PERFORMANCE ON VARIABLE-INTERVAL SCHEDULES ARRANGED SINGLY AND CONCURRENTLY¹

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Extensive parametric data were obtained from pigeons responding on variable-interval schedules arranged on three, two, and one response keys. Number of responses on the keys, the time spent responding on the keys, and the number of reinforcements obtained on the keys were measured. Response rates on each key were an increasing function of the reinforcement rate on that key, and an inverse function of the reinforcement rate on the other keys. In terms of preference, both response and time-allocation ratios undermatched ratios of obtained reinforcements, and the degree of undermatching was consistent both within, and between, two- and three-schedule data. When absolute response-rate data were analyzed according to Herrnstein's (1970) quantitative account, obtained values of assumed constants were not consistent either within or between conditions. However, a power-function modification of Herrnstein's account fitted the data well and provided similar exponent values to those obtained for the undermatching of preference ratios.

Key words: variable-interval schedules, concurrent schedules, response rate, preference, undermatching, pecking, pigeons

Preference on concurrent variable-interval (VI) schedules of reinforcement has been summarized in detail by Baum (1974), Catania (1966), and Herrnstein (1970). In brief, research carried out before this decade was taken to support the suggestion (Catania, 1963; Herrnstein, 1961) that the ratio of the number of responses on two keys equalled the ratio of the number of reinforcements obtained (Shull and Pliskoff, 1967) on the two keys. Thus, if P_1 and P_2 are the numbers of responses emitted, and R_1 and R_2 are the numbers of reinforcements obtained on keys 1 and 2, then

$$\frac{P_1}{P_2} = \frac{R_1}{R_2} \quad (1)$$

The same equality was thought to hold between the ratio of time spent responding on the two keys, T_1/T_2 (Catania, 1966). However, Lobb and Davison (1975) showed that the response matching law, Equation 1, may not be generally supported, in that replotting existing data generally showed undermatching.

Animals generally responded less to the higher reinforcement-rate schedule, and more to the lower reinforcement-rate schedule, than demanded by Equation 1. The situation for time allocation is much less clear, but these data are normally closer to strict matching than response data (Baum, 1975; Hollard and Davison, 1971; Lobb and Davison, 1975). The relations can be written:

$$\frac{P_1}{P_2} = c \left(\frac{R_1}{R_2} \right)^a \quad (2a)$$

and

$$\frac{T_1}{T_2} = c \left(\frac{R_1}{R_2} \right)^b \quad (2b)$$

The parameter c describes the bias between responses (Baum, 1974), and a and b describes the sensitivity (Lander and Irwin, 1968) with which performance changes with changes in reinforcement rate. A value of a less than 1.0 is called undermatching (Baum, 1974). The only major theory of schedule performance (Herrnstein, 1970) accepts strict matching (Equation 1), and not undermatching.

Catania (1966) suggested that the matching law (Equation 1) could be extended to choices between more than two manipulanda, and supported this argument with data reported by Reynolds (1963) obtained from three-key

¹The authors thank the cooperative of students who helped run this experiment, and also thank the University Grants Committee, which supported this research through grants to the first author. Reprints and tables of individual data are available from the authors, Department of Psychology, University of Auckland, Private Bag, Auckland, New Zealand.

concurrent chain performance. The extension implies that the addition of a third schedule and key would not affect preference (measured by ratios of responses) between two existing schedules (*c.f.* Luce, 1959). However, data have recently been reported which found that the addition of a third chain to a pair of concurrent chain schedules did change preference between the pair (Davison and Temple, 1974). In view of this finding, and in view of the recent support for Equation 2a (undermatching) as the best descriptor of preference, Catania's suggestion needs to be carefully checked.

