UNDERMATCHING: A REAPPRAISAL OF PERFORMANCE ON CONCURRENT VARIABLE-INTERVAL SCHEDULES OF REINFORCEMENT¹

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The extant data for pigeons' performance on concurrent variable-interval schedules were examined in detail. Least-squares lines relating relative pecks and time to the corresponding relative reinforcements were obtained for four studies. The between-study group slopes for time and pecks and five of seven within-study group slopes from individual studies were less than 1.00. This suggested the generality that pigeons respond less to the richer reinforcement schedule than predicted by matching. For pecks, a nonparametric test for distribution of points also supported this concept of *undermatching* (to the richer reinforcement schedule). In addition, using mean squared error as the criterion, a cubic curve fit the peck proportion data better than any line or other polynomial. This indicates that the relation between peck and reinforcement proportions may be nonlinear.

Key words: undermatching concurrent variable-interval schedules, matching, pigeons

The two-alternative concurrent schedule is commonly viewed as a procedure in which relative rates of two responses are sensitive to the relative reinforcement rates (*e.g.*, Catania, 1966; Herrnstein, 1970). By using least-squares regression analysis of extant data for pigeons' performance on concurrent variable-interval variable-interval (*conc* VI VI) schedules, this paper reviews the empirical findings and compares the obtained regression functions with several models for responding on concurrent schedules.

I. PROCEDURAL DETAILS

Two major stimulus arrangements have been used to present concurrent choice to a pigeon. Findley (1958) used the changeover-key procedure, in which pecks on one key (the changeover key) changed the color of the other key (the main key). Pecks to each of the two main-key colors were reinforced on independent schedules. Herrnstein (1961) used the

two-key procedure, in which each of two spatially separated responses (key pecks) produced food on independent VI schedules.

A procedure common to most concurrent studies involves the changeover delay (COD). The COD specifies the minimum interval (usually 1 to 3 sec) between the onset of responding to a schedule (defined as the onset of a main-key color in the changeover-key procedure, and the first peck to a key after pecking on the other key in the two-key procedure), and reinforcer presentation for a subsequent peck on that schedule. The COD is in effect any time a pigeon changes to a schedule.

II. THE MATCHING LAW

Herrnstein (1961) suggested that a linear function related peck proportions and reinforcement proportions in concurrent schedules. In his experiment, when independent variable-interval (VI) schedules arranged reinforcer availability on the two keys, the relative rate of responding tended to be the same as the relative rate of reinforcement. This *matching* relation is stated:

$$\frac{p_1}{p_1 + p_2} = \frac{R_1}{R_1 + R_2} \quad (2.1),$$

where *p* and *R* represent numbers of pecks and reinforcers to alternatives 1 and 2. The left side of the equality is called the response proportion (*y*), or the relative rate of respond-

¹Preparation of this paper was supported in part by NIMH grant MH-20752 to the University of California, San Diego, Edmund Fantino, principal investigator. The paper is based on Part I of a dissertation submitted by the senior author to the University of California, San Diego, in partial fulfillment of the requirements for the PhD degree. The authors thank Edmund Fantino, Robert Griffin, and Steven Hursh for their assistance with earlier versions of this paper. Reprints may be obtained from David Myers, Education Department, Mercer University, Macon, Georgia 31207.

ing, and the right side is the obtained reinforcement proportion (x).

Peck proportions as a function of reinforcement proportions for two of Herrnstein's (1961) pigeons are shown in Figure 1. Herrnstein's third pigeon was exposed to only two pairs of reinforcement schedules; those data were not considered here, since the regression of two points is trivial. The matching diagonal showing the relation of equation (2.1) is the dashed line in Figure 1.

Regression lines relating peck proportions to reinforcer proportions for Herrnstein's data were calculated to determine the support for the matching relation. The regression model used was:

$$y = ax + b \quad (2.2),$$

where y and x were defined for equation (2.1). Matching would be supported if the slope (a) and intercept (b) were respectively close to 1.0 and 0.0. Data values for the regression were

Fig. 1. Data, regression lines, and equations for peck proportions as a function of reinforcer proportions for Herrnstein (1961). Data for a third pigeon, exposed only to two schedule pairs, are not shown.

