PREFERENCE IN CONCURRENT VARIABLE-INTERVAL FIXED-RATIO SCHEDULES

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Five pigeons were trained on concurrent variable-interval fixed-ratio schedules in three experiments. Experiment 1 used two variable-interval schedules and one fixed-ratio schedule, and the ratio requirement was varied. Using the generalized matching law, sensitivity to reinforcement was close to 1.0, but performance was biased toward the variable-interval schedule with the lower reinforcement rate. In Experiment 2, which used one variable-interval and one fixed-ratio schedule, the interval schedule was varied. All birds showed sensitivities to reinforcement of less than 1.0 and of less than the values obtained in Experiment 1. The performance was also biased toward the fixed-ratio schedule. Because the generalized matching law could not account for the differences in the data from Experiments 1 and 2, an extension of this law was suggested and successfully tested in Experiment 3. The proposed dual-sensitivity model was also shown to clarify some previously reported results.

Key words: concurrent schedules, variable-interval schedules, fixed-ratio schedules, response allocation, time allocation, pecking, pigeons

The principle of indifference from irrelevant alternatives (Luce, 1959) states that the preference between two choice events should be unaffected by the addition of, and variation of, a third alternative. The principle was supported by Davison and Hunter (1976) who investigated the addition and variation of a third variable-interval (VI) schedule to concurrent VI VI schedules. Lobb and Davison (1977) extended this finding by showing that concurrent VI VI schedule preference was independent of schedules that were arranged successively to concurrent VI VI schedules. Davison and Temple (1974) showed that preference in a concurrent chain schedule was affected by the introduction of a third chain but was independent of the value of the third chain. The present experiments were designed to replicate systematically Davison and Hunter's findings when a fixed-ratio (FR) schedule, rather than a VI schedule, is added to concurrent VI VI schedules.

Independence of preference from the addition and variation of an FR alternative as well as from a VI alternative would provide general support for Herrnstein's (1970) quantitative law of effect. The added third alternative is an analogue of R_o , the rate of reinforcement obtained from unprogrammed sources, and in Herrnstein's system of equations, the size of R_o should not affect concurrent-schedule preference. Although Davison and Hunter showed that Herrnstein's assumption was correct for an added VI alternative, many of the types of behavior comprising R_o (such as grooming and scratching) may have a ratio, rather than an interval, character (Pear, 1975).

Performance in concurrent VI FR schedules was studied by Bacotti (1977). He analyzed his data using the generalized matching law:

$$\log \left(\frac{P_1}{P_2} \right) = a \log \left(\frac{R_1}{R_2} \right) + \log c, \quad (1)$$

where P denotes responses, R denotes obtained reinforcements, and the subscripts designate the two choices. Bacotti reported a large bias ($\log c$), averaging .39, towards responding on the VI schedule, and a sensitivity to reinforcement, a , averaging 1.12. The bias to the VI schedule is consistent with previous reports that preference is biased toward responding on a VI schedule when the alternative is a fixed-interval (FI) schedule (Lobb & Davison, 1975;

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Trevett, Davison, & Williams, 1972). However, response rates generated by FR schedules are usually higher than those generated by VI schedules (Ferster & Skinner, 1957), which might lead us to expect a bias to the FR schedule.

The present series of experiments began with a systematic replication of Bacotti's (1977) experiment using concurrent VI FR VI schedules rather than concurrent VI FR schedules. The VI schedules were kept constant, whereas the FR-schedule requirement was varied over a wide range.

EXPERIMENT 1

METHOD

Subjects

Five homing pigeons were maintained at $80\% \pm 15$ g of their free-feeding body weights. They were numbered 131, 133, 134, 135, and 136, and all had extensive previous experience of responding on concurrent schedules.

Apparatus

The experimental chamber, which was 31 cm wide by 33 cm high by 34 cm deep, contained three response keys 2 cm in diameter, 11.5 cm apart, and 25 cm from the grid floor. A food magazine, containing wheat, was situated below the center key and 12.5 cm from the floor. The chamber was sound attenuated and fitted with an exhaust fan and was situated remotely from the solid-state experimental control equipment. During the experimental sessions, except during reinforcement, the center key was illuminated red and the two side keys were illuminated white. Pecks to illuminated keys exceeding about .1 N were counted and produced a feedback click from a relay in the chamber. Responses emitted and reinforcements obtained on the three keys were accumulated on counters.

Procedure

Sessions ended in blackout when 50 reinforcements had been obtained or, more unusually, when 45 min had elapsed. The birds were then returned to their home cages where a supplementary feed of mixed grain was available if required to maintain their body weights. Water and grit were available at all times in the home cages.

A particular procedure remained in effect until a defined stability criterion had been met five, not necessarily consecutive, times. The criterion was that the median relative numbers of responses to the two VI keys over five sessions did not differ from the median from the previous five sessions by more than .05. When this criterion had been met five times by each bird, the experimental condition was changed for all birds.