The present paper is also addressed to theories that attempt to account for the absolute level of responding (number of responses divided by total time) on single schedules alone and on single schedules arranged concurrently with other schedules. The major account has again been given by Herrnstein (1970, 1974). It suggests that the rate of a response is a function of the rate of reinforcement for that response in the context of the rate of reinforcement for all possible responses. The formula is:

$$P_1 = \frac{kR_1}{R_1 + R_2 + R_0} \quad (3)$$

for one schedule comprising a two-key concurrent schedule. The constant k is the maximum response rate measured in pecks-equivalent, and R_0 is the rate of reinforcement, measured in equivalent reinforcements, for all behavior other than the defined operants. Equation 3 may be applied to single-schedule performance by putting $R_2 = 0$, or extended to performance on more than two schedules by adding terms R_3 , *etc.*, to the denominator. Clearly, this equation predicts strict matching (Equation 1) in preference. The equation is also dependent on strict matching because, in the single-schedule case, $k = P_1 + P_0$ where P_0 is the rate, in pecks-equivalent, of emitting operants other than those defined. Substituting for k in Equation 3, and simplifying, gives:

$$\frac{P_1}{P_0} = \frac{R_1}{R_0}$$

which is equivalent to Equation 1.

This experiment investigated whether preference between three concurrent schedules could be described in the same way as preference between two concurrent schedules, and

whether Herrnstein's (1970) quantitative description of absolute response rates accounted for behavior on single schedules arranged alone and concurrently with one or two further schedules.

METHOD

Subjects

Six homing pigeons numbered 141 to 146 were maintained at $80\% \pm 15$ g of their free-feeding body weights. Part way through the experiment, 143 died and was replaced by 143b. These birds had extensive training on concurrent chain schedules (Davison and Temple, 1974), except for 143b, which had training on simple concurrent schedules.

Apparatus

The experimental chamber, which was situated remote from solid-state control equipment, contained three response keys 2 cm in diameter, 9.5 cm apart, and 22.5 cm from the grid floor. Each key was operated by a peck exceeding approximately 0.1 N. The chamber was sound-attenuated, and some masking noise was provided by an exhaust fan. Two sources of feedback were arranged for pecks on illuminated keys: a 30-msec offset of the keylight and the click of a relay situated behind the keys. Pecks on keys that were not illuminated had no consequences. A grain hopper was situated below the center key and 10 cm from the grid floor. During reinforcement, the hopper was illuminated and the keys darkened. No other source of illumination was provided.

Procedure

The reinforcer consisted of a nominal 3-sec access to wheat. Sessions were terminated in blackout after a fixed number of reinforcements had been obtained. This number (Table 1) was 50 or less, and gave a session time of not more than 50 min.

Since all pigeons had previous schedule training, no response shaping was required and the animals were placed directly in the first experimental condition. During the session, the three response keys were illuminated white, and each was associated with various independently arranged VI schedules (or extinction). The VI schedules were all randomized from the arithmetic progression a , $a + d$, $a + 2d$, *etc.*, and contained 12 intervals. The

values of a used were: VI 15-sec, $a = 1.25$ sec; VI 30-sec, $a = 2.5$ sec; VI 60-sec, $a = 5$ sec; VI 120-sec, $a = 10$ sec; VI 180-sec, $a = 15$ sec; and VI 480-sec, $a = 40$ sec.

Performance was assumed stable when each animal had met a defined criterion five, not necessarily consecutive, times. The criterion was that the median of the relative numbers of responses for each of the three possible two-key comparisons (pecks on key 1 divided by pecks on keys 1 and 2; pecks on key 1 divided by pecks on keys 1 and 3; and pecks on key 2 divided by pecks on keys 2 and 3) in the last five sessions did not differ by more than 0.05 from the medians of the five sessions before these. When only one VI schedule was arranged, the criterion was not applied to the pair of keys under extinction. When the criterion had been met by all animals five times, the experimental conditions were changed for all animals as a group.

The design of the experiment included conditions in which the schedules on keys 2 and 3 were kept at VI 120-sec and VI 60-sec respectively, and the schedule on key 1 was varied from extinction to VI 15-sec, and conditions in which keys 1 and 2 were VI 60-sec and VI 120-sec respectively and the schedule on key 3 was varied. It also contained conditions in which keys 2 and 3 were VI 120-sec and extinction respectively, and the schedule on key 1 was varied, and conditions in which keys 1 and 2 were extinction and VI 120-sec respectively and the schedule on key 3 was varied. Finally, there was a set of conditions in which both keys 2 and 3 were under extinction and the schedule on key 1 was varied. The sequence of conditions, which was largely determined by the obtained data, is shown in Table 1.

