FEEDBACK FUNCTIONS FOR VARIABLE-INTERVAL REINFORCEMENT

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On a given variable-interval schedule, the average obtained rate of reinforcement depends on the average rate of responding. An expression for this feedback effect is derived from the assumptions that free-operant responding occurs in bursts with a constant tempo, alternating with periods of engagement in other activities; that the durations of bursts and other activities are exponentially distributed; and that the rates of initiating and terminating bursts are inversely related. The expression provides a satisfactory account of the data of three experiments.

Key words: variable interval, feedback function, correlation, contingency, molar analysis, reinforcement

Operant behavior, by its very nature, must be viewed as part of a feedback system. It affects the environment by producing certain experimentally arranged consequences, and is, in turn, affected by them. Consequences that increase the rate, probability, or "strength" of responding are termed reinforcers. Schedules of reinforcement are rules for presenting reinforcers in relation to responses. A variable-interval (VI) schedule is defined by a rule specifying that a reinforcer will follow the first response to occur after the passage of an irregularly varying time since the last reinforcer. VI schedules are widely employed in behavioral research because they establish stable yet sensitive baselines for the study of processes such as stimulus control and conditioned suppression. They have also been used extensively in parametric studies of the dependence of rate of responding on rate of reinforcement.

On VI schedules, the average rate of responding appears to be a monotonic increasing, negatively accelerated function of the rate of reinforcement, well approximated by the simple equation proposed by Herrnstein (1970):

$$B = \frac{k r}{r + r_o} \quad (1)$$

where B is the rate of responding, r is the rate of reinforcement obtained by the measured response, r_o is a parameter representing the rate of other, unspecified reinforcement for behavior other than the measured response, and k is the sum of all possible behavior in units of the measured response. The relevant research has been reviewed by de Villiers and Herrnstein (1976).

Equation 1 specifies the dependence of rate of responding on rate of reinforcement, which seems to operate as an independent variable. In fact, of course, it is not independent of behavior, because reinforcers are contingent upon responding. A complete understanding of operant behavior requires specification of the dependence of the rate of reinforcement on the rate of responding—the other half of the feedback loop.

Although the rules for presenting reinforcers may be stated with precision, the molar relations between rate of reinforcement and rate of responding implied by these rules are not always obvious. For ratio schedules, in which reinforcers are presented after some fixed or variable number of responses, the molar feedback function for reinforcement is straightforward. For all ratio schedules, the average rate of reinforcement is directly proportional to the average rate of responding:

$$r = cB \quad (2)$$

where r and B are defined as above, and the constant of proportionality, c , is the reciprocal

Preparation of this article was supported in part by NSF Grant BNS 76-11028 to John A. Nevin, and NSF Grant BNS 79-06852 to William M. Baum. Reprints are available from either author at the Department of Psychology, University of New Hampshire, Durham, New Hampshire 03824.

of the ratio. For continuous reinforcement, it is 1.0; for fixed-ratio or variable-ratio 50, it is .02; etc. For interval schedules, the feedback function is less obvious. Hypothesized feedback functions for VI schedules have figured importantly in recent theoretical papers on concurrent operants (e.g. Heyman & Luce, 1979; Staddon & Motheral, 1978) and in treatments of the differences between performances on interval and ratio schedules (Rachlin, 1978; Staddon, 1975, 1979; Baum, Note 1). The large number of experiments using VI schedules and the significance of their data in current theoretical work make it especially important to derive a feedback function for VI schedules from some general properties of operant behavior and to test it against empirical data.

The feedback function for VI reinforcement is constrained to pass through the origin (i.e., if the rate of responding is zero, the rate of reinforcement must also be zero), and to approach the scheduled rate of reinforcement asymptotically as response rate increases to high values.

Several researchers have recently suggested equations with these properties for the VI feedback function. Rachlin (1978) proposed a power function:

$$r = aI^m \quad (3)$$

where I is the proportion of session time spent responding, and a and m are parameters reflecting the maximum rate of reinforcement and the rate of approach to maximum, respectively. This form has the desired properties if m is less than 1.0. However, the power function form would not be appropriate for response rate, as opposed to proportion of time, because it implies that reinforcement rate could increase indefinitely as response rate goes up. In an appendix, Rachlin (1978) derived an approximation to a power function by assuming that subjects respond in regularly alternating bursts and pauses. The power function, however, is best viewed as a rough description, useful because of its simplicity.