In the first experimental condition (Table 1), the center key was neither lit nor operative, and nonindependent concurrent VI VI schedules (Stubbs & Pliskoff, 1969) were arranged on the two white keys. The left-key schedule was VI 120-sec and the right-key schedule was VI 30-sec. When either tape had set up a reinforcer, both tapes stopped until that reinforcer had been obtained. A changeover delay was in operation so that a reinforcer could not be produced by a response until 2 sec had elapsed since the bird commenced responding on one key after responding on the other key. This pair of VI schedules was arranged on the white side keys throughout Experiment 1.

In Condition 2, and in subsequent conditions, the center key was lit red and reinforcement was available at all times on a FR schedule. The ratio requirement was initially FR 8, and in subsequent conditions (Table 1) it was increased by small increments to FR 50. No changeover delay was arranged between the FR and the VI schedules, but the delay of 2 sec was maintained between the two VI schedules.

Table 1

Sequence of experimental conditions and number of sessions training on each condition in Experiment 1. The schedules on the left and right keys were, respectively, VI 120-sec and VI 30-sec.

Condition	Center key schedule	Sessions
1	None	32
2	FR 8	38
3	FR 10	22
4	FR 12	34
5	FR 15	21
6	FR 18	15
7	FR 20	17
8	FR 22	31
9	FR 27	31
10	FR 32	34
11	FR 36	21
12	FR 40	19
13	FR 45	22
14	FR 50	22

RESULTS

The mean numbers of responses emitted and reinforcements obtained on the three schedules averaged over the final five sessions of each

Fig. 1. Experiment 1. The relative number of VI responses emitted and VI reinforcements obtained on the left (VI 120-sec) key when the FR schedule requirement on the center key was varied. The data were calculated from the sums of responses and reinforcements over the last five sessions of each condition.

condition are shown in Table A of the Appendix. Data from only four birds were collected in Experiment 1 as Bird 135 failed to emit a substantial number of responses on the VI schedules in any condition. As the FR schedule requirement was raised from 8 to 50, the number of responses per session on the two VI schedules increased steadily, whereas the number emitted on the FR schedule first increased and then decreased. Over the same change in FR requirement, numbers of VI reinforcements increased and numbers of FR reinforcements decreased.

Figure 1 shows the proportion of responses emitted and reinforcements obtained on the left key (VI 120-sec) relative to the total VI responses and reinforcements as a function of the FR requirement. Data from conditions with fewer than about 150 VI responses per session were excluded from this graph as such measures are likely to be unreliable. No consistent trends in relative response allocation were shown as the FR requirement was changed, and there were no clear differences in performance when the FR schedule was absent (EXT). Obtained reinforcement proportions were constant, as determined by the nonindependent scheduling of the VIs, and these proportions were always more extreme than the response proportions.

The generalized matching law (Equation 1) can be used to analyze these data in a number of different ways. In the first analysis, both the VI schedules were taken together as a single operant, and overall choice between the VI and FR schedules was assessed. Figure 2 shows the log ratio of VI-to-FR responses for each of the final five sessions of each condition as a function of the log ratio of VI-to-FR reinforcements obtained. Data from conditions in which few responses were emitted or few reinforcements were obtained are not shown. For the four birds which responded to both the VI and the FR schedules, straight lines were fitted by the method of least squares, and the results are shown in Figure 2. The standard deviations of the obtained slopes (a in Equation 1) and intercepts ($\log c$) are shown below the equations of the best-fitting lines. The slopes averaged 1.14, and the intercepts $-.10$, showing an overall bias to responding on the FR schedules. The excellence of the fits is shown by the very small standard deviations of the slope and intercept estimates.

Fig. 2. Experiment 1. The logarithm of the ratio of total VI to total FR responses as a function of the logarithm of the ratio of total VI reinforcements to total FR reinforcements obtained. The data from each of the last five sessions of each condition, excluding infinite ratios, are shown. The best-fitting straight line by the method of least squares is shown and, beneath these, the standard deviations of the slopes and intercepts.

Fig. 3. Experiment 1. The logarithm of the ratio of VI 120-sec responses to FR responses as a function of the logarithm of the ratio of VI 120-sec reinforcements to FR reinforcements obtained. The data from each of the last five sessions, excluding infinite ratios, are shown. The best-fitting straight line by the method of least squares is shown and, beneath these, the standard deviations of the slopes and intercepts.

Figure 3 is similar to Figure 2 and shows the log ratios of VI-120 sec (left key) and FR (center key) responses and reinforcements. Again, straight lines were fitted by the method of least squares. The mean slope was .97 and there was a small average bias of .09 towards responding on the VI schedule. Again, the parameters were precisely estimated. Figure 4 shows the log ratios of the VI 30-sec and FR responses and reinforcements. The mean slope was 1.06, but the bias (mean $-.16$) was towards responding on the FR schedule. A comparison of the data obtained from the two VI versus FR binary choices (Figures 3 & 4) suggests no difference in slope (the value of a in Equation 1), but a consistent difference in the value of bias (log c). In each case, the birds showed a greater bias towards the lower reinforcement-rate VI schedule, the mean intercept difference being .25. Thus, for the same FR schedule value, preference was biased toward the VI 120-sec schedule and away from the VI 30-sec schedule.