After the daily experimental sessions, the birds were fed sufficient mixed grain to maintain their body weights. In all conditions, the number of pecks on the three keys, the time spent pecking the keys, taken from the first response on a key to the first on another key, and the number of obtained reinforcers on each key, were measured.

A changeover delay (Herrnstein, 1961) between each key arranged that a reinforcer could not be obtained, even if set up by the VI timers, until 1.5 sec had elapsed since the first response on that key after responding on another key.

RESULTS

The number of responses on each key, time spent responding on each key in seconds, and number of reinforcements obtained on each key are shown in Table 1 for each condition of the experiment. To conserve space, these data are the totals over the last five sessions of each condition averaged over the six birds. In these averages, no account was taken of the change from Bird 143 to 143b part way through the experiment. Tables of individual data can be obtained from the authors.

There are five sets of conditions from the present experiment. Two of these are when performance on all three keys was reinforced, and these are divided into conditions in which the varied schedule was on key 1 and those in which it was on key 3. There are two sets of conditions in which responses on only two keys were reinforced, and these are divided into the varied schedule on key 1 with extinction on key 3, and the varied schedule on key 3 with extinction on key 1. Finally, there is a set of conditions in which only responses on key 1 were reinforced, and this schedule was varied.

The absolute overall response rates (responses on a key divided by total session time) for each key for the five sets of conditions are shown in Figure 1. The abscissa of this figure is the number of reinforcements per minute obtained on the key with the varied VI schedule. Often, the obtained rates of reinforcement were higher than the nominal value that the VI schedule would apparently allow, due to the reinforcement time taking up 3 sec of each scheduled interval; the nominal VI 15-sec was thus effectively VI 12-sec, and so on for the other schedules. In Figure 1, the top graph shows an increasing function, which is the response rate on key 1 as the schedule on this key was changed progressively from extinction to VI 15-sec. The lower decreasing graph shows performance on key 2 (VI 120-sec), and the upper decreasing graph performance on key 3 (VI 60-sec), both as the schedule on key 1 was changed from extinction to VI 15-sec. Figure 1 does not show performance on keys on which extinction was arranged throughout a set of conditions.

The five graphs in Figure 1 show similar trends. Response rate is a negatively accelerating function of reinforcement rate for that response, and a negatively decelerating func-

Table 1

Sequence of experimental conditions, number of sessions training, number of responses and time spent responding and number of reinforcements obtained. All time data are in seconds. The data are the sums over the final five sessions of each condition averaged over the six birds. E denotes extinction.

VI Schedules (sec)				Number of Responses				Time Spent Responding				Reinforcement Obtained			
Key 1	Key 2	Key 3	Sessions	Key 1	Key 2	Key 3		Key 1	Key 2	Key 3		Key 1	Key 2	Key 3	
60	60	60	29	4622	4887	3822		2091	2191	1564		104.7	104.5	90.8	
60	120	60	23	6113	3560	4704		3009	1551	2424		126.8	59.2	114.0	
120	120	60	26	4923	4391	6600		2539	1707	3827		72.8	70.0	157.2	
E	120	60	28	75	7569	9398		60	3329	6725		-	94.5	205.5	
30	120	60	36	4350	1960	2734		2571	937	1373		159.5	40.8	49.7	
180	120	60	22	3618	6726	7453		1627	3038	4455		52.0	82.5	115.5	
60	120	180	25	6990	5069	3702		3398	2507	1863		136.2	69.0	44.8	
60	120	30	23	2049	1934	4580		976	742	2498		65.2	34.8	150.0	
60	120	120	28	6201	4477	4298		2836	1977	2344		124.7	61.7	63.7	
E	120	180	25	525	11921	8039		319	6664	4796		-	105.3	69.7	
E	120	30	19	123	3657	7039		74	1499	4133		-	49.7	200.3	
E	120	120	16	200	12086	10332		117	5938	6081		-	110.8	111.0	
30	120	E	34	6026	2081	10		3542	1317	9		184.2	40.8	-	
120	120	E	22	9895	11427	101		4600	6357	48		99.2	100.8	-	
180	120	E	17	7423	12706	20		3652	7883	16		68.5	106.5	-	
* 60	120	E	17	8604	5596	11		4546	2759	8		135.2	64.8	-	
480	120	E	23	4874	12999	14		2368	8416	11		23.7	101.3	-	
15	120	E	22	2558	506	0		1876	299	0		160.0	15.0	-	
15	E	E	19	2395	18	1		1881	20	0		150	-	-	
60	E	E	18	8432	179	4		7405	180	6		150	-	-	
180	E	E	18	11273	157	76		11125	175	53		75	-	-	
30	E	E	18	6887	7	0		5975	8	0		225	-	-	
480	E	E	14	11463	180	18		10503	197	26		25	-	-	
E	120	E	15	546	15168	27		526	11290	32		-	115.0	-	
15	120	60	28	2127	406	478		1524	196	286		142.2	11.2	21.6	
60	120	15	33	661	385	2247		312	168	1448		21.2	13.0	140.8	
480	E	E	23	9416	481	595		9380	402	460		25	-	-	