Table 1

Least-squares regression lines relating relative pecks and relative time to relative reinforcers for extant studies of concurrent VI VI with a COD in effect, where each pigeon was exposed to at least three pairs of concurrent schedules. Column (i) indicates studies, columns (ii) and (iii) present procedural data, and column (iv) gives subjects. Column (v) gives equations for the linear regression of peck proportions on reinforcer proportions. Column (vi) gives the regression equations for logarithms of peck ratios on logarithms of reinforcer ratios. Column (vii) regresses time proportions on reinforcer proportions. Column (viii) regresses logarithms of time ratios on logarithms of reinforcer ratios. Sample sizes for the equations are given in parentheses.

(i) Author	(ii) Procedure (COD, in sec)	(iii) Range of Schedules (in min)	(iv) Subjects	a. Least-squares lines— pecks on reinforcers		b. Least-squares lines— time on reinforcers	
				(v) Proportion Coordinates (N)	(vi) Log ratio Coordinates (N)	(vii) Proportion Coordinates (N)	(viii) Log ratio Coordinates (N)
Herrnstein (1961)	two-key (1.5)	VI 1.5- VI 9; extinction	055 231 both	$0.89x+0.07$ (6) $1.12x-0.07$ (6) $1.01x-0.01$ (12)	$0.80x+0.01$ (5) $1.38x-0.02$ (5) $1.11x-0.02$ (10)	Data not published	
Catania (1963)	CO-key (3)	VI 1.5- VI 12; extinction	117 243 294 all 3	$0.94x+0.04$ (10) $0.90x+0.08$ (10) $0.93x+0.04$ (10) $0.94x+0.04$ (30)	$0.79x+0.03$ (8) $0.72x-0.02$ (8) $0.74x-0.01$ (8) $0.75x$ (24)	$0.95x+0.05$ (10) $0.92x+0.03$ (10) $0.95x+0.06$ (10) $0.95x+0.04$ (30)	$1.03x+0.05$ (8) $0.76x-0.05$ (8) $0.87x+0.06$ (8) $0.89x+0.02$ (24)
Silberberg and Fantino (1970)	two-key (0.88-3.5)	VI 1.68- VI 13.5	all 6	$0.87x+0.07$ (18)	$0.85x+0.07$ (18)	$1.00x-0.03$ (18)	$1.05x-0.07$ (18)
Trevett <i>et al.</i> (1972)	two-key (3)	VI 1- VI 4	all 4	$0.75x+0.08$ (16)	$0.69x-0.10$ (16)	$0.87x+0.04$ (16)	$0.94x-0.08$ (16)
Group	both	VI 1 VI 13.5; extinction	all above	$0.90x+0.05$ (76)	$0.84x-0.02$ (68)	$0.95x+0.02$ (64)	$0.94x-0.08$ (58)

based on our estimates of the points in the original figure. The lines and their equations are shown in Figure 1. Although the individual functions varied from matching, the group line based on all 12 data values was $y = 1.01x - 0.01$, which suggested that $y = x$ was an accurate description of the performance. Thus, Herrnstein was correct in noting that there seemed "to be no regular pattern to the deviations from the theoretical function". But more data on concurrent VI performance have appeared since 1961, and it was possible to consider a larger data sample and carry out a more detailed analysis of the matching hypothesis.

III. RESPONSE MATCHING

To test for matching in the extant data where a COD was employed, regression lines were calculated for data where each pigeon was exposed to at least three different pairs of *conc VI VI* schedules. Table 1 compiles procedures (column ii), parameters (column iii), least-squares lines for peck proportions as a function of reinforcer proportions (column v), and least-squares lines for logarithms of peck ratios regressed linearly on logarithms of reinforcer ratios (column vi). The natural base was used for the logarithms. Equation (2.2) was used for column (v) calculations, and the following equation was used for column (vi) calculations.

$$\log (p_1/p_2) = a \log (R_1/R_2) + b \quad (3.1).$$

Included in the analysis were data from Catania (1963), and Herrnstein (1961), which we estimated from plots in the original articles, as well as published data by Silberberg and Fantino (1970), and Trevett, Davison, and Williams (1972).