DISCUSSION

The addition of, and variation of, a FR schedule concurrently available with two VI schedules changed the distribution of responses and reinforcements between the VI and FR schedules, but did not change the distribution of responses between the VI schedules. The present data therefore show that the principle of indifference from irrelevant alternatives (Luce, 1959) is correct under these conditions. The present results, with those reported by Davison and Hunter (1976) show that the assumption made by Herrnstein (1970) is correct whether R_0 has a VI or an FR character.

The only researchers to report that the principle of indifference from irrelevant alternatives did not apply in concurrent schedules were Prelec and Herrnstein (1978). They investigated preference in concurrent VI VR (variable-ratio) schedules and showed that the addition of a third VI-schedule alternative decreased preference for the VR schedule in the original VI VR choice. But their research did not constitute an adequate test of this principle because the VR-schedule reinforcement rate, and hence the VI-to-VR reinforcement ratio, changed with the addition and removal of the third VI alternative. The simplest test of the indifference principle requires a constant rein-

Fig. 4. Experiment 1. The logarithm of the ratio of VI 30-sec responses to FR responses as a function of the logarithm of the ratio of VI 30-sec reinforcements to FR reinforcements obtained. The data from each of the last five sessions of each condition, excluding infinite ratios, are shown. The best-fitting straight line by the method of least squares is shown and, beneath these, the standard deviations of the slopes and intercepts.

forcement ratio and, if there is no constancy, the third alternative is, by definition, not irrelevant. More detailed tests of the Prelec and Herrnstein data are precluded as data on obtained reinforcement rates were not reported. Thus, from the available data, we may assume that the indifference principle may generally be correct for concurrent-schedule performances.

The preference between the VI and FR schedules changed when the FR schedule requirement was varied, and this change was well-described by generalized matching lines relating log-preference ratio to log-reinforcement ratio (Figures 2, 3, & 4). As reported by Bacotti (1977), sensitivity to reinforcement (a in Equation 1) was close to 1.0 on average. But the values of $\log c$ or bias found here were very different from the large bias (.39) to the VI schedule found by Bacotti. In the comparisons here (Figures 3 & 4), responding was biased to the VI 120-sec schedule and away from the VI 30-sec schedule. These results do not support the simple conclusion made by Bacotti that preference in concurrent VI FR is biased to the VI alternative. Rather, the direction of the bias depends on the value of the VI schedule, and this conclusion is consistent with Bacotti's result. If bias to the VI alternative is inversely related to the VI reinforcement rate as the present results suggest, a large bias to a VI 240-sec schedule as used by Bacotti is predictable.

EXPERIMENT 2

The results obtained in Experiment 1 seemed counterintuitive. We would normally expect no change in bias between a pair of schedules to result from an absolute reinforcement-rate change on one of the schedules. For example, in concurrent VI VI schedules, changing the value of one schedule merely moves preference along a matching line, though an undermatching line ($a < 1$ in Equation 1) is consistent with bias to the lower reinforcement rate schedule increasing with the ratio of reinforcements.

The data from Experiment 1 (Figures 3 & 4) showed that the VI 120-sec vs. FR and the VI 30-sec vs. FR preference did not lie on the same straight line. It therefore follows that varying the VI schedule in concurrent VI FR must have an effect which has a different sensitivity to reinforcement from varying the FR

Fig. 5. A stylized graph of the way in which log VI-to-FR response ratios change with log VI-to-FR reinforcement ratios according to whether the VI or the FR schedules are varied. The continuous lines show the effects of varying the FR schedule with the constant VI schedule shown as the parameter. The broken lines show the effects of varying the VI schedules with the constant FR schedule shown as the parameter. The concurrent schedules at the labeled points are: A, VI 120-sec FR 40; B, VI 120-sec FR 20; C, VI 30-sec FR 20; and D, VI 30-sec FR 40.

schedule. When the VI schedule is changed, the bias as well as the reinforcement ratio changes, such that an increase in the VI-to-FR reinforcement ratio will be offset to some extent by an increased bias to responding on the FR schedule. Figure 5 shows a stylized plot of the implications of the data obtained in Experiment 1. I have assumed that the matching lines for varying the FR schedules are parallel with a slope of 1.0, an assumption consistent with the results of Experiment 1 and those reported by Bacotti (1977). When a change is made in the ratio requirement (solid lines) from, for example, FR 40 to FR 20 in the presence of VI 120-sec, the preference for the VI over the FR schedule would be expected to change along a line of slope 1.0 from point A to point B (as the FR reinforcement rate increases, the data point will move to the left of Figure 5). If the VI schedule is now changed to VI 30-sec, the data point will move to the right of Figure 5, but the bias to the VI schedule will decrease. The preference will now fall on a line parallel to AB but displaced downwards, and will change from B to C along the broken line. For reasons made clear later, the broken lines in Figure 5 are all parallel with a slope

of .6. Increasing the ratio requirement to 40 will move the data point on the line CD to D and, finally, changing the VI schedule to VI 120-sec will return the data point on the line DA (slope .6) to A. Thus, given the assumptions underlying Figure 5, sensitivity to reinforcement must be less in concurrent VI FR performance when the VI schedule is varied than when the FR schedule is varied.

