CONCURRENT FIXED-INTERVAL VARIABLE-RATIO SCHEDULES AND THE MATCHING RELATION

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Five rats responded under concurrent fixed-interval variable-ratio schedules of food reinforcement. Fixed-interval values ranged from 50-seconds to 300-seconds and variable-ratio values ranged from 30 to 360; a five-second changeover delay was in effect throughout the experiment. The relations between reinforcement ratios obtained from the two schedules and the ratios of responses and time spent on the schedules were described by Baum's (1974) generalized matching equation. All subjects undermatched both response and time ratios to reinforcement ratios, and all subjects displayed systematic bias in favor of the variable-ratio schedules. Response ratios undermatched reinforcement ratios less than did time ratios, but response ratios produced greater bias than did time ratios for every subject and for the group as a whole. Local rates of responding were generally higher on the variable-ratio than on the fixed-interval schedules. When responding was maintained by both schedules, a period of no responding on either schedule immediately after fixed-interval reinforcement typically was followed by high-rate responding on the variable-ratio schedule. At short fixed-interval values, when a changeover to the fixed-interval schedule was made, responding usually continued until fixed-interval reinforcement was obtained; at longer values, a changeover back to the variable-ratio schedule usually occurred when fixed-interval reinforcement was not forthcoming within a few seconds, and responding then alternated between the two schedules every few seconds until fixed-interval reinforcement finally was obtained.

Key words: concurrent schedules, fixed-interval schedules, variable-ratio schedules, matching relation, undermatching, bias, lever press, rats

Concurrent (conc) schedules are two or more schedules in effect simultaneously, each independently arranging reinforcement for different responses (Ferster & Skinner, 1957).

Herrnstein (1961, 1970) suggested that the distribution of responses between two schedules should equal the distribution of reinforcements obtained from the schedules. This matching relation is expressed as:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2} \text{ or } \frac{B_1}{B_2} = \frac{r_1}{r_2} \quad (1)$$

where B represents either responses or time spent on a schedule, r represents reinforcements obtained from a schedule, and the subscripts 1 and 2 represent reinforcement schedules.

Baum (1974) provided a more general formulation, expressed as:

$$\frac{B_1}{B_2} = b \frac{r_1^a}{r_2^a} \quad (2)$$

Equation 2 is equivalent to Equation 1 when the coefficient b and the exponent a are equal to 1. A logarithmic transformation linearizes Equation 2:

$$\log \frac{B_1}{B_2} = a \log \frac{r_1}{r_2} + \log b \quad (3)$$

Deviations from linear matching specified by Equation 1 can be assessed by fitting data to Equation 3 and obtaining estimates of b and a . The parameter b refers to bias for a schedule (Baum, 1974; Davison & Hunter, 1976); the parameter a refers to the sensitivity with

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which responding changes with changes in the obtained relative rate of reinforcement (Baum, 1979; Davison & Hunter, 1976). Unbiased matching requires that the parameters b and a each equal 1.

The four basic schedules of reinforcement—fixed ratio (FR), fixed interval (FI), variable ratio (VR), and variable interval (VI)—can be combined to form ten different pairs of concurrent schedules. Data from conc VI VI conform most closely to linear matching (de Villiers, 1977; Herrnstein, 1970). Although response and time ratios tend to undermatch (a is less than 1) reinforcement ratios somewhat (cf. Baum, 1974, 1979; Davison & Hunter, 1976; Hunter & Davison, 1978; Lobb & Davison, 1975; Myers & Myers, 1977), deviations from matching under conc VI VI are typically small compared to deviations from matching under other concurrent schedules. Undermatching has been reported in conc FI VI (Lobb & Davison, 1975; Nevin, 1971; Trevett, Davison, & Williams, 1972) and conc FI FR (LaBounty & Reynolds, 1973). In addition, considerable bias was evident in most cases. Matching occurred under conc FI FI only when the pattern of responding on each FI schedule was the same; otherwise, undermatching occurred (White & Davison, 1973).