For peck proportions regressed on reinforcer proportions, the group equation based on 76 data points from the above four studies was:

$$y = 0.90x + 0.05 \quad (3.2).$$

The group equation for the logarithms of ratios method of scaling was:

$$y = 0.84x - 0.02 \quad (3.3).$$

Whether the data were scaled as proportions or logarithms of ratios, three of the four within-study group slopes were less than 1.00. Herrnstein (1961) provided the exception in

Table 2

Least-squares regression lines of peck proportions and time proportions to reinforcer proportions for Catania (1963) and Findley (1958), with no COD in effect. Both studies used the changeover-key procedure to arrange reinforcement on *conc VI VI*.

(i) Author	(ii) Subject (N)	Least-Squares Lines (Proportions)	
		(iii) Responses	(iv) Time
Findley (1958)	2 (5)	0.96x+0.05	0.78x+0.11
	5 (5)	1.20x-0.03	0.92x+0.08
	6 (5)	0.79x+0.11	0.61x+0.18
	Group (15)	0.97x+0.05	0.77x+0.12
Catania (1963)	117 (5)	0.76x+0.07	
	243 (5)	0.87x+0.06	not
	294 (5)	1.02x+0.06	published
	Group (15)	0.88x+0.07	

each case. Thus, the between-study group slopes for both methods of scaling, as well as majority of the within-study group slopes, were less than 1.00. This response tendency has been called *undermatching* to the richer reinforcement schedule (Baum, 1974; Fantino, Squires, Delbruck, and Peterson, 1972; Myers, 1972).

Figure 2 presents the 76 data values for peck proportions as a function of reinforcer proportions that generated equation (3.2), and includes the line and equation for (3.2).

A nonparametric test for distribution of peck data points also was conducted to test the validity of the matching relation. In matching, experimentally obtained data should cluster about the matching diagonal ($y = x$), with roughly as many points above the diagonal as below it. That is, deviations of the data from matching in the direction of undermatching (data above the matching diagonal when reinforcer proportions were less than 0.50, and below the diagonal when greater than 0.50) should be as likely as deviations in the direction of overmatching (below the diagonal when reinforcer proportions were less than 0.50, and above the diagonal when proportions were larger than 0.50). Presuming matching, the expected proportion of data points favorable to undermatching is 0.50. The totals of the data points lying in the undermatching and overmatching regions are shown in the hexagons in Figure 2. Points falling on the lines $x = \frac{1}{2}$, $y = \frac{1}{2}$, or in the upper-left or lower-right quadrants were omitted. The obtained proportion of points favorable to un-

Fig. 2. Plot of the 76 data points showing peck proportions as a function of obtained reinforcer proportions from representative concurrent VI studies (Catania, 1963; Herrnstein, 1961; Silberberg and Fantino, 1968; Trevett, Davison, and Williams, 1972) involving a COD and at least three concurrent pairs per pigeon. The regression line [equation (3.3)] is solid line. The best fitting polynomial [equation (4.3)] is the alternating dashed-dotted curve. Matching diagonal is dashed line. Numbers in hexagons are numbers of points within eighths of the unit square, as discussed in text.

dermatching was 40/56 or 0.71. The probability of obtaining this binary distribution by chance is less than 0.005, using a two-tailed test. A point-count analysis of the data scaled as logarithms of ratios yields the same results as the data scaled as proportions, since an undermatching value in proportion coordinates is an undermatching value in logarithm-of-ratio coordinates.

This nonparametric test substantiates the between- and within-study regression analyses for peck proportions. Both statistical measures

indicate that, for pigeons, relative pecks (scaled as proportions or logarithms of ratios) undermatch relative obtained reinforcers on *conc* VI VI.

To digress a bit, it has been contended by Herrnstein (1961) that a changover delay will result in performance that comes closer to matching of relative responses to relative reinforcers, than when a COD is not in effect. Catania (1963) and Findley (1958) provide parametric data for the changeover-key procedure, when a COD was not in effect. Table 2

presents regression lines of peck proportions on reinforcer proportions for these data. Note that the within-study group slopes were close to 1.00, and that the slopes for the data without a COD approximate the matching slope of 1.00 as well as the data with a COD [Table 1, column (v)].

It should be stressed that the characterization of undermatching on concurrent VIs may be applicable only to the range of COD and schedule values studied. For instance, Fantino *et al.* (1972) found that when the COD was large relative to reinforcer rates, overmatching to the richer reinforcement schedules, and even maximizing, were obtained. However, on the basis of the extant data, no deviations from matching in the direction of undermatching appear to be attributable to either the two different concurrent procedures, or the several different COD or schedule values used.

