combinations are much more likely to arise from one of the nonreinforcement stimuli than from the S+ combination.

Extending the formulation of Boneau and Cole to the present case, one finds the line that divides the decision plane in Figure 14 into regions differentially associated with reinforcement. Sensory effects with a given probability of reinforcement, or more, lie on

the side of the line toward S+; those associated with less reinforcement lie on the side of the line away from the S+. To find the line, assume that the distribution of observations associated with reinforcement corresponds to the bivariate normal "mountain" centered where the stimulus axes cross. For simplicity, assume further that the distribution of observations associated with nonreinforcement is flat over a large area, but falls off at the edges according to a normal density function. This flat surface represents the sum of the 49 distributions associated with the several test stimuli. (This sum would not really yield a flat surface, even if both stimulus dimensions were scaled in equal intervals. However, using the sum would probably not make much difference in the criterion obtained.) The criterion line is defined by the intersection of the S+ "mountain" with the S- "plain", where the relative height of mountain and plain is determined by a constant that represents the pigeon's willingness to peck, given a fixed probability of reinforcement. (For details of this argument, see Boneau and Cole, 1967.)

Figure 14D represents the criterion line that results from the above assumptions, and shows how this line intersects with a single test stimulus distribution. It will be seen that the volume of the test distribution within this line is intermediate between those in Figures 14B and 14C, and so it is no surprise that the data predicted from this model are intermediate in form between the curving, convergent functions of the multiplicative rule and the linear, parallel functions of the simple detection model. An attempt to fit these functions to sample data is shown in Figure 15.

It is worth noting that the model summarized in Figure 14 can yield the city-block and Euclidean rules that were previously defined. The linear criterion of Figure 14C leads to city-block addition of stimulus effects: if a stimulus change on one axis alone causes a given response change (the response being measured on the cumulative normal scale), and a change on the other axis causes a second response change, the effect of both stimulus changes together will be the sum of the separate response changes. Likewise, the outcome of the criterion in Figure 14D approximates that of the Euclidean rule.

The criterion used to generate the curves in Figure 14D yields a better fit to the data

Fig. 15. Data points from Experiment 3B fitted by lines generated, by computer simulation, using the criterion shown in Figure 14D. This is not necessarily the best fit, but all curves using this and related criteria appear to be either too curved or too parallel.

Fig. 16. Same data as Figure 15 (plus three more curves from additional tone stimuli), fitted by eye with straight lines that meet at a point. Such a fit implies that tone and light multiply to produce their effect upon response probability.

than any rule so far considered (and, in fact, better than the exact Euclidean rule). Yet, again, the linearity of the data (Figures 8, 9, and 10) is slighted. Computer simulations with a variety of other criteria fail to improve this situation adequately. It is hard to escape the conclusion that the stimuli interact either with each other or with the criterion. Such interaction will now be considered.

5. *Multiplicative interaction.* Linearity and convergence characterize the data of Figures 8, 9, and 10. These features suggest a new multiplicative rule, differing from the one discussed above because it applies to normalized data. This rule requires the curves in these figures to converge upon a single point, and, as it turns out, this requirement does surprisingly little violence to the data. Figure 16 shows a fit to data of Bird 476, Experiment 3B, of curves drawn by eye to intersect at a single point. This arbitrary "zero" turns out to be at about 0.01% response, reached at a stimulus wavelength of about 575 nm. (Were the tone dimension scaled, one could presumably find a frequency that also would correspond to "zero".) A computer-aided method

was used to find convergence points like those of Figure 16 for all of the data sets from Experiments 3 and 4, with the point of convergence altered step by step until there was little improvement in a least-squares fit of the lines to the data points. The results of this procedure are summarized in Figure 17 for the two birds that were used in all of the experiments. The fits for the other birds were comparable, though the computed points of convergence for these birds tended to be at lower wavelength and lower response levels than the others. This fit, it should be noted, required the estimation of scale values for the second stimulus dimension (frequency or tilt), in addition to the two main parameters that specify the point of convergence of the curves.