Data from concurrent schedules that do not include FI show good sensitivity of responding to reinforcement (a is approximately equal to 1). For example, Bacotti (1977) found that response and time ratios approximated reinforcement ratios under conc FR VI, although all subjects displayed bias in favor of the VI schedule. Herrnstein and Loveland (reported by de Villiers, 1977) found that response and time ratios approximated reinforcement ratios under conc VI VR, although responding was biased in favor of the VR.

Previous research is consistent with the generalization that bias occurs whenever qualitatively different schedules are arranged concurrently, and undermatching is most likely whenever one of those schedules is FI (cf. Baum, 1979). The present study supplied data from conc FI VR, a combination likely to produce both systematic bias and undermatching by the above generalization. Rats were trained under concurrent schedules with FI values ranging from 50 to 300 sec and VR values from 30 to 360.

METHOD

Subjects

Five experimentally naive male albino rats were maintained at approximately 80% of their free-feeding weights. The rats were obtained from Midcontinent Research Animals, Shawnee, Kansas. All were 3 to 5 months old at the start of the experiment.

Apparatus

A two-lever Lehigh Valley Electronics (LVE) rat chamber enclosed in an LVE sound-attenuating chest was used. The chamber measured 30.6 cm long, 25.3 cm wide, and 26.0 cm high. The levers were 2.7 cm wide and 17.2 cm apart, center to center. Above each lever were three miniature lamps (GE 313). Reinforcers (45 mg Noyes Precision Food Pellets) were dispensed into a tray equidistant between the levers. A houselight provided general illumination during experimental sessions, and the chamber was ventilated by a fan attached to the chest.

Schedules were programmed and data were collected by electromechanical equipment located in the same room as the chamber. Noise from this equipment was masked by white noise generated inside the chamber, noise from the fan, and noise from equipment controlling other experiments in the room.

Pretraining

The lamps above each lever were illuminated throughout pretraining. After magazine training, lever pressing was shaped. When responding to each lever was established, alternation training began, during which reinforcement was available for a response to one lever, but responses to the other lever had no scheduled consequences. Which of the levers was to provide reinforcement was determined on an irregular basis from reinforcement to reinforcement; no more than three consecutive reinforcements could be obtained from the same lever. All rats responded to each lever, changing from one to the other frequently, by the end of two sessions of 60 reinforcements each.

Experimental Procedures

The rats responded under conc FI 50-sec VR 30 after pretraining. The individual ratios comprising all VR schedules were derived ac-

cording to the formula described by Catania and Reynolds (1968; Appendix b), substituting responses for seconds. The lamps above the FI lever were lit continuously; the lamps above the VR lever flashed on and off every .5 sec. A changeover delay (COD) prevented reinforcement from occurring immediately following a CO to either lever. The COD was increased gradually from 1 sec to 5 sec during the first nine experimental sessions and was kept at 5 sec for the remainder of the experiment.

Time spent responding on each schedule was recorded by running-time meters. The first response in a session on either schedule started its respective timer, which continued to operate until a response to the other lever occurred, thereby starting its timer.

Responses on each schedule were recorded in successive tenths of the scheduled fixed interval and summed across fixed intervals within sessions.

Relative response rates were computed by dividing the number of responses on the FI schedule by the total number of responses on both schedules in a session. All experimental conditions continued until the relative response rates did not deviate in any of six consecutive sessions by more than five centiles (.05) of the mean relative response rate for those six sessions and until no consistent trend in relative responding was evident. The values of the FI and VR components and the number of sessions in each condition are given for each rat in Table 1.

Sessions were terminated with the first FI reinforcement after 40 min, or after 60 min, whichever occurred first. Sessions were conducted six or seven days per week at the same time each day.

RESULTS

Average number of responses, reinforcements, COs, and time spent in each component for the last six sessions in each condition are given in Table 2.

Figure 1 shows the logarithms of the ratios of responses (FI/VR) correlated with the logarithms of the ratios of reinforcements (FI/VR) for the last six sessions in each condition. The solid lines in the figure were derived by the method of least squares; the value of r^2 is the coefficient of determination. The dashed

Table 1

Sequence of Conc FI VR schedules and number of sessions each schedule was in effect. FI values are given in seconds.