Other equations based on behavioral considerations have also been proposed. Baum (1973a) suggested that the obtained rate of reinforcement depended on the rate of responding according to the function

$$r = \frac{1}{t + .5\left(\frac{1}{B}\right)} \quad (4)$$

where r and B are defined as above and t is the average scheduled interval between reinforcers. The meaning of this equation is clearest if both sides are inverted:

$$\frac{1}{r} = \frac{1}{t} + .5\left(\frac{1}{B}\right) \quad (5)$$

The left side of the equation, $1/r$, is the average time between reinforcers actually obtained. It equals the average time from presentation of one reinforcer until availability of the next, plus the delay from then until the next response. Baum's (1973a) estimate of this delay was half the average interresponse time, $1/B$. This estimate is approximately correct if reinforcement is scheduled randomly in time, and responses occur at a high rate (relative to reinforcement rate) with no variation in the interresponse time.

Staddon and Motheral (1978) assumed that responses are randomly distributed in time, implying that at any instant the expected time to the next response is simply the reciprocal of the average response rate. Their feedback function, then, is

$$r = \frac{1}{t + \frac{1}{B}} \quad (6)$$

with the symbols defined as above. The form in which they stated their feedback function was

$$r = \frac{aB}{a + B} \quad (7)$$

where a is the scheduled rate of reinforcement. Substituting $1/t$ for a reveals the equivalence of the two forms.

Prelec and Herrnstein (1978) advanced a feedback function similar to those of Baum (1973a) and Staddon and Motheral (1978):

$$r = \frac{aB}{B + \alpha a} \quad (8)$$

where α is a parameter representing the extent to which responding departs from randomness. If α is .5, as for Baum (1973a), responses are evenly spaced; if α is 1.0, as for Staddon and Motheral (1978), responses are randomly distributed. Values of α greater than 1.0 would correspond to departures from random responding attributable, for example, to the changeover delay in concurrent schedules.

If the value of α is to remain constant when average response rate changes, the form of the distribution of interresponse times (IRTs) must remain constant while its mean changes. If α is less than 1.0, the stereotyped modal IRT must increase as response rate decreases; if α is 1.0, the time constant of the exponential distribution of IRTs must change; and if α is greater than 1.0, suggesting more long IRTs than expected under an exponential distribution, the relative number of long and short IRTs must remain constant while the lengths of both increase. In fact, data from a variety of studies in which average response rates changed while the contingencies of reinforcement remained constant suggest that modal IRTs remain fairly stable while long IRTs change disproportionately. For example, cumulative records of responding on VI schedules at various body weights presented by Ferster and Skinner (1957, p. 367) often suggest a fairly constant running rate, with the amount of irregular responding and pausing increasing as body weight increased. Likewise, IRT distributions for responding on unchanged VI schedules in studies of behavioral contrast often show little change except in long IRTs (e.g., Spealman & Gollub, 1974, Fig. 4; Marcucella & MacDonall, 1977, Fig. 2). A feedback function should reflect these aspects of free-operant behavior.

The Model

We now outline a simple model of behavior that leads to a new feedback function. In essence, the model combines Baum's (1973a) suggestion that responses are regularly spaced with Staddon and Motheral's (1978) assumption that responding is randomly distributed in time. We conceive of free-operant behavior as consisting of episodes or bursts of responding, alternating with periods of engagement in other behavior. During a burst, responses occur at a constant tempo; that is, interresponse times are approximately constant. Thus, the overall average rate of responding varies only when the proportion of time allocated to bursts varies.

We assume that while the subject is responding, there is a constant probability of terminating the burst, with the result that the probability density function for burst duration is exponential. Likewise, given that the subject is not responding, there is a constant proba-

ity of initiating a burst, with the result that pause duration is also exponentially distributed. Although there are no forced constraints on these probabilities, we assume the probabilities of initiating and terminating bursts to be inversely related. Thus, if the probability of initiating a burst is high, the probability of terminating it is low, and the subject spends most of its time responding. Conversely, if the probability of initiating a burst is low, the probability of terminating it is high, and the subject spends most of its time engaged in other activities.