The new combination rule, which I call "multiplicative interaction" to distinguish it from the multiplicative rule abandoned earlier, appears from Figure 17 to provide a good description of the data from Experiments 3 and 4. Assuming that the equal-interval wavelength scale from Experiment 1 is accepted, the form of this rule is rather firmly fixed. A

Fig. 17. Here, the process exemplified in Figure 16 has been carried out with most of the data from Experiments 3 and 4. Each line in Figure 16 produces a predicted response for each wavelength; the corresponding dots in Figure 16 represent obtained response. These predictions arise from a least-squares fit; fits for birds not shown are comparable.

logarithmic transformation cannot convert it to an additive rule, in the manner discussed previously, because the wavelength step sizes must be made unequal if the transformed data are to fit the rule. Other transformations that might be suggested by the signal-detection model will not work for the same reason. For example, if sensory events along a decision axis are assumed to follow a Poisson rather than a normal distribution, the convergent ROC curves of Figures 12 and 13 can be accounted for within the detection model, but only if we make the assumption, contrary to Experiment 1, that equal wavelength steps produce unequal displacements of the signal distribution along the decision axis.

The present experimental paradigm was chosen because it seemed to represent, in a simple form, the process by which a single stimulus pattern is picked out of a complex environment: hence the single-response, single-reinforcement stimulus combination, and steady-state test procedure. Animal data from somewhat more complex situations have led other authors to different combination rules. The data of Heinemann and Chase (1970) and Chase and Heinemann (1972) are the

most extensive. The former concerns conditional stimulus control and is probably most like the present study. Pigeons were trained with the four possible combinations of two light intensities and two sound intensities, one combination leading to reinforcement of one response, the other three combinations to reinforcement of the other response. Heinemann and Chase argue that their results support the multiplicative rule (independent criteria of Figure 14B), though some data deviated in the direction shown in Figure 11 of the present report. Though their data, which came from transient generalization tests, lack the detail necessary to enable a choice among models predicting small differences, it is quite possible that steady-state testing of a single S+ pattern against a field of nonreinforcement stimuli (as in the present study) generates a different pattern of results. The more recent study of Chase and Heinemann (1972) incorporated a symmetrical test procedure that may involve the recognition of two stimulus patterns, one for each of the two responses that were reinforced. The results of that study were consistent with the city-block rule of signal-detection theory (linear criterion of Figure 14C).

The only other animal data that appear to come close to testing the sort of models under discussion are those from a single-stimulus experiment (Blough, 1967). That paper presents ROC curves based on the pigeons' "ratings" of stimuli varying in wavelength. If the unknown factors affecting the pigeons' criterion in that study are called a "second stimulus dimension", for comparison purposes, then multiplicative interaction receives further support from those data. Those ROC curves were as markedly convergent as the ones reported in the present study.

6. *The functional significance of multiplicative interaction.* In the preceding sections, various combination rules were discarded, and with them several models for the recognition of compound stimuli. The surviving rule, multiplicative interaction, describes the data well. What might it mean? Since stimulus combination, not response measurement, is the matter at issue, clarity is gained by plotting the rule in terms of the values along the two dimensions that yield a constant response. A sample of such combinations can be derived from Figure 16 by drawing horizontal lines at arbitrary response levels, and noting the stimulus combinations at each intersection of this line with one of the data curves. Figure 18 shows the resulting points, fitted by curves for several response levels. This is the view that one would see if he looked straight down on the generalization "mountain" in Figure 7, assuming that the frequency dimension there were rescaled in equal intervals; the curves represent contours of constant height across the slopes of the mountain (these are the "isosimilarity contours" of Shepard, 1965). The contours belong to a family of hyperbolas of the form $x \cdot y = R_i$, where x and y represent the wavelength and rescaled frequency dimensions, measured from an estimated zero at lower left, and R_i is a fixed response level.