Experiment 2 was designed to measure sensitivity to reinforcement in concurrent VI FR schedules when the FR requirement was constant and the VI schedule was varied.

Subjects and Apparatus

These were the same as in Experiment 1.

Procedure

The procedure was identical to that used in Experiment 1 except that the schedule on the right key (which was illuminated) was Extinction. As a result, the stability of the behavior was assessed, using the same criterion, on relative left (VI) and center (FR) key responses. The FR schedule on the center key always required 30 responses, whereas the VI schedule on the left key was varied from VI 15-sec to VI 120-sec in five experimental conditions (Table 2). Responses emitted and reinforcements obtained on each key were measured.

RESULTS

The numbers of responses emitted and reinforcements obtained on the left and center keys, summed over the final five sessions of each condition, are shown in Appendix B. After the first five sessions, less than 10 responses per session were normally emitted on the right (Ext) key, and these data have therefore not been reported. The higher the arranged reinforcement rate on the VI schedule, the more responses were emitted and reinforce-

ments obtained on that schedule relative to the FR schedule.

In Figure 6, the logarithm of the ratio of the numbers of responses emitted on the left (VI) and the center (FR) keys has been plotted as a function of the logarithm of the ratio of the numbers of reinforcements obtained on these keys. Straight lines were fitted by the method of least squares and the equations, with the standard deviations of the parameter estimates, are shown on the graphs. The straight lines fitted the data well and most parameter values were accurately estimated. The mean slope (sensitivity to reinforcement) was .69 and all slopes were less than 1.0 (significant on a Sign test at $p < .05$). All curve fits gave negative intercepts (but only slightly so for Bird 136), averaging $-.36$, showing a consistent bias to responding on the FR schedule.

DISCUSSION

The results found in Experiment 2 were consistent with the results of Experiment 1 and the results reported by Bacotti (1977). They also confirmed the interpretation of the results of Experiment 1 given in Figure 5. For every subject for which data were obtained in Experiments 1 and 2, the sensitivity to VI schedule reinforcement-rate variation was less than for FR schedule reinforcement-rate variation (Figures 3, 4, & 6).

An undermatching line describes the fact that as one reinforcement rate is decreased, a greater preference is shown to that schedule than is predicted by matching or a strict equality between response and reinforcement ratios. At points on such a line which lie below the line of strict matching, variation of some other choice-affecting independent variable (for instance, response force) will give data which are biased to the denominator. At points which lie above the strict matching line, the same manipulation will give data which are biased to the numerator. The data obtained in Experiment 2, a negatively biased undermatching line such as line BC in Figure 5, show that varying the FR-schedule requirement when the VI-reinforcement rate is high (to the right of the Figure) would give a bias to the FR schedule (e.g., line DC). But when the VI reinforcement rate is low (the left of the Figure), varying the FR schedule will give data with a bias to the VI schedule. Thus, as found in Experi-

Table 2

Sequence of experimental conditions and number of sessions training on each condition in Experiment 2. The schedule on the right key was Extinction, and that on the center key was FR 30, throughout.

Condition	Left-key schedule	Sessions
1	VI 120-sec	31
2	VI 30-sec	20
3	VI 90-sec	20
4	VI 15-sec	20
5	VI 60-sec	18

Fig. 6. Experiment 2. The logarithm of the VI-to-FR response ratios as a function of the logarithm of the obtained reinforcement ratios. The data are each of the last five sessions of each experimental condition. Straight lines were fitted by the method of least squares, and the equations are shown for each bird. Beneath the equations are the standard deviations of the slopes and intercepts.

ment 1, and by implication by Bacotti (1977), the lower the VI reinforcement rate the greater the bias to the VI schedule when the FR schedule is varied. Figure 5 is thus a correct stylization of the effects of varying the FR and the VI schedules in both concurrent VI FR VI and concurrent VI FR schedules.

How may the present data be described in a quantitative model? Clearly, if in concurrent VI FR schedules, varying each of the schedules changes response allocation with a different sensitivity, the generalized matching law (Equation 1) cannot account for the data. The simplest extension of this law which could account for these data has a separate sensitivity parameter for each independent variable:

$$\log\left(\frac{P_{VI}}{P_{FR}}\right) = m \log(R_{VI}) - n \log(R_{FR}) + \log g. \quad (2)$$

This equation is of interest because it is related to a "chemical" model of behavior allocation discussed by Staddon (1977). The model he described was the hypothetical "reaction":

which, at stability, identifying k_1 and k_2 with reinforcement rates, gives the equation:

$$m \log(P_1) - n \log(P_2) = \log(R_1/R_2).$$

The model underlying Equation 2 is:

$$mR_{VI} \xrightleftharpoons[k_2]{k_1} nR_{FR},$$

where k_1 and k_2 are identified with response rates. Equation 2 thus describes an "environmental" reaction (the concentration of reinforcers available from two sources according to the response rates emitted by the organism) rather than the "behavioral" reaction discussed by Staddon. I have used Equation 2 simply because it fits more comfortably with the generalized matching law (Equation 1).