There is both theoretical precedent and empirical support for these ideas. Gilbert (1958) proposed that operant behavior consists in alternation between responding at a constant tempo and engagement in other activities, and presented some supporting data for rats and humans. Premack and his associates (e.g., Premack, 1965; Premack & Schaeffer, 1962) have argued that freely occurring, recurrent activities such as running, eating, and drinking in rats occur in bursts at a constant speed or tempo within bursts. More recently Pear and Rector (1979) have reported evidence of constant tempo in pigeons' key pecking.

Blough (1963) and Schaub (1967) have reported many examples of interresponse time distributions for pigeons that show sharply peaked modes at short interresponse times, which remain invariant when average response rate changes during extinction or stimulus generalization testing. Both researchers concluded that changes in average response rate result from changes in long interresponse times—in our terms, the duration of engagement in other activities.

The view that operant behavior is composed of alternating periods of responding and other activity is in accord with Herrnstein's (1970, 1974) formulation of the law of effect. Herrnstein assumes that behavior is partitioned among various classes of action in proportion to the rates of reinforcement they produce. For the conventional single operant situation, this notion reduces to partitioning behavior between the experimentally defined and reinforced class and all other activities, where the latter are maintained by unmeasured reinforcers. In effect, Herrnstein's formulation treats the single operant as a case of concurrent reinforcement of responding and not responding, and assumes that it has the same

properties as concurrent reinforcement of two (or more) experimentally specified responses. Extending Herrnstein's approach, we assume that the subject alternates between responding and not responding in the same way that it would alternate between two explicitly reinforced concurrent responses. Recently, Heyman (1979) has shown that the probability of switching from one concurrent operant to the other is constant and independent of the number of responses emitted before switching. If responses occur at a steady rate, a constant-probability process of this sort implies an exponential distribution of times between switches from one activity to another, as we assumed above.

Research on concurrent schedules is also consistent with the assumption of an inverse relation between the probabilities of initiating and terminating bursts. Suppose, contrary to our assumption, that the probability of terminating a burst remains constant, so that average burst length remains constant. Then, average response rate can change only if the duration of pausing changes. Low average response rates arise when pauses are long and, consequently, bursts are infrequent, high rates arise when pauses are short and bursts are frequent. Thus, there must be a monotonic increasing relation between average response rate and rate of alternating between bursts and pauses, up to the point at which bursts occupy the entire session. The assumption of constant pause length implies instead a decreasing relation over the same range. However, in concurrent schedules, changeover rate is maximal when responding is equally distributed between alternatives and decreases as responding to either alternative approaches exclusive preference (e.g., Baum, 1976; Stubbs & Pliskoff, 1969). As we will show formally below, the assumption of an inverse relation between the probabilities of initiating and terminating bursts implies a relation between alternation rate and average response rate similar to that between changeover rate and preference in concurrent performance.

With this background, we can develop our feedback function more formally. The rate of reinforcement obtained on a VI schedule may be construed as the sum of two rates of reinforcement: one for reinforcers that become available during bursts, calculated over the duration of bursts, and one for reinforcers

that become available during pauses and are presented when the next burst begins, calculated over the duration of pauses. Assuming that reinforcers are scheduled every t min on the average, and that a constant tempo of k responses per min prevails during bursts, the rate of reinforcement for reinforcers that become available during bursts is approximately

$$\frac{1}{t + 1/2k},$$

because $1/2k$ approximates the average delay from availability to the occurrence of the next response (see discussion of Equation 5). Assuming that pauses are exponentially distributed with average length $1/x$, the rate of reinforcement for reinforcers that become available during pauses is

$$\frac{1}{t + 1/x},$$

because $1/x$ is the average delay from availability to the initiation of the next burst (see discussion of Equation 6). Weighting these two rates of reinforcement by the proportions of session time devoted to bursts or pauses, we obtain

$$r = p\left(\frac{1}{t + 1/2k}\right) + (1 - p)\left(\frac{1}{t + 1/x}\right) \quad (11)$$

where p is the proportion of session time spent responding.