In review, Herrnstein's (1961) group function provided the initial impetus for the matching hypothesis on concurrent VIs. However, the point count and linear regressions of extant concurrent VI data have shown that pigeons respond less to the richer reinforcement schedule than predicted by matching; this has been termed undermatching. More specific parametric cases of undermatching are considered below.

IV. HIGHER-ORDER MODELS

Figure 2 suggested a nonlinear trend in the peck-proportion data, and it seemed appropriate to consider more general polynomial models:

$$y = B_0 + B_1x + B_2x^2 + \dots + B_kx^k \quad (4.1).$$

For convenience in calculations in this section, the data were translated from the unit square (all points in the plane whose coordinates are between 0 and 1) to the square of the same area centered at the origin. The new coordinates were designated $(u, v) = (x - \frac{1}{2}, y - \frac{1}{2})$. For purposes of the regression analysis, there was no loss of generality in this, as any polynomial in x may be written as a polynomial of the same degree in $(x - \frac{1}{2})$, and *vice versa*.

The first analysis utilized ninth degree stepwise polynomial regression on the 76 peck-proportion data points in the new $(x - \frac{1}{2}, y - \frac{1}{2})$ coordinate system. That is, the program had access to the dependent variable $(y - \frac{1}{2})$ and the nine "dummy" variables $x_1 = (x - \frac{1}{2})$,

$x_2 = (x - \frac{1}{2})^2, \dots, x_9 = (x - \frac{1}{2})^9$. It introduced the independent variables one at a time, not necessarily in the natural order, but in order of importance. Importance was defined as follows. The most important variable x_{k_1} was the x_k for which the mean squared error (m.s.e.) associated with the least-squares function $B_0 + B_{k_1}x_{k_1}$ was smallest. The second most important variable was the one that provided the greatest drop in the m.s.e.; that is, among the least-squares functions $B_0 + B_{k_1}x_{k_1} + B_{k_2}x_{k_2}$, $k_2 \neq k_1$, the second most important variable was that x_k producing the smallest associated m.s.e. in the second-order equation, and so on. This analysis was performed on the same 76 values that contributed to equation (3.2).

In the analysis, the linear term $(x - \frac{1}{2})$ was most important, with least squares-equation

$$(y - \frac{1}{2}) = 0.90 (x - \frac{1}{2}) \quad (4.2),$$

and m.s.e. 0.0041. This undermatching equation is equivalent to the group line (3.2).

The second most important variable was the cubic term $(x - \frac{1}{2})^3$, and the least-squares equation at this point was

$$(y - \frac{1}{2}) = 1.26 (x - \frac{1}{2})^3 + 0.70 (x - \frac{1}{2}) - 0.01 \quad (4.3),$$

with m.s.e. 0.0035. This is a 15% drop in m.s.e., under the first order approximation, and is compelling evidence that *the relation between x and y is nonlinear*. If the regression assumptions held (*i.e.*, if for all values of x the error random variable were normally distributed with mean 0 and constant variance σ^2), it would be possible to reject the hypothesis that the dependence of y on x (for pecks) is linear at the 1% significance level.

None of the remaining variables could produce a reduction in m.s.e. That is, once the first and third terms were removed, none of the remaining variables had any predictive power. Put another way $(x - \frac{1}{2})$ and $(x - \frac{1}{2})^3$ contain all the information in the sample, and the deviations of the data about equation (4.3) are due to experimental error and/or individual difference.

Incidentally, the benefit of the translation [subtraction of $(\frac{1}{2}, \frac{1}{2})$] is apparent. Operating in the original (x, y) coordinate system necessitates the inclusion of all the first three powers of x (x, x^2, x^3) in order to get an m.s.e. as low as that associated with (4.3).

Table 3

Stepwise curvilinear regression of peck proportions on reinforcer proportions for individual studies and group data. Except for one of Catania's pigeons (*), passage beyond the second step did not yield any reduction in mean squared error. Data were transformed as $v = y - \frac{1}{2}$; $u = x - \frac{1}{2}$. Mean squared error (m.s.e.) for each step is shown in column (v).