Although in Equation 1 the variables can be interpreted as either numbers or rates of responses and reinforcements, reinforcement rates must be used in Equation 2 (Staddon, 1977). If $m \neq n$, and if reinforcement *numbers* are used on the right-hand side of Equation 2, then the longer the experimental session the greater the difference between the log-reinforcement numbers and the greater the pre-

dicted preference. On the other hand, the difference between the logarithms of the reinforcement rates is unaffected by session duration.

Equation 2 appears to be necessary to describe the results of Experiments 1 and 2 with $m < n$, but the data collected in these experiments cannot be used to test Equation 2 because time allocation, and hence session time, was not measured. Experiment 3 was designed to provide the necessary data through extensive variations of both VI and FR reinforcement rates.

EXPERIMENT 3

Subjects and Apparatus

These were the same as in Experiments 1 and 2.

Procedure

The procedure was identical to that used in Experiment 1 (Concurrent VI FR VI schedules) except that the time spent responding to each of the three keys was also measured. Time allocation to a key commenced with the first response to that key and ended with the first response to another key. The sequence of experimental conditions is shown in Table 3. In the first five conditions, which were conducted between Experiments 1 and 2, the center key was FR 45, and the VI schedules on the left and right keys were varied, keeping the total

VI reinforcement rate constant at 2 per min. The next nine conditions, done after Experiment 2, first kept the FR schedule at 20 responses and varied the VI schedules over the same range as Conditions 1 to 5, and then kept the VI schedules at 37.5 sec and 150 sec and varied the ratio requirement from 20 to 60 responses in steps of 10. The stability of the performance, using the same criterion as Experiment 1, was assessed on the relative number of VI responses.

RESULTS

Responding was well maintained on all three keys in all conditions for all subjects except Bird 135. This bird emitted very few responses on the VI schedules in Condition 1, ceased responding again on these schedules when the FR requirement was decreased to 20 (Condition 6) and did not commence responding on the VI schedules again until Condition 11 when the FR requirement was raised to 30 responses. Otherwise, the responses emitted and reinforcements obtained (Appendix C) varied in a way similar to the results of Experiments 1 and 2. Time allocation to the three keys (Appendix C) covaried with response allocation.

The data from Experiment 3 were analyzed according to Equation 2 using multiple linear regression. Reinforcement rates were measured per minute. The dependent variables investigated were the log-response ratio and, because the generalized matching law also applies to time allocation (e.g., Baum, 1979), log time-allocation ratio. Each experimental condition could provide two data points for analysis, one from the left and center keys and one from the right and center keys. The results of the multiple linear regression analyses, with the standard deviations of the estimated parameters and the proportion of data variance accounted for, are shown in Table 4. The number of data points shown for Bird 135 was 15 because Conditions 1 and 6 to 10 provided no data, and Condition 11 provided only one data point. The data from Bird 136 in Condition 6 were also discarded.

In most cases, the data fitted Equation 2 very well, with large proportions of data variance accounted for and parameters estimated with small standard deviations. The relatively large standard deviations of parameter estimates for Bird 135 were due to the very small

Table 3

Sequence of experimental conditions and number of sessions training on each condition in Experiment 3. All VI schedule values are in seconds.

Condition	Schedules:			Sessions
	Left key	Right key	Center key	
1	VI 60	VI 60	FR 45	33
2	VI 90	VI 45	FR 45	27
3	VI 45	VI 90	FR 45	28
4	VI 150	VI 37.5	FR 45	22
5	VI 37.5	VI 150	FR 45	24
6	VI 60	VI 60	FR 20	23
7	VI 90	VI 45	FR 20	40
8	VI 45	VI 90	FR 20	32
9	VI 150	VI 37.5	FR 20	28
10	VI 37.5	VI 150	FR 20	27
11	VI 37.5	VI 150	FR 30	32
12	VI 37.5	VI 150	FR 40	24
13	VI 37.5	VI 150	FR 50	27
14	VI 37.5	VI 150	FR 60	21

Table 4

The results of multiple linear regression analyses of the data from Experiment 3 according to Equation 2. Standard deviations of parameter estimates are given in parentheses. VAC is the proportion of data variance accounted for, and N is the number of data points used. m is the sensitivity to VI reinforcement rate and n is the sensitivity to FR reinforcement rate.