	Ses-				Ses-		
	FI	VR	sions		FI	VR	sions
Rat 102	50	30	26	Rat 103	50	30	22
	50	90	56		50	90	27
	50	180	28		50	180	21
	50	360	45		50	360	71
	100	90	16		100	90	16
	200	90	27		200	90	22
	300	90	39		300	90	18
	300	180	14		300	180	29
	100	180	26		300	90	10
	200	180	13		100	180	14
					200	180	19
Rat 104	50	30	43	Rat 105	50	30	30
	50	90	28		50	90	41
	50	180	17		50	180	58
	50	360	58		50	360	19
	100	180	51		100	180	29
	200	180	18		200	180	39
	300	180	18		300	180	27
	300	90	15		300	90	15
	100	90	18		100	90	12
	200	90	20		200	90	20
					300	360	17
	Ses-				Ses-		
	FI	VR	sions		FI	VR	sions
Rat 106	50	30	29				
	50	90	55				
	50	180	58				
	100	180	41				
	200	180	68				
	300	60	40				

lines represent perfect matching of response ratios to reinforcement ratios. The linear regression equation for each subject had a slope less than one and a negative intercept: that is, each subject undermatched response ratios to reinforcement ratios and displayed bias in favor of the VR. The least-squares linear regression equation relating response ratios to reinforcement ratios for the group is:

$$y = .85x - .67; r^2 = .891.$$

The group equation was obtained by applying the least-squares method to all 48 data points from all five subjects.

Figure 2 shows the logarithms of the ratios of time spent in each component of the concurrent schedules (FI/VR) correlated with the logarithms of the ratios of reinforcements

Table 2

Average number of responses, reinforcements, changeovers, and time spent in each component of conc FI VR for the last six sessions in each condition. Ranges are given in parentheses.