Equation 11 is essentially identical to the feedback function for one alternative of independent concurrent VI VI schedules derived by Heyman and Luce (1979).¹ Like ourselves, they assumed exponentially distributed times between switches from one alternative to the other. Their expression for alternative 1 is

$$E(R_1) = \frac{p}{V_1} + \frac{1 - p}{V_1 + 1/\mu_2} \quad (12)$$

where $E(R_1)$ is the expected rate of reinforcement, V_1 is the average scheduled time between reinforcers, p is the proportion of time spent responding on alternative 1, and $1/\mu_2$ is the average time spent at alternative 2 between switches. Their equation differs from ours

¹We are indebted to Gene Heyman for pointing out the relations between the Heyman-Luce (1979) formulation and our approach in his comments on an earlier version of this paper.

only in that they do not consider within-burst delays, $1/2k$, which are negligible when k is large. Time spent responding to the other defined alternative in the Heyman-Luce formulation corresponds to time spent engaged in other activities in our formulation. Even in concurrent performance, however, there may be periods of time spent responding to neither alternative. Application of our model to both alternatives of independent concurrent schedules would permit explicit inclusion of such time. For the present, however, we confine ourselves to the single-response case.

We defined the average delay introduced by periods of not responding as $1/x$ to make its similarity to the delay term in Equation 6 obvious. This definition implies that x is the rate of initiating bursts, given that the subject is pausing. Stated as a rate, it is defined as the number of burst initiations divided by time spent pausing. We now define y as the rate of initiating pauses, given that the subject is responding; that is, the number of pause initiations divided by time spent responding. We consider two possibilities suggested by the notion that x and y are inversely related:

- 1) $x + y = c$
- 2) $x \cdot y = c$

where c is a constant representing the overall tendency to alternate between responding and not responding. The first possibility can be called the *additive* model, the second the *multiplicative* model. Both have the property that the number of switches per unit time is maximal when $x = y$, and the subject spends exactly half of its time responding (see Appendix for proof). As noted above, this property corresponds to the empirical finding that the average changeover rate is maximal when responses are equally distributed across alternatives in two-key concurrent schedules (Baum, 1973b, 1974, 1975, 1976; Stubbs & Pliskoff, 1969).

In both cases, the average rate of responding, B , is equal to pk , so $p = B/k$ and $1 - p = (k - B)/k$. As stated above, we assume k to be constant; thus, B can change only when p changes. We now derive the relations between r and B expected under the additive and multiplicative models.

For the additive model, $x = pc = Bc/k$. Substituting into Equation 11, we obtain

$$r = \frac{B}{k} \left(\frac{1}{t + 1/2k} \right) + \left(\frac{k - B}{k} \right) \left(\frac{1}{t + k/Bc} \right) \quad (13)$$

For the multiplicative model, $p = x^2/(c + x^2)$, so that $x = \sqrt{pc/(1 - p)} = \sqrt{Bc/(k - B)}$. Substituting into Equation 11, we obtain

$$r = \frac{B}{k} \left(\frac{1}{t + 1/2k} \right) + \left(\frac{k - B}{k} \right) \left(\frac{1}{t + \sqrt{\frac{k - B}{Bc}}} \right) \quad (14)$$

Figure 1 presents illustrative curves from Equations 13 and 14 along with the curves given by the simpler Equations 3 and 6. For all four equations, we assumed that t , the scheduled interreinforcement interval, was 2 min, and that k , the tempo (or the maximum overall response rate) was 100 responses per min. Equation 6 has no other parameters. For Equation 3, we let $I = B/k$, $a = 1/t$, and $m = .1$. We assumed $c = 10$ for Equation 13, and $c = 3$ for Equation 14. These various param-

Fig. 1. Illustrative plots of four feedback functions, assuming a VI 2-min schedule of reinforcement with overall response rate varying from 100 to .1 responses per min. The curve labelled POW represents Rachlin's (1978) power function, with an exponent of .1, which plots as a straight line in these double-logarithmic coordinates. The curve labelled HYP represents Staddon and Motheral's (1978) hyperbolic function, which assumes that interresponse times are exponentially distributed. The curves labelled ADD and MUL represent the additive and multiplicative models advanced in this paper. Both are derived from the assumptions that responding occurs in bursts of constant tempo alternating with pauses; that both burst length and pause length are exponentially distributed; and that the probabilities of initiating and terminating bursts are inversely related.

eter values were chosen to illustrate the similarities of the four functions at high response rates, and their increasing divergence at low response rates. The use of logarithmic scales makes the divergence at low response rates especially clear.