(i) <i>Author</i>	(ii) <i>Subject(s)</i>	(iii) <i>n</i>	(iv) <i>Least-Squares Functions</i>	(v) <i>step 1 m.s.e. step 2</i>
Herrnstein (1961)	055	6	$v = 0.89u + 0.01$	(0.00125)
			$0.71u^3 + 0.77u + 0.01$	(0.00100)
	231	6	$1.12u - 0.02$	(0.00150)
			$-1.91u^4 + 1.20u + 0.02$	(0)
	both	12	$1.01u$ no improvement	(0.00280)
Catania (1963)	117	10	$0.94u + 0.01$	(0.00288)
			$0.38u^3 + 0.87u - 0.01$	(0.00228)
	243*	10	$0.90u + 0.02$	(0.00225)
			$1.19u^3 + 0.67u + 0.01$	(0.00172)
	294	10	$0.93u + 0.01$	(0.00313)
			$0.40u^2 + 0.86u - 0.02$	(0.00228)
	all 3	30	$0.94u + 0.01$	(0.00289)
			$0.39u^2 + .87u - 0.02$	(0.00174)
	all 6	18	$0.87u + 0.01$ no improvement	(0.00338)
Silberberg and Fantino (1970)				
Trevett, Davison, and Williams (1972)	all 4	16	$7.27u^3 - 0.01$	(0.00414)
			$4.43u^3 + 0.31u - 0.03$	(0.00400)
Group	all of above	76	$0.90u$	(0.0041)
			$1.26u^3 + 0.71u - 0.01$	(0.0035)

Figure 2 includes the curve of equation (4.3), the best fitting polynomial, along with the 76 data points, and the regression line (3.2).

The stepwise regression analysis was performed on data for individual pigeons and studies, and the results are presented in Table 3. In all cases but one [Trevett *et al.* (1972) for which the cubic term entered first], the linear term was the most important. Furthermore, except for one pigeon of Catania (indicated by *), no significant improvement in mean squared error was obtained by passing beyond the second step.

Table 3 shows that for individual studies and pigeons, the second stage always yielded a function of the form

$$(y - \frac{1}{2}) = a(x - \frac{1}{2})^k + b(x - \frac{1}{2}) + c \tag{4.4}.$$

The frequencies of the different values of k that arose are:

- k = 4 1 case
- 3 4 cases (including group function)
- 2 3 cases (all Catania)
- 1 2 cases.

Thus, the individual stepwise regression analysis provided support for the group analysis, in that the linear term was the most important, and a cubic and a linear term were sufficient, in a plurality of cases, for a polynomial description of the data.

In only two cases [the group data of Herrnstein (1961), and Silberberg and Fantino, 1970] did the best relation appear linear. For Silberberg and Fantino, this might be attributed to the absence of extinction points. The effect of Herrnstein's overmatching pigeon in cancelling the more typical undermatching subject may explain the lack of higher-order effects in his group data.

Thus, a curvilinear function is a better description than any line for extant peck proportion data on *conc VI VI*. This curvilinear function resembles empirical and theoretical functions obtained in other studies. For instance, Lander and Irwin (1968) and Reynolds (1963) used a curvilinear model to describe responding on multiple VI VI schedules. Herrnstein (1970) proposed a function visually resembling the cubic as the general form of

the relation between peck and reinforcer proportions in the two-alternative situation (multiple or concurrent schedules), although he did predict that matching would be the particular case for concurrent schedules.

V. TIME MATCHING

In the same study discussed above (peck data shown in Figure 2), Catania (1963), using the changeover-key procedure, measured the relative proportion of time (t) an animal spent in the presence of each of two food-reinforced stimuli. If matching holds for time spent in a stimulus, then equation (2.1) applies here, where time (t) is substituted for (p) in that equation. Regression equations for time proportions as a function of reinforcer proportions using data estimated from Catania (1966) are presented in column (vii) of Table 1. All slopes are less than 1.00.

Two other previous experiments had sufficient time and reinforcement data for a regression analysis. The group choice function of Silberberg and Fantino (1970) was $t = 1.00x - 0.03$. For Trevett *et al.* (1972), the group regression line was $t = 0.87x + 0.04$. These regression lines are presented in Table 1.