		<i>Responses</i>	<i>Reinforcements</i>	<i>Changeovers</i>	<i>Minutes</i>
Rat 102	FI 50	15.8 (79)	1.0 (6)	2.3 (10)	.29 (1.43)
	VR 30	5340.3 (1961)	176.5 (66)	2.7 (10)	59.71 (1.43)
	FI 50	1049.2 (213)	42.7 (4)	45.3 (11)	12.96 (2.36)
	VR 90	3731.2 (659)	43.8 (12)	45.2 (10)	27.03 (1.63)
	FI 50	1795.3 (455)	46.8 (4)	47.8 (9)	19.40 (1.56)
	VR 180	3764.5 (410)	20.8 (4)	47.7 (9)	20.92 (1.99)
	FI 50	1448.7 (762)	47.0 (2)	5.5 (7)	39.24 (1.47)
	VR 360	38.0 (78)	.2 (1)	4.8 (6)	0.79 (.98)
	FI 100	597.2 (273)	22.8 (2)	31.0 (9)	21.16 (2.71)
	VR 90	1826.0 (546)	20.7 (7)	30.3 (8)	19.35 (2.53)
	FI 200	456.3 (206)	10.5 (1)	25.7 (10)	10.14 (4.05)
	VR 90	3251.0 (1127)	36.7 (13)	25.0 (10)	30.67 (6.57)
	FI 300	260.8 (163)	6.5 (1)	15.7 (5)	4.78 (1.71)
	VR 90	5544.5 (2196)	62.0 (25)	15.5 (5)	38.56 (8.36)
	FI 300	350.3 (121)	7.2 (2)	19.7 (8)	5.84 (2.27)
	VR 180	4664.7 (935)	26.5 (7)	19.3 (9)	35.19 (6.99)
	FI 100	680.2 (126)	23.5 (1)	29.0 (5)	20.43 (3.34)
	VR 180	2740.7 (905)	16.0 (9)	28.5 (6)	19.45 (5.91)
	FI 200	612.0 (125)	11.8 (1)	28.7 (8)	14.19 (3.14)
	VR 180	3510.2 (731)	19.8 (8)	28.2 (7)	26.80 (4.41)
RAT 103	FI 50	2.8 (11)	0.0 (0)	1.5 (6)	.17 (0.55)
	VR 30	5808.0 (544)	192.5 (22)	1.8 (7)	59.84 (0.55)
	FI 50	251.2 (118)	23.8 (7)	27.3 (5)	6.27 (3.36)
	VR 90	3898.5 (697)	44.0 (8)	27.3 (5)	34.24 (3.18)
	FI 50	549.3 (169)	41.0 (5)	43.7 (11)	14.34 (2.22)
	VR 180	2922.0 (573)	16.7 (7)	43.7 (11)	26.01 (2.24)
	FI 50	401.8 (180)	45.3 (1)	8.3 (7)	38.81 (1.02)
	VR 360	17.3 (11)	0.0 (0)	8.3 (7)	1.27 (1.17)
	FI 100	230.2 (100)	19.5 (5)	25.0 (9)	19.09 (4.65)
	VR 90	1589.5 (473)	17.2 (7)	25.0 (9)	22.16 (3.23)
	FI 200	115.5 (28)	9.3 (2)	19.5 (3)	9.19 (2.64)
	VR 90	3124.5 (361)	34.2 (11)	19.5 (3)	33.32 (5.38)
	FI 300	53.7 (13)	5.5 (2)	9.8 (4)	5.25 (1.82)
	VR 90	3888.3 (820)	44.2 (14)	9.8 (4)	38.62 (11.21)
	FI 300	107.5 (113)	7.7 (1)	16.7 (18)	21.16 (5.39)
	VR 180	1511.5 (783)	8.8 (9)	16.7 (18)	22.49 (7.07)
	FI 300	49.7 (21)	6.3 (1)	10.2 (8)	4.43 (2.63)
	VR 90	3574.5 (1036)	40.8 (9)	10.2 (8)	37.26 (5.83)
	FI 100	182.7 (60)	20.0 (2)	20.7 (2)	18.34 (5.20)
	VR 180	2115.3 (879)	12.0 (3)	20.7 (2)	21.84 (6.18)
	FI 200	224.0 (51)	11.2 (1)	25.7 (10)	17.80 (6.51)
	VR 180	2258.5 (756)	13.0 (3)	25.7 (10)	23.35 (7.91)
RAT 104	FI 50	242.3 (136)	16.3 (3)	21.3 (12)	4.07 (2.14)
	VR 30	4079.5 (986)	135.0 (30)	21.3 (12)	36.84 (4.60)
	FI 50	1400.7 (289)	45.8 (2)	55.0 (7)	17.43 (2.90)
	VR 90	2964.5 (466)	30.2 (3)	55.0 (7)	22.69 (3.00)
	FI 50	1676.5 (198)	45.8 (1)	51.7 (10)	19.66 (1.11)
	VR 180	2371.5 (253)	13.5 (2)	51.7 (10)	20.16 (1.27)
	FI 50	1487.2 (611)	47.7 (1)	52.2 (7)	27.51 (2.48)
	VR 360	1125.8 (211)	4.2 (5)	52.2 (7)	13.07 (2.33)
	FI 100	1081.0 (110)	24.0 (0)	37.3 (8)	20.67 (3.03)
	VR 180	1572.3 (485)	9.3 (4)	37.3 (8)	19.54 (2.91)
	FI 200	689.0 (296)	11.8 (1)	49.3 (18)	14.74 (3.15)
	VR 180	2420.0 (681)	14.5 (6)	49.3 (18)	26.88 (5.71)
	FI 300	520.5 (326)	7.8 (1)	36.5 (7)	11.66 (3.59)
	VR 180	2853.0 (1007)	16.2 (7)	36.5 (7)	29.96 (5.13)