The interaction of the stimulus dimensions is expressed, in Figure 18, by the fact that stimuli trade off differently, depending upon the amount by which they depart from S+. The 80% contour results from small stimulus changes, relative to S+. This contour approximates a straight line, and thus the city-block rule: along this contour, one small step away from S+ on one dimension is compensated by an almost equal step toward S+ on the other dimension. In contrast, the 1% contour, far

Fig. 18. Selected points from Figure 16, replotted to show the trade-off between stimulus components that will maintain a constant response. The multiplicative relation requires that these contours be hyperbolic; they pass exactly through the points because the points are estimates based on the fitted lines of Figure 16. This sort of plot cannot be based on raw data, because only rarely will actual data points fall along the same response contour.

from S+, nearly corresponds to a segment of a circle. Along most of this contour, each stimulus affects response approximately in proportion to the square of its value—an approach, thus, to the Euclidean combination rule. Consider, for example, the point at which the 1% contour meets the upper edge of the figure. Here, wavelength changes are more effective than frequency changes, for, to maintain a constant response, a large change in frequency is necessary to compensate a slight change in wavelength. (The relative weighting of the dimensions is, of course, reversed at the other end of the contour.) Data are lacking below 1% response, but were multiplicative interaction to hold, the influence of one dimension would become progressively more extreme, as suggested by the "0.05%" contour in Figure 18. This process terminates at the axes; here, the "dominance rule" holds, as defined in Section 1 above.

The essential message of the data, summarized in Figure 18, is that each component contributes to the recognition of the compound S+, but that the more one component differs from S+, the more potent that component becomes. This outcome raises a number

of questions. For example, are these observations related to "overshadowing"? This concept has reappeared in recent animal discrimination literature, having developed from Pavlov's observation that the weaker element of a compound stimulus may be ineffective when tested alone. The present dimensions are not intensive, but if a "strong" stimulus is analogous to one markedly different from S+, the present results show overshadowing in a novel context. The relative weighting of the components does not seem due to associative mechanisms, or to a lasting attentional shift, since the components are weighted differently from trial to trial, depending upon their relative values on the individual trials. Should this result prove generally true, it would provide further support for a "perceptual processing" interpretation of overshadowing (*cf.* Kamin, 1968).

The nature of the processing remains to be specified more exactly. Does the multiplicative rule strictly hold, such that the estimated "zero" stimulus values have empirical meaning? It is perhaps equally plausible that these zero points do not really exist. Instead, the relative potency of the stimulus components might asymptote, such that, for example, the Euclidean rule might represent stimulus interaction for all stimuli sufficiently distant from S+. (In this regard, it is well to note that the widespread interest in the Euclidean rule seems justified mainly on grounds of mathematical convenience, rather than by compelling arguments that an organism might be expected to operate this way.) These questions might best be answered by trying to discover variables that affect the apparent "zero" points, which are out of empirical reach in the present study, lying as they do near a response level of 0.01% or less.

Still more tenuous is the possible connection between the present results and superficially similar data from human experiments on recognition and stimulus similarity. Yet, the sort of interaction found here has a certain intuitive appeal, and it might be worth more consideration than it usually receives. For example, the convergence of ROC curves (*cf.* Figures 12 and 13) is generally explained by non-interactive mechanisms (Green and Swets, 1966). Yet, the interaction of bias and effective signal is not really unlikely; subjects know that they are sometimes "sharp" and

sometimes "fuzzy", and they may tend to relax their criteria when signal input is being poorly processed. Likewise, it is not unreasonable that a stimulus component might carry relatively increasing weight in similarity judgement as it departs from a standard; in commonsense terms, we would say that it "attracted the subject's attention", without necessarily meaning that "attention" is an all-or-nothing affair.

Some theorists would probably feel that any appeal to "attention" must imply the separability of stimulus components. In this regard, it is interesting that the wavelength-tilt compounds generated about the same pattern of results as did the wavelength-tone compounds. Though color and form dimensions have been called "integral" in some human work (Gardner, 1970), color and pitch are certainly separable. Even the results of the wavelength-time compound (Experiment 2) follow the same general form as those of the other experiments, leading to my earlier suggestion that all of the experiments may reflect the same combination process, modified in the case of Experiment 2 by the action of attention or set.