EVALUATION OF THE MODELS

Empirical feedback functions may be obtained from experiments in which responding is reinforced according to some constant schedule, while the rate of responding is caused to vary by some variable other than the reinforcement schedule. Three such studies are described below. For each study, the data were transformed to logarithms and compared with the predictions of Staddon and Motheral's (1978) simple hyperbolic function which assumes random responding in time (Equation 6), and with our additive and multiplicative models, Equations 13 and 14. Our equations have two free parameters: k , the tempo of responding within bursts, and c , the tendency to alternate between bursts and pauses. B and r are taken from the data, and t from the specification of the reinforcement schedule. In some cases, it was necessary to estimate the value of t , because some reported rates of reinforcement were higher than the maximum permitted by the nominal schedule value. For each of the studies to be described, a value of k was selected slightly higher than the *highest* average response rate exhibited by any subject. We then determined, for Equations 13 and 14, the value of c that maximized the proportion of variance explained.

McSweeney (1975)

McSweeney (1975) trained pigeons on concurrent VI VI schedules of food reinforcement until their performances were stable, and then varied their body weights. She reported both rates of responding and rates of reinforcement obtained at various body weights for individual subjects on *conc* VI 30-sec VI 2-min, *conc* VI 1-min VI 4-min, *conc* VI 1.5-min VI 6-min, and *conc* VI 2-min VI 8-min schedules. A 2.5-sec changeover delay was in effect throughout the experiment. The data were averaged for 10 sessions (400 reinforcers). Only the VI 2-min and VI 4-min schedules provided data for all subjects over a sufficient range of response and reinforcement rates to permit

characterization of the empirical feedback functions.

Inspection of McSweeney's (1975) Table 1 suggested that her schedules sometimes arranged a higher rate of reinforcement than the nominal schedule value would permit. To estimate the correct value of t , we plotted the reciprocal of reinforcement rate as a function of the reciprocal of response rate in double-logarithmic coordinates, as shown in Figure 2. In this form, t is the horizontal asymptote as the reciprocal of response rate becomes very small. It appears that the data from the VI 2-min schedule approach a lower asymptote of

Fig. 2. The relation between the reciprocals of obtained reinforcement rates and response rates reported by McSweeney (1975), in double-logarithmic coordinates. The upper panel presents data obtained on a VI 2-min schedule, and the lower panel presents data obtained on a VI 4-min schedule, for all subjects and body weights. The horizontal lines represent asymptotes estimated by inspection.

Table 1

Parameter values, number of data points, and proportions of variance explained by Equations 13, 14, and 6.

Experiment	Nominal schedule value	t	k	N	c		Proportion of variance		
					Eq. 13	Eq. 14	Eq. 13	Eq. 14	Eq. 6
McSweeney (1975)	VI 2	2.0	100	25	6.2	2.0	.93	.79	-.56
	VI 4	3.2	100	19	6.2	2.0	.90	.70	-.79
Rachlin & Baum (1972)	VI 3	2.8	150	35	22	8	.88	.72	.30
Baum (1976)									
All rats	VI 2	2.0	70	70	8.3	4.0	.75	.69	-.79
Rat 1	VI 2	2.0	70	15	6.0	2.0	.93	.77	-.70

2 min, as they should. However, the VI 4-min data appear to approach an asymptote lower than the nominal value of 4 min. Since the data permit no sure definition of the asymptotic value of t , we guessed at 3.2 min as the lower limit toward which the data tend.

Those same data, expressed as response and reinforcement rates, are plotted in Figure 3 together with the best-fitting predictions of Equations 13 and 15. The parameter values of these equations are given in Table 1, together with the proportions of variance explained by these equations and by Equation 6. The additive model, Equation 13, provides a better fit to the data than do the other equations, with the same value of the parameter c for both schedules. Inspection of Figure 3 shows that the additive model does far better in approximating the curvature in the data. In view of this consistent curvature, we can rule out Rachlin's (1978) power function, which would plot as a straight line in these coordinates.² A negative proportion of variance explained by Equation 6 implies that this equation actually does worse than a horizontal line through the overall average rate of reinforcement.

Rachlin and Baum (1972)

Rachlin and Baum (1972) maintained key pecking by pigeons on a VI 3-min schedule, and arranged concurrent reinforcement at variable intervals either independently of re-

Fig. 3. The relation between obtained reinforcement rates and response rates reported by McSweeney (1975), in double-logarithmic coordinates. As in Figure 1, the upper panel presents data obtained on a VI 2-min schedule, and the lower panel presents data obtained on a VI 4-min schedule, for all subjects and body weights. The smooth curves are the best-fitting predictions of the additive and multiplicative models described in the text.