	<i>Responses</i>	<i>Reinforcements</i>	<i>Changeovers</i>	<i>Minutes</i>	
	FI 300	287.2 (118)	7.2 (2)	29.0 (9)	5.46 (2.50)
	VR 90	3749.2 (1170)	42.8 (12)	29.0 (9)	36.94 (3.81)
	FI 100	753.0 (348)	22.5 (2)	29.0 (4)	14.94 (3.19)
	VR 90	2857.7 (474)	30.3 (11)	29.0 (4)	25.38 (3.27)
	FI 200	305.3 (196)	10.0 (2)	26.8 (10)	5.81 (3.98)
	VR 90	4968.5 (1131)	57.8 (8)	26.8 (10)	35.58 (3.52)
RAT 105	FI 50	78.8 (37)	11.8 (6)	14.2 (9)	2.84 (2.11)
	VR 30	7935.8 (2776)	263.5 (94)	14.3 (9)	44.73 (16.20)
	FI 50	484.0 (201)	33.8 (5)	36.5 (5)	7.73 (1.25)
	VR 90	5514.0 (768)	64.2 (6)	36.5 (5)	32.60 (2.74)
	FI 50	1110.5 (318)	46.5 (1)	43.3 (6)	20.71 (3.93)
	VR 180	2742.8 (976)	15.3 (5)	43.3 (6)	19.65 (3.56)
	FI 50	1471.7 (166)	47.8 (2)	19.7 (9)	25.83 (4.24)
	VR 360	1736.0 (409)	5.3 (4)	19.7 (9)	14.46 (4.57)
	FI 100	903.3 (299)	23.5 (1)	27.3 (4)	17.71 (2.03)
	VR 180	2813.3 (415)	16.3 (1)	27.3 (4)	22.83 (1.32)
	FI 200	658.3 (189)	12.0 (0)	21.2 (7)	14.79 (5.36)
	VR 180	3313.2 (967)	18.7 (6)	21.0 (7)	26.35 (6.63)
	FI 300	520.7 (404)	7.8 (1)	29.5 (14)	11.05 (2.14)
	VR 180	4155.7 (1037)	22.5 (10)	29.5 (14)	30.64 (1.09)
	FI 300	171.5 (70)	6.5 (1)	18.0 (8)	4.86 (2.39)
	VR 90	5049.2 (1279)	57.2 (8)	18.0 (8)	36.77 (2.85)
	FI 100	382.0 (145)	19.7 (4)	34.7 (10)	10.00 (3.26)
	VR 90	4286.7 (704)	49.5 (9)	34.7 (10)	31.00 (2.20)
	FI 200	282.8 (185)	10.5 (2)	32.0 (5)	7.86 (5.14)
	VR 90	4051.2 (1350)	45.0 (20)	32.0 (5)	32.49 (5.18)
	FI 300	463.3 (154)	7.8 (1)	43.8 (15)	13.26 (8.78)
	VR 360	3739.3 (2389)	11.0 (11)	43.8 (15)	27.73 (11.33)
RAT 106	FI 50	896.2 (229)	38.5 (8)	38.7 (16)	16.82 (2.12)
	VR 30	2206.5 (544)	72.2 (18)	38.5 (16)	23.31 (2.47)
	FI 50	1316.7 (445)	47.0 (3)	41.3 (9)	20.20 (2.69)
	VR 90	2423.5 (304)	27.3 (7)	41.3 (9)	19.93 (3.41)
	FI 50	910.7 (414)	47.2 (2)	5.3 (3)	39.72 (0.74)
	VR 180	37.5 (55)	0.0 (0)	5.3 (3)	.70 (0.65)
	FI 100	994.7 (306)	24.2 (1)	13.8 (9)	32.38 (4.95)
	VR 180	296.0 (209)	1.3 (4)	13.8 (9)	8.42 (4.77)
	FI 200	384.2 (366)	12.2 (1)	6.0 (8)	33.04 (6.21)
	VR 180	44.3 (52)	.2 (1)	6.0 (8)	5.91 (6.58)
	FI 300	258.5 (205)	7.3 (2)	20.5 (11)	8.30 (8.51)
	VR 60	1986.8 (418)	33.7 (8)	20.5 (11)	33.77 (6.95)

(FI/VR) obtained from the last six sessions in each condition. The solid lines through the data points were derived by the method of least squares, and the dashed lines represent perfect matching. Again, the slopes were all less than one and the intercepts were negative. The least-squares linear regression equation relating time ratios to reinforcement ratios for the group is:

$$y = .71x - .24; r^2 = .927.$$

The slope of the line fitting response ratios to reinforcement ratios was closer to one than that of the line fitting time ratios to reinforcement ratios for each subject; the intercept of the line fitting time ratios to reinforcement ratios was closer to zero than that fitting response ratios to reinforcement ratios. That is,

response ratios produced greater sensitivity to reinforcement ratios but greater bias in favor of the VR than did time ratios for each subject.