In summary, where the dependent variable is the time proportion, only one line has a slope as large as 1.00. All other slopes are less than 1.00, and the group regression line is $t = 0.95x + 0.02$. Figure 3 presents a plot of the 64 data points for time involved in the group equation in column (vii) of Table 1 (from Catania, 1966; Silberberg and Fantino, 1970; Trevett *et al.*, 1972). Also included is the group time-regression line, which is noted to be another instance of undermatching. Thus, the data indicate the same trend for time as for pecks. Column (viii) of Table 1 gives regression equations for logarithms of time ratios on logarithms of reinforcer ratios; again, undermatching is the rule. Along with Lobb and Davison (1975), it is noted that the time functions (columns vii and viii) have slopes closer to 1.00 (matching) than the corresponding peck functions (columns v and vi of Table 1).

In a somewhat different choice procedure, which is perhaps a better test of time matching, Baum and Rachlin (1969) and Brownstein and Pliskoff (1968) studied the relative time a pigeon spent in the presence of each of two houselight colors, as a function of the relative

rate at which response-independent food was presented in each color on a variable-time or VT schedule.

In Brownstein and Pliskoff (1968), each peck to the one key available altered the color of house illumination. Food was delivered in each of two house colors if the variable-time (VT) schedule associated with that color had arranged a reinforcer delivery, and the COD had elapsed. With the schedules at *conc* VT 1-min VT 3-min, three pigeons were exposed to increasing COD values over several sessions until matching was obtained. Each pigeon required a different COD (2, 5, and 7.5) for matching on the *conc* VT 1-min VT 3-min.

With the COD now held constant (at 2, 5, 7.5 sec), relative reinforcement rates of 0.50 and 1.00 were presented to each pigeon, and a fourth rate of 0.67 was presented to one of the pigeons. The slopes for all three subjects were close to 1.00. However, basing the generality of time matching on this study is questionable for three reasons: (1) each pigeon required a different COD for matching; (2) the number of reinforcement pairs was small; and (3) relative reinforcement rates of 1.00 and 0.50 do not constitute a fair test of matching *versus* undermatching. The *a priori* (predictive) utility of time matching is restricted to those parameter values (in this case, COD) at which matching has been empirically demonstrated. Shull and Pliskoff (1967) and Silberberg and Fantino (1970) did not obtain matching at all COD values.

Baum and Rachlin (1969) measured the time spent on each of two sides of a shuttlebox as a function of the relative rate of response-independent variable-time reinforcers obtained on each side, with a COD of 4.5 sec. The authors obtained the best fit of their data to the matching diagonal only when the changeover times were excluded.

The present authors analyzed the data of Baum and Rachlin with and without the changeover times, and with relative measures scaled as proportions and as logarithms of ratios. Table 4 presents the four resultant regression lines for each of the six subjects of Baum and Rachlin. Of the four methods of calculating relative time from the data of Baum and Rachlin (1969), regression analyses of three of these sets of data supported undermatching. Only where the changeover data were excluded and logarithm of ratio calcu-

Fig. 3. Plot of 64 points from representative concurrent VI studies (Catania, 1966; Silberberg and Fantino, 1968; Trevett *et al.*, 1972), showing the relation between time and reinforcer proportions. Group regression line and equation are shown. Matching diagonal is the dashed line. Numbers in hexagons refer to numbers of points within eighths of the unit square, as discussed in text.

Table 4

Regression lines of relative time on relative reinforcers for data of Baum and Rachlin (1969). Data were analyzed both with changeover times excluded (a), as in the original article, and with changeover times included (b). Each of these two types of data were computed as logarithms of ratios (columns iii and v) and as proportions (columns ii and iv). There were 19 data values used to compute each subject's regression line.

(i) Subject	a. CO Time Excluded		b. CO Time Included	
	(ii) Proportions	(iii) Ratios	(iv) Proportions	(v) Ratios
488	1.02x-0.14	1.11x-0.32	0.70x+0.07	0.60x-0.17
489	0.86x-0.17	1.10x-0.61	0.59x+0.08	0.54x-0.25
490	1.19x-0.07	1.35x-0.01	0.90x+0.06	0.79x
496	0.93x-0.08	0.98x-0.28	0.49x+0.18	0.41x-0.14
334	0.83x	0.84x-0.21	0.36x+0.28	0.28x-0.08
360	0.67x+0.10	0.61x-0.12	0.25x+0.34	0.20x-0.05
Group (N = 114)	0.92x-0.06	1.00x-0.26	0.55x+0.17	0.47x-0.12

lations performed was there support for matching. In this case, although the group slope is 1.00, the individual subjects' slopes vary over a wide range, 0.61 to 1.35, rendering the group matching function an inaccurate predictor of individual subject data.