Bird	$m(SD)$	$n(SD)$	$\log g(SD)$	VAC	N
Responses					
131	.50(.09)	1.24(.06)	-.06(.04)	.97	28
133	.71(.09)	.90(.05)	.03(.03)	.95	28
134	.51(.09)	.79(.10)	-.09(.04)	.82	28
135	.49(.11)	2.28(.36)	.57(.18)	.89	15
136	.74(.07)	1.18(.05)	.07(.03)	.98	26
Time					
131	.68(.09)	1.09(.05)	.11(.04)	.97	28
133	1.02(.07)	.86(.04)	.27(.02)	.98	28
134	.86(.07)	.79(.08)	.21(.03)	.92	28
135	.90(.10)	1.44(.33)	.58(.16)	.92	15
136	.90(.08)	.99(.06)	.22(.04)	.97	26

obtained variation in FR reinforcement rate in the available data (see Appendix C).

DISCUSSION

The data analyses confirmed the results of Experiments 1 and 2 and showed that Equation 2 described the data accurately. For response allocation between VI and FR schedules, preference was more sensitive to FR reinforcement rate for every bird (mean n , excluding Bird 135, 1.03) than to VI reinforcement rate (mean m , including Bird 135, .59), and this difference was significant on a Sign test at $p < .05$. The sensitivities to VI reinforcement rate were all less than 1.0 (significant on a Sign test at $p < .05$), but the sensitivities to FR reinforcement rate were not different from 1.0.

The sensitivities of time allocation to the VI and FR reinforcement rates were not significantly different from each other (mean $m = .87$ and mean $n = .93$ excluding Bird 135) nor were they significantly different from a sensitivity of 1.0. Comparing response and time allocation for the VI reinforcement rate, response-allocation sensitivity was significantly less than time-allocation sensitivity (Sign test, $p < .05$), the common finding for concurrent VI VI schedules (Baum, 1979; Lobb & Davison, 1975). This relation was reversed, however, for response- and time-allocation sensitivities to FR reinforcement rate. Here, taking

the data of Bird 134 to three decimal places, response-allocation sensitivities were significantly greater than time-allocation sensitivities (Sign test, $p < .05$). A tendency for response-allocation sensitivity to be greater than time-allocation sensitivity in concurrent ratio schedules was also noted by Rider (1979).

The values of the intercept, $\log g$, obtained from the multiple linear regressions, were generally close to zero for response allocation, but for time allocation they were all positive and significantly larger (Sign test, $p < .05$) than the intercepts for response allocation. The interpretation of the intercept values ($\log g$) shown in Table 4 is problematical. In the generalized matching law (Equation 1) the intercept (bias) is dimension free. In the model proposed here, for the equation to be dimensionally consistent, $\log g$ must have the dimensions T^{m-n} . As a result, the obtained value of $\log g$ will be different depending on the timebase (reinforcements per minute or per hour) used to measure the independent variable. $\log g$ is thus a time constant. But logic dictates an additional interpretation of $\log g$. It should also measure any constant proportional preference for an alternative, independent of the types and values of the schedules arranged, that is, bias as described by the generalized matching law:

$$\log g = \log t + \log c,$$

where $\log t$ is the time constant and $\log c$ is bias. Multiple linear regression cannot provide separate estimates of the values of such additive constants. Thus, the reason that $\log g$ was larger for time allocation than for response allocation is presently unclear.

Considering Figure 5, it is clear that a bias reported in a generalized matching law (Equation 1) analysis may, under some circumstances, be accounted for and better understood using Equation 2. Furthermore, if data which conform to Equation 2 are analyzed using Equation 1, poor fits and unaccountable bias and sensitivity changes would be likely to be reported. There are a number of such reports in the literature. For example, Trevett, Davison, and Williams (1972) and Lobb and Davison (1975) reported that concurrent FI VI schedule performance was biased to the latter schedule. Trevett et al. found the same sensitivity to reinforcement as in a concurrent VI VI control, but Lobb and Davison found a clear sensitivity difference from their concur-

rent VI VI control. These incompatible results might be better understood in terms of Equation 2.

A reanalysis was carried out of Lobb and Davison's (1975) data using Equation 2 and taking concurrent FI VI and concurrent VI VI performances separately. For the former, using group data, the analysis gave a sensitivity to FI reinforcement rate of .41 (SD .15) and to VI reinforcement rate of .78 (SD .07). For the concurrent VI VI performance, the sensitivity to both VI reinforcement sources was .76. Analyzing their data using Equation 1, Lobb and Davison reported that the reinforcement sensitivity for concurrent FI VI schedules (mean .68) was less than for concurrent VI VI schedules (mean .82), but they were unable to give a satisfactory explanation for the difference. The reanalysis shows that sensitivity to VI reinforcement sources was the same in both schedule combinations, and that the low overall sensitivity to reinforcement in concurrent FI VI arose from a low sensitivity to the FI reinforcement source alone.