*143b entered the experiment in this condition.

Fig. 1. The overall response rate (number of responses divided by session time) on each key as a function of the overall rate of reinforcement obtained on the key with the varied VI schedule. The schedules arranged on each key are specified in seconds on each graph, and x denotes the varied VI schedule.

tion of the reinforcement rate on alternative keys (Herrnstein, 1970). Figure 1 suggests that the former, which we shall term the direct effects of reinforcement, is a more sharply curved function than the latter, which we shall call the indirect effect of reinforcement (Davison and Temple, 1974). As would be expected, the response rate on the VI 60-sec schedule in the three-schedule conditions is always higher than that on the VI 120-sec schedule.

The data in Table 1 show that response rates on keys on which no reinforcement was arranged were low, and therefore that the changeover delays of 1.5 sec were sufficient to prevent maintenance of responding on one

key by the production of reinforcement on another (Catania and Cutts, 1963; Herrnstein, 1961).

Figure 2 shows the ratio of responses emitted on pairs of keys in the various sets of conditions as a function of the ratio of numbers of reinforcements obtained on the pairs of keys. Only two such functions are shown for the three-schedule conditions, because it is not possible to plot in this way such ratios for the pair comprising two constant schedules. Straight lines were fitted to the logarithmic data (Baum, 1974; Staddon, 1968) by the method of least squares, and the equations of these lines are shown on each graph. In these

Fig. 2. The logarithm of the ratio of the numbers of responses on two keys as a function of the logarithm of the ratio of the number of reinforcements obtained on the two keys. The schedules arranged on each key are specified in seconds on each graph, and x denotes the varied VI schedule. The equations of the best-fitting straight lines, in logarithmic terms, are shown by each graph.

equations, the slope of the line is the exponent a in Equation 2a, and the constant is the logarithm of the multiplier c in the same equation. Figure 2 shows that the preference between the schedules was described by Equation 2a with a bias c (the antilog of the constant to the fitted line) of close to 1.0 and an exponent a of between 0.63 and 0.73. When the performance of the individual animals (Table 2), excluding Subjects 143 and 143b, is treated in the same way, sign tests (Siegel, 1956) allow the following conclusions with p better than 0.05: the slopes of the functions were significantly less than 1.0 (two contrary out of 30); no slope for any set of conditions, either within or between three- and two-schedule conditions, was significantly different from any other. Across all conditions, the slopes for Bird 141 were significantly higher than those for Birds 142 and 144. In the analysis of individual performance, the correlations between data and the fitted line were between 0.71 and 1.0 (mean 0.94); in the analysis of the group data they were between 0.95 and 1.0 (mean 0.98).

The ratios of time spent responding on the two keys were treated in the same way as the ratios of numbers of responses. For the group data, the slopes of the fitted lines (b in Equation 2b) were between 0.73 and 0.85 and the bias c was again close to 1.0. For the individual animals (Table 2), the slopes varied between 0.57 and 1.11 and sign tests allow the following conclusions at p better than 0.05: the slopes of the fitted lines were less than 1.0 (five contrary out of 30), and no slope for any set of conditions, either within or between three- and two-schedule conditions, was significantly different from any other. Across all conditions, the slopes for Birds 141 and 145 were smaller than those for Bird 144. In the analysis of individual performance, the correlations between data and the fitted line were between 0.78 and 1.0 (mean 0.95); for the group data, correlations were between 0.96 and 0.99 (mean 0.98).