²Rachlin (1978) also examined McSweeney's (1975) data and concluded that they were approximated by a power function. However, Rachlin (1978, Fig. 8) plotted only the data of two pigeons from the VI 4-min schedule, and one of the key low-rate points was misplotted. Examination of the full data set makes their curvature in double-logarithmic coordinates evident.

sponding, or contingent upon the passage of 2 sec without a response. They found that response rates decreased systematically as the rate of concurrent reinforcement increased, and that the decrease was similar in the two procedures. Figure 4 shows rates of key pecks and peck-produced reinforcers for the final 10 sessions of training at each concurrent VI schedule, conditions IIIa and IIIb. We include all six birds, even though response rates for Subject 2 were exceptionally high and require a large value of k . Figure 4 shows also the best-fitting curves from Equations 13 and 14. The parameters are given in Table 1. Some rates of reinforcement exceeded the scheduled maximum and we estimated 2.8 min as an appropriate value of t . Once again, the additive model, Equation 13, provides a better fit to the data, although much of this superiority depends on the two data points at the lower left. Equation 6 gives a poorer fit than either Equation 13 or 14.

Baum (1976)

Baum (1976) trained rats in a two-lever chamber with *conc* VI VI schedules of milk reinforcement. One or the other lever was always correlated with a VI 2-min schedule, while the value of the second schedule was varied. Rate of responding on the lever correlated with the VI 2-min schedule was inversely related to the rate of reinforcement provided by the second schedule. Average re-

sponse and reinforcement rates were determined for stable performance on each schedule value for each subject; the number of sessions used varied from three to 31, with a mean of 11. The data for all five rats appear in the upper panel of Figure 5. Those for a single subject appear in the lower panel. Curves representing Equations 13 and 14 are also shown. By inspection, the data are noisier than those already presented. Nevertheless, both for combined data and for a single subject chosen for orderly data over a wide range of response and reinforcement rates, Equation 13 again provides a better fit than Equation 14. The parameters are given in Table 1.

Fig. 4. The relation between obtained reinforcement rates and response rates reported by Rachlin and Baum (1972) in double-logarithmic coordinates. The smooth curves are the best-fitting predictions of the additive and multiplicative models described in the text.

Fig. 5. The relation between obtained reinforcement rates and response rates reported by Baum (1976) in double-logarithmic coordinates. Data for all subjects are presented in the upper panel, and for a single subject in the lower panel. The smooth curves are the best-fitting predictions of the additive and multiplicative models described in the text.

It is evident from the graphs that the additional curvature of Equation 13 is important for its relatively greater success in accounting for the data. Again, Equation 6 performs poorly.

Baum (1976) also reported rates of changing over between levers, an aspect of performance which our model addresses. The parameter c represents the sum or product of the rate of burst initiation and the rate of burst termination, and changing over between bursts of responding and other activities should be most frequent when these two rates are equal (see Appendix). For the additive model, Equation 13, the maximum rate of alternation between responding and other activities is $c/2$, whereas for the multiplicative model, Equation 14, it is $\sqrt{c}$. In concurrent schedules, many of the changeovers from one reinforced response are switches to other reinforced responses. The additional changeovers to unmeasured activities go uncounted. Because the parameter c includes all changeovers, the recorded changeover rate should underestimate c . It should be less than or equal to $1/2$ of the estimated value of c for the additive model, or the square root of the estimated value of c for the multiplicative model. For Rat 1, the maximum changeover rate was 1.5 per min, well below half the value of c estimated according to the additive model, but slightly higher than the square root of the value estimated by the multiplicative model. The average maximum changeover rate for all five rats was 2.12 per min, clearly less than the value allowed by the additive model, but slightly higher than the value allowed by the multiplicative model. The additive model therefore suggests that a substantial number of changeovers to unmeasured activities occurred, a plausible result for rats lever-pressing at moderate rates, whereas the multiplicative model cannot accommodate even the number of changeovers between the levers.

