THEORETICAL NOTE

IMPLICATIONS OF THE MATCHING LAW FOR
RATIO RESPONDING

An interesting empirical property of ratio-schedule responding is that responding ceases at some finite upper limit of the ratio of responses to reinforcements. For example, referring to the pigeon's variable-ratio (VR) key pecking, Ferster and Skinner (1957) stated: "As in all schedules requiring a number of responses, the bird will stop responding altogether if the average number goes beyond a certain value" [p. 391]. This fact is implied by Herrnstein's (1970) formulation of the matching law, even though that formulation was developed primarily in the context of interval-schedule responding. Herrnstein's (1970, p. 255) equation for simple schedules is

$$P = \frac{kR}{R + R_0}, \quad (1)$$

where P is responses (e.g., key pecks), R is reinforcement for responding, " R_0 is the unknown, aggregate reinforcement for other alternatives", and k is a constant. R_0 typically is assumed to remain constant across values of R , and for specific values of k and R_0 Equation 1 generates curves that closely fit data produced by varying the reinforcers delivered in variable-interval (VI) schedules.

In interval schedules, R is determined almost entirely by the value of the schedule and P decreases monotonically as R decreases. In ratio schedules, on the other hand, not only is P determined by R , but R is determined by P as follows:

$$R = P/n, \quad (2)$$

where n is the average number of responses required for reinforcement. Substituting as indicated by Equation 2 for R in Equation 1 we obtain:

$$P = \frac{k P/n}{P/n + R_0} = \frac{kP}{P + nR_0}. \quad (3)$$

Equation 3 holds only if P is equal to zero or if

$$P = k - nR_0. \quad (4)$$

Assume that for n greater than a certain value R_0 has a lower bound greater than zero (which is reasonable, because R_0 should either remain constant or increase as n increases). Then, Equation 1 predicts that beyond some finite n , a subject on a ratio schedule will not respond.

A similar analysis can be applied to Herrnstein's (1970, p. 258) equation for concurrent schedules, where concurrent schedules operate simultaneously for two or more different responses. For two concurrent schedules the equation is

$$P_1 = \frac{kR_1}{R_1 + R_2 + R_0}, \quad (5)$$

where P_1 is responses on one schedule, R_1 and R_2 are reinforcers delivered respectively by schedules 1 and 2, and R_0 and k are as defined for Equation 1. If P_1 -responding is reinforced on a ratio schedule and P_2 -responding is reinforced on an interval schedule, then, from reasoning similar to the preceding, either P_1 is equal to zero or

$$P_1 = k - n(R_2 + R_0), \quad (6)$$

where n is the average number of responses required for reinforcement in the ratio schedule. Again, there is a finite n beyond which the subject will not respond on the ratio schedule (assuming that either R_2 or R_0 has a lower bound that is greater than zero for n greater than some number). In addition, P_1 -responding will not be sustained beyond some finite value of R_2 . Both implications are consistent with Herrnstein's (1970, p. 252) finding that subjects respond exclusively on the VI schedules in concurrent VR VI schedules when the relative reinforcement rate in the VI schedule exceeds a certain value.

When both concurrent schedules are ratio schedules requiring different numbers of responses for reinforcement, Equation 5 predicts that the subject will not respond on one of the schedules. From Equation 5

$$P_1 = \frac{kP_1/n_1}{R_1 + R_2 + R_0} \text{ and } P_2 = \frac{kP_2/n_2}{R_1 + R_2 + R_0}, \quad (7) \text{ and } (8)$$

where n_1 and n_2 are the respective number of P_1 -responses and P_2 -responses required for reinforcement. From Equations 7 and 8,

$$\frac{P_1}{P_1 + P_2} = \frac{P_1/n_1}{P_1/n_1 + P_2/n_2}. \quad (9)$$

From Equation 9, except for n_1 equal to n_2 , a subject on concurrent ratio-ratio schedules can comply with Equation 5 only by not responding on one of the schedules. Herrnstein (1958) indicated that subjects do comply with the equation by exclusively preferring one of the schedules.

The present treatment may be completed by considering that some portions of R_0 almost certainly occur according to ratio-schedule requirements (although unspecified by the experimenter). When paired with interval schedules, the ratio portions of R_0 must decrease as the density of reinforcement in the interval schedules increases (cf. Equation 6). When paired with ratio schedules, however, the ratio portions of R_0 typically must be zero when responding occurs on the ratio schedules (cf. Equation 9). The constant k must be the same for ratio and interval schedules, because matching (i.e., relative responses equal to relative reinforcements) occurs in concurrent ratio-interval sched-

ules (e.g., Herrnstein, 1970, p. 252). It therefore follows from Equation 1 that through a wide range of reinforcement densities, simple ratio schedules generate higher response rates (because R_0 is smaller) than do simple interval schedules when equated for reinforcement density. This is supported by Ferster and Skinner (1957, pp. 399-407).

In conclusion, Herrnstein's (1970) formulation makes some interesting predictions concerning ratio responding. Considering that the formulation was developed primarily in the context of interval schedules, the strong empirical support for these predictions adds to its plausibility.

REFERENCES

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