It was possible to apply to time proportions the same analysis of "point count" used on the peck proportions. Looking first at the pooled data from the studies of Catania, Silberberg and Fantino, and Trevett *et al.*, 25/49 or 0.51 of the admissible data values (points interior to the upper-right and lower-left quadrants, excluding the matching diagonal) lie in undermatching regions. Thus, the time-point counts support matching.

For Baum and Rachlin, proportions with changeover times included were more in the direction of undermatching than the other time data, and it seemed best to consider their data separately. Their 114 data values produced the proportion 69/88 or 0.784; the probability of this occurring by chance is less than 10^{-5} .

In summary, the time-proportion regressions from two of three studies (Catania, 1966; Silberberg and Fantino, 1970; Trevett *et al.*, 1972) indicate undermatching. Further, the two studies using time spent as the contingent response provide questionable support for a time-matching proposition: Brownstein and Pliskoff (1968) used a different COD for each subject, and Baum and Rachlin (1969) required a particular method of data selection and analysis to support matching.

VI. SOME ADDITIONAL EMPIRICAL CONSIDERATIONS

Based on the foregoing regression analysis of relative pecks and time spent, the best general representation of *conc* VI VI choice behavior should involve some form of *undermatching*. However, this contention may be limited to a particular procedure or set of parameters. It may be, as implied by some investigators (Herrnstein, 1961; Shull and Pliskoff, 1967), that certain procedures, parameter values, CODs, performance measures or data-analysis procedures, are necessary to generate a matching function. This means that, while matching may be found, it is not as general as implied by Herrnstein (1970) and Rachlin (1971). Matching may be one of a family of

functions, depending on several of the above parameters.

Also, matching of relative performance to relative reinforcements apparently fails to occur in some other types of concurrent schedules. The studies reviewed here used VI schedules to arrange reinforcement. Recent studies examining performance on concurrent variable-interval fixed-interval (FI) schedules (Lobb and Davison, 1975; Myers, 1972; Nevin, 1971; Trevett *et al.*, 1972), and *conc* FI FI schedules (Myers, 1972; White and Davison, 1973) indicate that undermatching, as well as systematic preference for the aperiodic schedule, occurs on these types of schedule pairs.

Rachlin (1971) contended that relative responding matches relative magnitude of reinforcement (duration of magazine presentation to pigeons) in concurrent schedules. Reinforcement presentations are typically arranged by two identical VI schedules. Fantino *et al.* (1972), however, obtained undermatching of responding to relative magnitude. They questioned the generality of other findings that indicated matching of relative responding to magnitude.

In addition to providing a poor description of the data, matching has limited utility as a predictor of choice proportions, since it is stated in terms of the obtained, rather than the scheduled rates of reinforcement. Often the obtained reinforcement rates differ markedly from the scheduled rates (Fantino *et al.*, 1972; Shull and Pliskoff, 1967). The use of obtained, rather than scheduled rates of reinforcement, leaves the experimenter in the undesirable position of having no independent variable, in the sense of an experimenter-controlled variable, from which to predict the dependent variable. Since data are required to obtain the independent variable, the data are also available to construct an individual regression function that is a more accurate description of choice behavior than matching. This virtually eliminates the need for an *a priori* predictor, since the most accurate description of the choice behavior is based on the obtained data.

Stubbs and Pliskoff (1969) introduced a procedure in which the obtained relative rate of reinforcement is controlled, thus ensuring that the obtained reinforcement proportion will be the same as the scheduled rate. The slopes for logarithms of response ratios as a function of

logarithms of reinforcements for each of their three pigeons were 0.89, 1.25, and 0.97. The group slope was 1.04, in support of matching.

In contrast, Schneider (1974), using this forced-choice procedure, obtained undermatching of responding to both relative amount and relative rate of reinforcement at two unequal values of amount and two unequal values of rate of reinforcement.

The preceding discussion has emphasized that concurrent schedules with different procedures, schedule types, and parameter values also tend to yield data that deviate from matching. While some of these data are examined in more detail elsewhere, the general finding is that undermatching is the trend in most of these studies also.