Beautrais and Davison (1977) compared performance on concurrent second-order VI (FR:s) VI (FR:s) schedules under conditions of equal and unequal FR requirements. They found a bias to the smaller FR requirement when the VI schedules were varied, and also lower Equation 1 reinforcement sensitivities when the FR requirements were different (FR 5 vs. FR 10) from when they were the same (both FR 5). The unequal FR data were analyzed according to Equation 2 and gave a reinforcement sensitivity to the VI (FR 5:s) schedule of .8 (SD .21) and to the VI (FR 10:s) schedule of .44 (SD .22). Thus, here again, Equation 2 makes more sense of the data than did Equation 1. Similarly, Davison and Ferguson (1978) found a smaller reinforcement sensitivity for a greater response requirement (treadle pressing versus key pecking in pigeons).

Reanalyses of previous data require that responses, time spent responding, and reinforcements obtained are reported, and also that both independent variables (schedules or other requirements) are varied over a reasonable range. Very few such reports exist. Herrnstein and Heyman (1979) reported data on concurrent VI VR schedule performance, but they varied the VI schedule rather little. A reanalysis using Equation 2 was again consistent

with the results reported above. Sensitivity to VI reinforcement rate was .87 (SD .15) and to VR reinforcement rate it was 1.04 (SD .02).

Equation 2 is therefore applicable to a number of experimental results and clarifies some previously unaccountable differences. Under what conditions will Equation 2, rather than Equation 1, apply? The indications from the present experiments and from reanalyses suggest that differential sensitivity to reinforcement sources may occur when either response requirements or schedules are qualitatively different. Further research will be necessary to demonstrate whether these are both the necessary and sufficient conditions.

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Appendix A

Experiment 1. Numbers of responses emitted and reinforcements obtained on the two VI and the FR schedules summed over the last five experimental sessions of each experimental condition.

Bird	Condition	Responses:			Reinforcements:		
		Left key	Right key	Center key	Left key	Right key	Center key
131	1	5021	7570	—	46	204	—
	2	0	1	1980	0	0	250
	3	10	0	2457	0	3	247
	4	0	0	2954	0	0	250
	5	0	1	3698	0	0	250
	6	0	67	4384	0	5	245
	7	56	263	4443	2	24	224
	8	449	1843	4924	8	49	193
	9	325	415	4956	9	55	186
	10	1638	3182	3310	29	116	105
	11	3255	6610	1084	43	175	32
	12 ¹	2006	4287	487	22	115	13
	13	4071	8502	128	49	198	3
	14	2400	9476	0	48	202	0
133	1	5290	9502	—	57	193	—
	2	0	4	1994	0	0	250
	3	0	0	2488	0	0	250
	4	0	0	2987	0	0	250
	5	14	78	3550	1	11	238
	6	1	65	4419	0	3	247
	7	15	163	4903	0	13	237
	8	3	233	5921	0	2	248
	9	329	933	5101	11	50	189
	10	1185	3768	3369	29	121	106
	11	1024	3131	4151	21	113	116
	12 ¹	1195	3173	1165	23	98	29
	13	2645	6125	1436	43	174	33
	14	1736	5148	3671	32	144	74
134	1	3834	5160	—	50	200	—
	2	2	112	1887	0	14	236
	3	5	142	2340	0	15	235
	4	0	154	2927	0	10	240
	5	9	200	3576	0	6	244
	6	6	156	4424	0	3	247
	7	521	1104	3323	16	66	168
	8	598	1243	3866	19	68	163
	9	840	1619	3474	24	97	129
	10	939	2213	3360	24	115	111
	11	1217	3090	2792	35	134	81
	12 ²	570	1768	726	15	69	16
	13	1694	5122	1981	35	171	44
	14	1797	4803	1566	46	174	30
136	1	2999	7704	—	49	201	—
	2	0	1	2000	0	0	250
	3	0	0	2499	0	0	250
	4	0	7	2994	0	0	250
	5	97	10	3657	5	0	245
	6	56	120	4289	3	8	239
	7	6	358	4698	0	14	236
	8	0	210	5688	0	12	238
	9	459	780	4962	14	49	187
	10	434	1024	5522	15	59	176
	11	1448	5091	3190	34	132	84
	12	1729	5340	2574	36	149	65
	13	1471	6010	2582	39	153	58
	14	2636	8612	876	48	182	20

¹Data from three sessions²Data from two sessions

Appendix B

Experiment 2. Number of responses emitted and reinforcements obtained on the VI and FR schedules summed over the last five sessions of each experimental condition.