DISCUSSION

We have presented a model of operant behavior that leads to a feedback function for variable-interval reinforcement. The model assumes that free-operant behavior occurs in episodes or bursts of responding at a constant tempo, alternating with periods of engagement in other activities. We assume further

that the rates of burst initiation and burst termination are inversely related, such that either their sum or their product is constant. Of these two possibilities, when tested against data, the additive model (Equation 13) is clearly superior.

The three experiments considered were relatively diverse in method. In McSweeney's (1975) experiment, key pecking by pigeons on independent concurrent schedules was caused to vary by changing body weight. In Rachlin and Baum's (1972) experiment, key pecking by pigeons was maintained by a single schedule, and caused to vary by arranging other sources of concurrent reinforcement. Finally, in Baum's (1976) experiment, lever pressing by rats on one of two concurrent schedules was caused to vary by changing the value of the second schedule. For each, the dependence of rate of obtained reinforcement on rate of responding was well described by our feedback function. The diversity of species, reinforcers, and methods for causing response rate to vary suggest that our proposed function may have considerable generality.

The two parameters of the equation, c and k , both have straightforward behavioral interpretations. Although c differs from rate of changing over between explicitly reinforced alternatives, it does have a direct algebraic relationship to it (see Appendix). Most important, it sets a theoretical upper limit on rate of changeover ($c/2$). Observed rates higher than this would falsify the model. Whereas Baum's (1976) reported rates of changeover fall well below the maximum predicted by Equation 13, the upper limit predicted for McSweeney's (1975) experiment (about 3 changeovers per min) would be unusually low, if correct.

The parameter k , the tempo of responding (response rate while responding), can be estimated from the first major mode of the distribution of interresponse times. In practice, however, its lower limit may be satisfactorily estimated from the maximum average rate of responding observed in the experiment.

The tempo and changeover parameters covary in the determination of best-fitting functions. For example, doubling the value of k leads to approximate doubling of the value of c with only slight decrement in proportion of variance explained by Equation 13. Further increases in k require progressively higher

values of c , with consequent flattening of the feedback function so that it no longer provides an acceptable account of the data at low response rates. For the future, more exact methods for estimating k may be devised, and careful observation may lead to improved methods for estimating c as well. Indeed, explicit measurement of bursts and pauses would permit direct tests of our assumptions.

Our conceptualization of operant behavior accords well with a number of previous observations and with current thinking about behavior in experiments on concurrent schedules. Our approach gains support from a feedback function for variable-interval reinforcement that accords well with a variety of data.

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Received October 8, 1979

Final acceptance April 8, 1980

APPENDIX

For both the additive and multiplicative models, the rate of changing over between bursts of responding and other activities is maximal when the subject spends half its time engaged in responding. The proofs follow:

a. Common Notation

Let x be the rate of burst initiation, y the rate of burst termination, p the proportion of time spent responding, and z the rate of changing over from responding to pausing or vice versa. The average pause duration is $1/x$, and the average burst duration is $1/y$. By definition,

$$p = \frac{1/x}{1/x + 1/y}.$$

The average length of a complete cycle of alternation between pausing and responding is $1/x + 1/y$. Because there are two changeovers per alternation cycle, $z = 2/(1/x + 1/y)$.

b. Additive Model

$x + y = c$, assumed.

Substituting for y in the expressions for p and z ,

$$p = x/c \text{ or } x = cp \quad (1A)$$

and

$$z = \frac{2(xc - x^2)}{c} \quad (2A)$$

Substituting for x in Equation 2A,

$$z = 2cp - 2cp^2 \quad (3A)$$

Differentiating with respect to p ,

$$dz/dp = 2c - 4cp \quad (4A)$$

Setting Equation 4A equal to zero and solving for the maximum, $p = 1/2$, and from Equation 3A, $z = c/2$.

c. Multiplicative Model

$x \cdot y = c$, assumed.

Substituting for y in the expressions for p and z :

$$p = \frac{x^2}{c + x^2} \quad (5A)$$

and

$$z = \frac{2cx}{c + x^2} \quad (6A)$$

Solving Equation 5A for x ,

$$x = \sqrt{\frac{cp}{1-p}} \quad (7A)$$

Substituting for x in Equation 6A and simplifying,

$$z = 2\sqrt{cp - cp^2} \quad (8A)$$

which, like Equation 3A, is maximal when $p = 1/2$. From Equation 8A, $z = \sqrt{c}$ at its maximum.