Staddon and Motheral (1978) showed that the relation between ratio and interval response rates under Bacotti's (1977) conc FI VR schedules was approximately linear. Data were fit to the equation

$$x + cy = k \quad (4)$$

where x and y are response rates under the interval and ratio schedules, respectively. Staddon and Motheral found that the mean value of c was .40 for Bacotti's data, favoring the ratio schedule, and the median r^2 was .92, with a range of .14 to 1.00. Data from the present experiment do not fit this linear model well. The mean value of c for the five subjects was

Fig. 1. Logarithms of the ratios of responses under concurrent FI VR plotted as a function of the logarithms of the ratios of reinforcements obtained from the schedules. Data are from the last six sessions of each condition, excluding conditions in which all reinforcements were obtained from one schedule. The solid lines represent the least-squares linear regressions, and the corresponding equations are given for each rat. The dashed lines represent perfect matching—slopes of 1 and intercepts of 0.

Fig. 2. Logarithms of the ratios of time spent responding under concurrent FI VR plotted as a function of the logarithms of the ratios of reinforcements obtained from the schedules. Data are from the last six sessions of each condition, excluding conditions in which all reinforcements were obtained from one schedule. The solid lines represent the least-squares linear regressions, and the corresponding equations are given for each rat. The dashed lines represent perfect matching—slopes of 1 and intercepts of 0.

Fig. 3. Number of responses to each schedule plotted as a function of the time allocated to each schedule, averaged over the last six sessions of each condition. The solid lines represent the least-squares linear regression, and the corresponding equations are given for each rat.

.16, again favoring the ratio schedule, but the median r^2 was only .19, with a range of .05 to .74.

Figure 3 displays the number of responses to each schedule plotted with respect to the time allocated to that schedule and averaged over the last six sessions for each condition. Local response rates (responses to a schedule divided by time allocated to that schedule) generally were higher on the VR than on the FI for all subjects, as indicated by the steeper slopes for the VR data. Also, local response rates were more constant across conditions on the VR than on the FI for most subjects.

Patterns of Responding

When responding was maintained exclusively by the VR, high rates and little or no pausing after reinforcement occurred. When responding was maintained exclusively by the FI, lower rates and postreinforcement pauses of about half to two-thirds of the fixed interval were typical. Positively accelerated patterns of responding within interreinforcement intervals were evident occasionally for most subjects, but a break-run pattern more often prevailed.

Although scalloped patterns of responding, in contrast to break-run, rarely occurred under the FI, temporal control of responding by the FI was evident for all subjects. Generally, the probability that responding on the VR would be replaced by responding on the FI increased with elapsed time since the previous FI reinforcement. Figure 4 shows response rates in successive tenths of the scheduled fixed interval, averaged over the last six sessions in a condition. Responses on each schedule were recorded in ten separate bins corresponding to successive tenths of the fixed interval. Average response rates for each bin were calculated by dividing the total number of responses on each schedule in each bin during the last six sessions in a condition by the total time represented by each bin (that is, one-tenth of the FI value times the number of FI reinforcements during those six sessions). Time in the tenth bin includes one-tenth of the scheduled fixed interval plus any differences between the time FI reinforcement was available and the time FI reinforcement was obtained. The conditions for which data are displayed in Figure 4 were selected to show typical changes in responding as a function of changes in the parameters of the FI and VR

components, where some responding was maintained by each component.

In each panel of Figure 4, response rate on the FI increased with elapsed time since the previous FI reinforcement. Response rate on the VR, however, increased with elapsed time since FI reinforcement early in the interval and then decreased as FI reinforcement became imminent.

Figure 4 shows that the interaction between FI and VR responding was largely independent of the parameters of the FI and VR components. The panels for Rats 102 and 103, for example, are for conditions in which VR 180 was in effect and the FI value was varied. Terminal FI response rates were lower with longer FI values, and VR response rates declined more rapidly with shorter FI values. However, in each condition, FI response rate generally increased and VR response rate approached an asymptote and then decreased with elapsed time since FI reinforcement.