A note on latencies. Response latency varied markedly among subjects and across the many months spanned by these experiments. Visual observation of the birds suggests that this variance can be attributed to the development, and topographic drift, of superstitious behaviors preceding the key-peck response. These included such things as pecking the chamber wall, pacing, turning, and head-waving. I do not report extensive latency data, because it appears that differences between the experiments may be due largely to this irrelevant variation. A critic could fairly ask if these latency differences, or any other time-related effects, might have affected the response probability data reported above. I have tried to correct for two cases where this might be so: short-latency responses (which, in Bird 556 were correlated with pre-response nibbling near the key) and response failure through inattention. For the rest, I can only argue that despite remarkable differences in latency distributions, and the death and replacement of one bird, the effects reported above were all consistent across all birds. (A nagging doubt remains in my mind that perhaps the results of Experiment 2, which was

Fig. 19. Latency distributions from Experiment 3B that exemplify an effect seen in all birds. The two stimulus combinations produced almost the same response (circles—40.2%; dots—39.6%), yet the birds took relatively longer to respond to stimulus combinations that included a visual stimulus differing from S+. Such latency differences did not appear systematically in Experiment 4 (wavelength and tilt variable).

conducted first, may owe some of their special characteristics to their primacy; I think it unlikely enough to balk at the months required for a repetition.)

Despite the predominant variability, several interesting latency results were consistent across birds. Two I have mentioned: the poor control of short-latency responses in all experiments (Figure 2), and the marked shortening of latency as the time for reinforcement drew near in Experiment 2 (Figure 6). In addition to these effects, there were small but consistent increases in latency as the visual stimulus (either tilt or wavelength) departed from S+. By contrast, latency did not change consistently with auditory variation. One outcome of this was that for stimulus combinations that yielded about the same overall response output, those that involved a large auditory change together with a small visual change yielded the shorter latency. This finding, true for each bird, is illustrated in Figure 19 with data from Bird 476 selected because each of the two stimulus combinations shown gave almost exactly the same number of responses. A possible interpretation of this finding is that recognition time increases as the stimulus departs from S+, but that visual recognition is slower in this situation and thus is the factor limiting latency.

REFERENCES

- Anderson, N. H. Functional measurement and psychophysical judgement. *Psychological Review*, 1970, 77, 153-170.
- Blough, D. S. Spectral sensitivity in the pigeon. *Journal of the Optical Society of America*, 1957, 47, 827-833.
- Blough, D. S. The shape of some wavelength generalization gradients. *Journal of the Experimental Analysis of Behavior*, 1961, 4, 31-40.
- Blough, D. S. Definition and measurement in generalization research. In D. I. Mostofsky (Ed.), *Stimulus generalization*. Stanford: Stanford University Press, 1965. Pp. 31-37.
- Blough, D. S. The reinforcement of least-frequent interresponse times. *Journal of the Experimental Analysis of Behavior*, 1966, 9, 581-591.
- Blough, D. S. Stimulus generalization as signal detection in pigeons. *Science*, 1967, 158, 940-941.
- Blough, D. S. Generalization gradient shape and summation in steady-state tests. *Journal of the Experimental Analysis of Behavior*, 1969, 12, 91-104. (a)
- Blough, D. S. Attention shifts in a maintained discrimination. *Science*, 1969, 166, 125-126. (b)
- Boneau, C. M. and Cole, J. L. Decision theory, the pigeon, and the psychophysical function. *Psychological Review*, 1967, 74, 123-135.
- Butter, C. M. Stimulus generalization along one and two dimensions in pigeons. *Journal of Experimental Psychology*, 1963, 65, 339-346.
- Chase, S. and Heinemann, E. G. Choices based on redundant information: an analysis of two-dimensional stimulus control. *Journal of Experimental Psychology*, 1972, 92, 161-175.
- Clark, W. A. and Molnar, C. E. The LINC: A description of the Laboratory Instrument Computer. *Annals of the New York Academy of Sciences*, 1964, 115, 653-668.
- Cross, D. V. Metric properties of multidimensional stimulus generalization. In D. I. Mostofsky (Ed.), *Stimulus generalization*. Stanford: Stanford University Press, 1965. Pp. 72-93.
- Dodwell, P. C. *Visual pattern recognition*. New York: Holt, Rinehart and Winston, 1970.
- Estes, W. K. The problem of inference from curves based on group data. *Psychological Bulletin*, 1956, 53, 134-140.
- Feller, W. *An introduction to probability theory and its applications*. New York: John Wiley & Sons, Inc., 1950.
- Garner, W. R. The stimulus in information processing. *American Psychologist*, 1970, 25, 350-358.
- Gibson, J. J. The concept of the stimulus in psychology. *American Psychologist*, 1960, 15, 694-703.
- Green, D. M. and Swets, J. A. *Signal detection theory and psychophysics*. New York: John Wiley & Sons, Inc., 1966.
- Guttman, N. The pigeon and the spectrum and other perplexities. *Psychological Reports*, 1956, 2, 449-460.
- Heinemann, E. G. and Chase, S. Conditional stimulus control. *Journal of Experimental Psychology*, 1970, 84, 187-197.