VII. DISCUSSION

Since the preponderance of regression slopes for pecks and time are less than 1.00, and time matching to response-independent reinforcers is limited to specific parameters or data-analysis procedures, the empirical evidence contradicts the matching law of equation (2.1). In evaluating the relation between these data and the matching law, it is helpful to consider Killeen's (1972) distinction between the empirical and the theoretical matching laws. While the empirical matching law [equation (2.1)] was a constructive first approximation in the description of concurrent performance, that statement has become obsolete in light of the undermatching trends indicated in this paper.

Killeen (1972) proposed a theoretical matching law, in which a functional relation could be obtained between relative responses and some concatenation of the reinforcer variables. Killeen's general model is:

$$\frac{t_1}{t_2} = \frac{f_1(r_1)}{f_1(r_2)} \cdot \frac{f_2(a_1)}{f_2(a_2)} \dots \quad (6.1)$$

where $f_1, f_2, \dots$ are operations on the respective independent variables. Killeen's model does not predict a precise match between relative responses and reinforcers. The model does state that a function can be obtained relating the dependent and independent variables. The present thesis requires that a transformation [e.g., equation (3.2) or (4.3)] be performed on the independent variables, and is in accord with Killeen. In terms of Killeen's formu-

lation, undermatching would require that the concatenation operations reduce the relative importance of the richer reinforcement schedule.

There are some indications that the flexibility inherent in a formulation such as Killeen's may be necessary. For instance, functions relating responses to reinforcer rate may differ from functions relating responses to reinforcer magnitude. Schneider (1973) obtained undermatching of responding to both amount and rate of reinforcement, reporting greater undermatching (*i.e.*, greater deviations from matching) to amount than to rate. Schneider described the same trend in Todorov (1973).

Several specific functions related to Killeen's model have been proposed, and are considered below.

Herrnstein (1970) advocated the following model for behavior in multiple (sequential) or concurrent (simultaneous) choice situations:

$$\frac{p_1}{p_1 + p_2} = \frac{1}{1 + \frac{R_2(R_1 + mR_2 + R_0)}{R_1(R_2 + mR_1 + R_0)}} \quad (6.2),$$

where m and R_0 are free parameters. While Herrnstein presented a family of theoretical functions (in his Figure 22) involving different values of m and R_0 , m must be 1.00 (and R_0 can then take any value) for matching in concurrent schedules. With R_0 fixed, other values of m generate curves resembling the inverted 0-give of Figure 2 in this paper. Since the curved regression function [equation (4.3)] shown in Figure 2 accounts for more of the variability in the extant data than the regression line, let alone the matching diagonal, the contention that m is 1.00 for concurrent schedules is directly contradicted by these extant data. In order to use Herrnstein's model to account for concurrent performance, m will take a value other than 1.00.

Rachlin (1971) advanced a multiplicative model in which the ratio of choice responses to the two alternatives will match the corresponding ratio of reinforcement parameters. This model is:

$$\frac{t_1}{t_2} \text{ or } \frac{p_1}{p_2} = \frac{r_1}{r_2} \cdot \frac{a_1}{a_2} \cdot \dots \cdot \frac{x_1}{x_2} \quad (6.3).$$

Baum (1974) has advocated a particular regression model that is related to (6.3). Baum

takes the logarithms of both sides of (6.3) and arrives at the general regression equation.

$$\log(t_1/t_2) = a \log(r_1/r_2) + b + \dots \quad (6.4).$$

As shown in this paper, the use of Baum's model [equation (3.2) or (6.4)] has proven fruitful, in that linear regression functions have been obtained for most parametric data on conc VI VI schedules. However, in using Baum's model we have also shown that the parameter (a) [slope of equations in columns (vi) and (vii) of Table I] consistently assumes values less than the 1.00 demanded by the matching formulation. That is, (a) assumes values consistent with undermatching.

The present analysis of extant data has shown that the empirical matching of equation (2.1) does not hold. Recent discussions have used the matching law as a yardstick in evaluating concurrent performance (Baum, 1974; Herrnstein, 1970; Rachlin, 1971). Regarding undermatching on concurrent VIs as a deviation from some ideal function approximates a hypothetico-deductive approach to concurrent theory.

On the other hand, perhaps "undermatching" is at best a descriptive label for certain response-reinforcer relationships. Research will enumerate the conditions under which matching, or undermatching occur. Laws of concurrent performance would emerge as generalizations of regularities in the data. This inductive approach has typified the experimental analysis of behavior.

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Received 9 August 1972.

(Final Acceptance 23 July 1976.)