Comparing response and time-allocation measures, a sign test shows that the exponent in Equation 2 required by response measures was less than that required by time measures (five contrary out of 30). However, the bias c required by the two measures was not significantly different (response biases were greater than time biases in 19 out of 30 comparisons).

The ratios of response rates on the keys on which the schedules remained constant when

Table 2

The obtained values of a or b and c in Equation 2, and the correlation (r) between data and fitted line, for all conditions in which VI schedules were arranged on more than one key. The key on which the VI schedule was varied is shown by x , and the underlined schedule specifications show which pair of keys provided the values of a , c , and r . The data for Bird 143 are incomplete for all sets of schedules and are calculated from at least three experimental conditions. Bird 143b did not provide sufficient data for any curve fits.

Bird	<u>x, 120, 60</u>			<u>x, 120, 60</u>			<u>60, 120, x</u>			<u>60, 120, x</u>			<u>E, 120, x</u>			<u>x, 120, E</u>		
	a	c	r	a	c	r	a	c	r	a	c	r	a	c	r	a	c	r
1. RESPONSE DATA																		
141	0.85	0.71	0.98	0.72	0.94	0.96	0.73	0.77	0.97	1.02	1.49	0.97	0.72	1.83	0.97	0.93	0.53	1.00
142	0.79	0.75	0.99	0.65	1.21	0.88	0.60	1.15	0.79	0.72	1.32	0.86	0.66	1.52	0.71	0.71	0.96	0.91
143	0.72	0.89	0.97	0.70	0.91	0.96	0.63	1.35	0.93	0.64	1.08	0.94	0.54	1.09	0.89	0.92	0.89	0.97
144	0.58	1.49	0.97	0.64	1.58	0.98	0.67	1.28	0.96	0.66	1.17	0.92	0.68	1.48	0.94	0.72	1.02	0.98
145	0.62	1.03	0.91	0.71	1.40	0.87	1.04	1.00	0.98	0.89	1.19	0.97	0.85	1.22	1.00	0.65	1.20	0.97
146	0.99	0.68	1.00	0.88	0.61	0.95	0.42	0.60	0.98	0.62	0.68	0.98	0.60	0.88	0.95	0.59	1.00	0.96
2. TIME DATA																		
141	1.01	0.66	0.98	0.88	0.85	0.96	0.72	0.60	0.95	1.11	1.37	0.97	0.83	1.26	1.00	0.87	0.55	0.99
142	0.98	0.87	0.93	1.03	0.79	1.00	0.67	0.86	0.90	0.74	0.90	0.96	0.66	1.03	0.97	0.80	0.93	0.99
143	0.92	0.83	0.99	0.87	0.71	0.94	0.77	1.10	0.96	0.86	1.06	0.97	0.90	0.97	0.95	0.86	0.57	0.97
144	0.62	1.96	0.85	0.57	1.76	0.94	0.71	1.38	0.95	0.77	0.94	0.97	0.78	1.22	0.95	0.87	1.09	0.99
145	0.67	1.16	0.78	0.83	1.13	0.83	0.93	0.66	0.89	0.87	0.82	0.93	1.00	1.03	0.99	0.91	0.85	0.99
146	1.10	0.73	1.00	1.11	0.70	0.96	0.67	0.66	1.00	0.82	0.76	0.99	0.66	0.73	0.97	0.82	0.80	0.98

Fig. 3. The logarithm of the numbers of responses emitted on the constant schedule keys when the schedule on the third key was varied. A: the response ratio on keys 2 and 3 when VI 120-sec and VI 60-sec were arranged on these keys and the schedule on key 1 (abscissa) was varied. B: the response ratio on keys 1 and 2 when VI 60-sec and VI 120-sec were arranged on these keys, and the schedule on key 3 (abscissa) was varied.

three concurrent VI schedules were arranged can be shown as a function of the reinforcement rate on the key on which the schedule was varied (Figure 3). It can be seen that these ratios showed no trend as reinforcement rate on the third key was varied.

DISCUSSION

Preference

Catania (1966) suggested that Luce's (1959) principle of indifference from irrelevant alternatives applies to concurrent VI schedule performance. This principle would predict that changing the rate of reinforcement on one key would not change the response ratios between two other, constant reinforcement rate, keys. Figure 3 shows that this prediction is correct for the present data. The principle would also predict that the *function* relating response to reinforcement ratios would be the same whether or not reinforcements were available on a third key, and this effect is shown in Figure 2. The present data, then, give strong support to Luce's principle.