When responding was maintained well by both schedules (arbitrarily, when the proportion of responses on either schedule did not exceed 90%), the absolute rate of COs generally decreased as the value of the FI increased (see Table 2). The ratio of COs to FI reinforcements, however, generally increased as the FI value increased. A period of no responding on either lever after FI reinforcement typically was followed by high-rate responding on the VR. With short FI values, a CO often did not occur until the entire fixed interval had elapsed, and responding usually continued on the FI until reinforcement occurred. As the FI value increased, COs before the fixed interval had elapsed became more frequent. When FI reinforcement was not forthcoming within a few seconds, a CO back to the VR occurred, and this pattern of alternation continued until FI reinforcement finally was obtained.

DISCUSSION

The present study supplied data from several conc FI VR schedules. Relative rates of responding and allocation of time on one schedule increased as the requirement of the other schedule increased. Equation 3 describes the relation between reinforcement ratios obtained from the schedules and response and time ratios. All subjects undermatched with re-

Successive Tenths of the Fixed Interval

Fig. 4. Averaged response rates on each schedule in successive tenths of the fixed interval over the last six sessions in a condition. See text for details.

spect to both response and time allocation, and all subjects displayed bias in favor of the VR schedules. Response ratios produced greater sensitivity to reinforcement ratios than did time ratios, but with greater bias in favor of the VR. Although these differences were fairly small, they were consistent across all subjects. Matching occurs most readily under conc VI VI (cf. de Villiers, 1977; Herrnstein, 1970; but see Baum, 1974, 1979, and Myers & Myers, 1977 for qualifications), in which little temporal control over responding is exerted, and in which local response rates are nearly equal (Catania, 1966). Baum (1974) noted that one possible source of bias is the use of quali-

tatively different reinforcement schedules. Previous research is consistent with this contention, as bias has been observed whenever qualitatively different schedules have been arranged concurrently. Baum later (1979) suggested that a possible source of undermatching is the inconstancy of preference for a schedule which provides reinforcement at fixed intervals. Response and time ratios have approximated reinforcement ratios, although with considerable bias, under qualitatively different schedules that did not include FI (conc VI VR: Herrnstein & Loveland, reported by de Villiers, 1977; and conc FR VI: Bacotti, 1977), but undermatching has occurred whenever FI has

been paired with another schedule (conc FI VI: Lobb & Davison, 1975; Nevin, 1971; Trevellett et al., 1972; conc FI FR: LaBounty & Reynolds, 1973). The present study adds conc FI VR.

The VR schedules generated high response rates in the present study with virtually no temporal control exerted by VR reinforcement. The FI schedules, by contrast, generated much lower response rates and FI reinforcement exerted considerable temporal control over both FI and VR responding, as Figure 4 indicates. Responding on the FI generally did not occur within the first half of the fixed interval; responding on the VR usually reached its highest rate within the first half of the fixed interval.

Schneider (1969) described FI performance as two state, in which little or no responding occurs during the first half to two-thirds of the fixed interval, followed by comparatively high response rates until reinforcement is delivered. Because response and reinforcement probabilities are nearly zero early in the fixed interval, the FI may function like a multiple schedule in which extinction (EXT) alternates with VI reinforcement. Baum (1979) extended this analysis to conc FI VI, which may function like a multiple schedule in which conc EXT VI alternates with conc VI VI. Baum suggested that this characterization might account for undermatching obtained under conc FI VI:

"If, following each FI reinforcement, extra time and responses were given to the VI alternative, then the poorer the VI relative to the FI, the more performance will deviate from matching in apparent favor toward the VI. A line fitted to the choices (FI relative to VI) will lie below the matching line, but converging on it as the VI becomes more favorable relative to the FI" (1979, p. 278).

Baum's analysis can be extended to conc FI VR, and the data from the present study are consistent with such an analysis: the lines fitting response and time ratios to reinforcement ratios (Figures 1 and 2) lie below the matching lines for all subjects, but converge on the matching lines as the VR becomes more favorable relative to the FI.

This analysis is given further credence by data from conc FI FI (White & Davison, 1973). Matching was observed when both schedules generated similar patterns of responding, whether response rates between successive re-

inforcements were constant or positively accelerated. However, when different patterns were generated by the schedules—that is, when temporal control was exerted by only one schedule—undermatching was observed.

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