<i>Bird</i>	<i>Condition</i>	<i>Responses:</i>		<i>Reinforcements:</i>	
		<i>Left key</i>	<i>Center key</i>	<i>Left key</i>	<i>Center key</i>
131	1	598	5992	29	221
	2	1103	3261	133	117
	3	903	5489	45	205
	4	1442	1799	181	59
	5	1590	4595	72	178
133	1	265	6205	24	226
	2	564	4726	81	169
	3	707	5842	39	211
	4	1180	994	213	37
	5	1003	5130	60	190
134	1	772	5479	29	221
	2	1781	2964	137	113
	3	1285	5079	41	209
	4	1722	1572	183	67
	5	1203	4593	57	193
135	1	231	6697	9	241
	2	672	5357	57	193
	3	698	6420	20	230
	4	730	4443	90	160
	5	778	5911	34	216
136	1	920	5746	26	224
	2	2806	3592	111	139
	3	937	5659	32	218
	4	3450	1746	184	66
	5	1837	4986	52	198

Appendix C

Experiment 3. Number of responses emitted, time in seconds spent responding, and reinforcements obtained on the two VI (left and right) and the FR (center) schedules. The data have been summed over the last five experimental sessions of each Condition. Conditions 1 to 5 were completed before Experiment 2, and Conditions 6 to 12 after Experiment 2. For Bird 135, data from Conditions 6 to 10, in which the bird made very few VI responses, have been omitted.

Bird	Condition	Responses:			Times:			Reinforcements:		
		Left	Right	Center	Left	Right	Center	Left	Right	Center
131	1	4241	4381	3956	1971	2565	1596	82	83	86
	2	3789	4390	2914	1956	3179	1172	53	131	66
	3	3894	2514	4432	2283	1405	1786	108	44	101
	4	2884	7485	735	1640	5452	362	48	184	18
	5	5665	2533	1570	4234	1811	905	172	43	35
	6	238	78	4147	229	64	2190	10	9	231
	7	150	265	4055	138	191	2003	7	23	220
	8	262	255	3886	387	131	1905	25	11	214
	9	153	422	3953	159	245	1887	6	26	218
	10	139	163	3962	188	90	1780	24	7	219
	11	944	386	4912	936	239	2236	58	11	181
	12	1971	937	5108	1782	472	2099	93	20	137
	13	3248	1479	5567	2224	849	2403	73	15	161
	14	5627	2465	2714	3795	1166	1158	159	36	55
133	1	6126	7658	2387	2231	4203	814	83	117	50
	2	1663	4255	4208	911	3149	1739	45	113	92
	3	5646	3027	3123	3223	1830	1182	127	56	67
	4	2130	5477	3491	1060	3521	1258	38	137	75
	5	4446	2076	1552	5546	958	574	174	43	33
	6	798	835	3603	389	836	1682	26	29	197
	7	870	759	3558	355	739	1609	20	36	194
	8	1522	328	3649	755	365	1738	34	18	198
	9	319	908	3611	145	863	1612	8	42	200
	10	1012	257	3925	437	260	1816	28	9	213
	11	1450	674	5098	1097	330	2093	56	14	180
	12	4334	1540	4525	2848	648	1847	108	24	118
	13	5355	1910	4087	3470	647	1541	129	24	107
	14	9071	3670	1050	5577	1318	384	189	41	20
134	1	5739	6955	2546	2754	3747	1084	85	112	53
	2	1820	3055	4616	1016	2487	1378	44	99	107
	3	2246	1520	5919	1842	791	1677	80	32	138
	4	1436	3488	4550	788	3066	1600	26	118	106
	5	2574	1272	4788	2855	719	1728	114	25	111
	6	605	506	3454	801	308	1819	28	24	198
	7	874	701	3228	469	804	1686	22	43	185
	8	1479	703	3348	960	438	1737	39	23	188
	9	721	1311	3220	460	980	1705	11	56	183
	10	1265	428	3304	1078	254	1751	49	12	189
	11	2137	896	3975	2023	550	1778	84	20	146
	12	2715	1188	4782	2314	550	1845	96	25	129
	13	3466	1074	5485	2815	478	2099	82	10	158
	14	3231	1199	4873	3195	621	1946	126	27	97
135	1	0	61	12070	0	112	2464	0	2	248
	2	1548	1751	8340	390	1080	1786	23	48	179
	3	2488	850	8518	1070	457	1707	50	17	184
	4	1525	1586	8230	486	1067	1661	14	62	174
	5	2176	831	8433	1100	343	1761	59	13	178
	11	227	20	6555	260	19	1732	18	0	232
	12	295	92	8882	299	21	1906	18	2	230
	13	1402	676	8444	793	221	1840	32	6	213
	14	1621	639	9961	947	287	1891	52	11	187

Appendix C continued

136	1	8142	8370	1326	3055	3462	470	100	122	28
	2	1798	2628	6072	718	1414	2114	29	77	144
	3	4470	1936	4807	1908	863	1810	92	44	114
	4	2679	8158	2925	1254	3759	1037	36	149	65
	5	9529	2259	1961	4789	1099	743	168	37	45
	6	225	23	4601	203	11	2394	1	1	248
	7	265	246	4175	247	227	1939	9	16	225
	8	331	203	4022	403	144	1872	21	8	221
	9	224	367	3823	292	317	1802	9	29	212
	10	400	127	3817	489	59	1842	32	4	214
	11	2442	673	4244	1510	324	1903	68	23	159
	12	5088	1693	4244	2613	705	1529	107	26	117
	13	3649	744	5813	2469	304	2057	62	11	177
	14	5020	1099	4633	2871	523	1662	115	29	106