On the other hand, these data do not support the notion of matching (Equation 1) between response and reinforcement ratios (Herrnstein, 1970), nor matching between time allocation and reinforcement ratios (Baum, 1975). The data are characterized by undermatching in both measures, and join the increasing number of reports suggesting that

undermatching in both time and response measures is the norm (Baum, 1974; Lobb and Davison, 1975). These data also strongly support the proposition that time-allocation functions are characterized by larger exponents than response functions (Lobb and Davison, 1975), and hence that local response rates (number of responses divided by time spent responding) do not remain constant when concurrent VI schedules are changed (Catania, 1966). Averaging all the data obtained here, response functions have a slope of about 0.69 and time functions a slope of about 0.81. Thus, the relation between the ratio of *local* response rates and the ratio of *overall* reinforcement rates requires an exponent of about -0.12 .

Absolute Response Rates

We have shown in the introduction that Herrnstein's (1970) predictions for absolute response rates on each key in concurrent schedule performance are not compatible with undermatching between response and reinforcement ratios. Since the present data show strong undermatching, it is of interest to analyze the data according to Herrnstein's account to determine whether undermatching produces a serious deviation from this account. Accordingly, response rates on single keys were analyzed using Cohen's (1973) method, and appropriate values of k and R_0 were obtained. This analysis was carried out for each of the five sets of conditions and for all keys except those on which extinction was arranged throughout a set. Thus, in conditions of *conc* VI x-sec VI 120-sec VI 60-sec, the response rates on all three keys were analyzed separately. The results are shown in Table 3. Values of k using group data range from 70 to 325 responses per minute, and values of R_0 from -0.92 to 1.73 equivalent grain reinforcements per minute. Two comments may be made: first, both the values of k and R_0 should have remained approximately constant; second, it is difficult to conceive how R_0 , the rate of reinforcement for emitting other than key-peck responses, could acquire a negative value.

A closer look at Table 3 shows that the problem with k arises in the indirect cases in which the reinforcement rate on another key was varied. For the key on which the reinforcement rate was varied, k was between 70 and 81 responses per minute, a reasonable value for these birds (Table 1). The problem with

Table 3

Obtained values of k and R_o (Herrnstein, 1970), values of k' and a for the present analysis of absolute response rates (Equation 4b), and values of a obtained from the preference analysis (Equation 2a) for each set of conditions. Varied schedules are denoted by X and extinction conditions by E.

VI Schedules (sec)			Key 1		Key 2		Key 3	
Key 1	Key 2	Key 3	k	R_o	k	R_o	k	R_o
<i>A: Herrnstein, 1970</i>								
X	120	60	81	-0.73	142	-0.62	82	-0.35
60	120	X	126	0.33	147	-0.12	75	-0.92
E	120	X	-	-	325	1.73	78	-0.18
X	120	E	77	-0.25	145	-0.16	-	-
X	E	E	70	0.03	-	-	-	-
<i>B: Present analysis</i>								
			k'	a	k'	a	k'	a
X	120	60	76	0.62	108	0.80	80	0.72
60	120	X	95	0.83	125	0.87	82	0.62
E	120	X	-	-	79	0.49	81	0.74
X	120	E	83	0.68	89	0.72	-	-
*X	E	E	70	0.73	-	-	-	-
<i>C: Preference analysis: values of a</i>								
			Key 1/Key 2		Key 1/Key 3		Key 2/Key 3	
X	120	60	0.71		0.67		-	
60	120	X	-		0.66		0.73	
E	120	X	-		-		0.63	
X	120	E	0.71		-		-	

*The value of R_o in this condition was 0.04 reinforcements per minute.

the indirect cases relates to Figure 1, in which it was shown that the absolute response rates on keys with constant reinforcement rates changed in a different way from the response rates on keys with varied reinforcement rates. Much the same sort of effect can be seen in Herrnstein's (1970) Figure 11. We conclude that Herrnstein's account does not adequately predict response rates for concurrent schedule data that demonstrate undermatching in preference.