- Heinemann, E. G., Avin, E., Sullivan, M. A., and Chase, S. Analysis of stimulus generalization with a psychophysical method. *Journal of Experimental Psychology*, 1969, 80, 215-224.
- Johnson, D. F. Determiners of selective stimulus control in the pigeon. *Journal of Comparative and Physiological Psychology*, 1970, 70, 298-307.
- Kamin, L. J. Selective association and conditioning. In N. J. Mackintosh and W. K. Honig (Eds.), *Fundamental issues in associative learning*. Halifax: Dalhousie Univ. Press, 1969. Pp. 42-64.
- Luce, R. D. and Tukey, J. Simultaneous conjoint measurement: a new type of fundamental measurement. *Journal of Mathematical Psychology*, 1964, 1, 1-27.
- Miller, L. and Ackley, R. Summation of responding maintained by fixed-interval schedules. *Journal of the Experimental Analysis of Behavior*, 1970, 13, 199-203.
- Reberg, D. and Black, A. H. Compound testing of individually conditioned stimuli as an index of excitatory and inhibitory properties. *Psychonomic Science*, 1969, 17, 30-31.
- Shepard, R. N. Approximation to uniform gradients of generalization by monotone transformations of scale. In D. I. Mostofsky (Ed.), *Stimulus generalization*. Stanford: Stanford University Press, 1965. Pp. 94-110.
- Sutherland, N. S. and Mackintosh, N. J. *Mechanisms of animal discrimination learning*. New York: Academic Press, 1971.
- Swets, J. A. and Kristofferson, A. B. Attention. *Annual Review of Psychology*, 1970, 21, 339-366.
- Torgerson, W. S. *Theory and methods of scaling*. New York: John Wiley & Sons, Inc., 1958.
- Weiss, S. J. Summation of response strengths instrumentally conditioned to stimuli along different sensory modalities. *Journal of Experimental Psychology*, 1964, 68, 151-155.
- Weiss, S. J. Free-operant compounding of variable-interval and low-rate discriminative stimuli. *Journal of the Experimental Analysis of Behavior*, 1967, 10, 535-540.
- Wolf, M. M. Some effects of combined S^ps. *Journal of the Experimental Analysis of Behavior*, 1963, 6, 343-347.
- Wright, A. A. *Psychometric and psychophysical hue discrimination functions for the pigeon*. Doctoral dissertation, Columbia University, 1971 (Unpublished.)

Received 4 October 1971.
(Final acceptance 18 July 1972.)