What description of absolute response rates would both fit these data and predict undermatching in preference? Mathematically, three absolute response-rate functions could satisfy undermatching. They are:

$$P_1 = \frac{k'R_1^a}{\Sigma R} \quad (4a)$$

$$P_1 = k' \left(\frac{R_1}{\Sigma R} \right)^a \quad (4b)$$

$$P_1 = \frac{k'R_1^a}{\Sigma (R^a)} \quad (4c)$$

Of these, Equation 4a may be dismissed as being dimensionally incorrect. Equation 4b is easily solved, but Equation 4c is difficult to

solve. Equation 4b also has a compelling logical advantage, as it states that the preference between the reinforcement rate on one key and the reinforcements available from all other sources (including R_o) has the same form as the preference between any two schedules taken together (Equation 2a). A further point may be made: if the value of the exponent a in Equation 2a is a descriptor of the sensitivity of behavior to changes in reinforcement, we might expect the same sensitivity value when a bird was choosing between response 1 and all other responses (Equation 4b) as when it was choosing between any two defined responses (Equation 2a). We would also expect the same sensitivity when a bird was choosing between emitting a defined response and emitting other undefined and unmeasured behavior. The only difference in the latter case is that P_1 and P_o are different responses and R_1 and R_o are different reinforcers. There are no experimental data on different responses maintained on concurrent schedules, but different reinforcers only change the bias c between the responses (Hollard and Davison, 1971).

The data on absolute response rates were therefore analyzed according to Equation 4b

by fitting a line between the logarithm of the absolute response rate on a key and the logarithm of the ratio of the reinforcement rate for that response divided by the total obtained reinforcement rate. Consideration of R_0 was omitted from the analysis of three- and two-schedule data in the hope that its value would be small compared with the other obtained reinforcement rates. For ease of analysis, it was also assumed that the rates of reinforcement for keys with fixed schedules were as arranged.

A slightly different procedure was used for the single-schedule data because if R_0 is not considered, Equation 4b predicts a constant response rate. In this analysis, various values of R_0 were tried and that which fitted best to the results for the three- and two-schedule data was accepted.

Table 3 shows the values of k' and a for the absolute response-rate data, and for comparison purposes also the values of a from the analysis of preference. The fits of the lines for the three- and two-schedule data were good, with correlations between 0.93 and 1.0 (mean 0.97), but were not so good for the single-schedule data (correlation 0.73 on six data points). Table 3 shows first that analysis by Equation 4b, even neglecting the contribution of R_0 , gave much more consistent values of the multiplier k' than did the analysis by Herrnstein's (1970) equation. It also produced consistent values of a that are not significantly different from the values of a obtained from the preference analysis ($t = 0.1$, $n = 63$) when individual performance was analyzed. The value of R_0 for the single-schedule data was that obtained for an a value of 0.73, the average a for the three- and two-schedule conditions.

Because of the greater consistency in the parameters obtained from Equation 4b, in contrast to those obtained from Herrnstein's (1970) equation, and because of the consistency between the values of a obtained from absolute response rates and preference, Equation 4b is clearly superior to Equation 3. The former is, of course, only a more general statement of the latter. This analysis suggests therefore that the sensitivity in preference between two defined reinforcement rates may also apply to the preference between a defined reinforcement rate and other, possibly undefined, reinforcement rates; that is, to absolute response rates.

A similar analysis can be carried out for measures of absolute time spent responding, rather than absolute response rate, but the extension of the analysis to single-schedule performance is difficult because, by our measures, the animals spent the total session time responding on the one schedule. Since this analysis is successful for absolute times in two- and three-schedule performance, it supports recent attempts to focus on time allocation as a measure of performance (e.g., Baum, 1975).

In summary, the present experiment has again demonstrated undermatching between response and time-allocation ratios and reinforcement ratios in concurrent VI schedule performance; it has demonstrated that preference functions that apply to two-schedule performance also apply to three-schedule performance, and it has shown that absolute response rates in three-, two-, and one-schedule performance can be described by a modification of Herrnstein's (1970) account of absolute response rates. As Baum (1974) stated, it now remains to elucidate the conditions in which undermatching, rather than matching, occur.

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Received 17 July 1975.

(Final Acceptance 8 January 1976.)