PRACTICAL IMPLICATIONS OF THE MATCHING LAW

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Many problem situations in applied settings are best conceptualized as choice situations. In addition, applied behavior analysts create choice situations when they reinforce a competing response to decrease inappropriate behavior. When such situations are analyzed using the matching law, variable interval (VI) schedules of reinforcement prove to be a superior intervention strategy regardless of the nature of the schedule maintaining other, less appropriate behavior. This conclusion is robust in that VI schedule superiority is observed in situations in which choice behavior is highly biased or shows pronounced undermatching as well as those in which the matching law holds precisely. Our analysis demonstrates the potential practical value of mathematical descriptions of behavior.

DESCRIPTORS: matching law, concurrent reinforcement schedules, operant choice behavior, basic and applied behavior analysis

The matching law (Herrnstein, 1961) describes operant choice behavior on concurrent schedules in which subjects can distribute their behavior between two or more simultaneously available schedules of reinforcement (see de Villiers, 1977, for a review). Results obtained with the concurrent schedules procedure are of importance for the analysis of human behavior because of the ubiquitous presence of concurrent schedules in the world outside the laboratory. Reinforcements are often available for more than one response class or from more than one source or both. Thus, concurrent schedules represent a more realistic laboratory analogue than do single reinforcement schedules, and the results of concurrent experiments may have correspondingly greater relevance to the applied setting. Even more important, the applied behavior analyst must frequently create concurrent schedules. Often reinforcement for inappropriate behavior cannot be eliminated, or extinction is inadvisable because of the aggressive or other emotional behavior that might be elicited (McDowell, 1981). Punishment may be ethically unacceptable or contraindicated

because of the behavioral by-products of aversive control (Hutchinson, 1977). The behavior analyst does, however, usually have the option of reinforcing a competing response. When this is done without completely eliminating reinforcement for inappropriate behavior a concurrent schedule situation is created.

Given that applied situations may often be analyzed as concurrent reinforcement schedules, how well established is the matching law as a description of operant choice behavior and especially of human behavior? Simply stated, the matching law asserts that an individual will distribute his or her behavior between alternatives in the same ratio that reinforcements have been obtained for these alternatives. That is, if there are two alternative behaviors, B1 and B2, and an individual has received twice as many reinforcements for alternative B1 as for alternative B2, then that individual will allocate twice as much of his or her behavior (either in terms of time or number of responses) to B1 as to B2. This relation may be expressed mathematically as:

$$\frac{B1}{B2} = \frac{N1}{N2} \quad (1)$$

Where N1 and N2 are, respectively, the numbers of reinforcements obtained for the alternative behaviors B1 and B2. Equation 1 may also be rewritten in terms of proportions:

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$$\frac{B1}{B1 + B2} = \frac{N1}{N1 + N2} \quad (2)$$

The adequacy of the matching law as a description of operant choice behavior on concurrent probabilistic, i.e., variable interval (VI) or variable ratio (VR), schedules of reinforcement has been confirmed repeatedly. Matching of the allocation of behavior to the distribution of reinforcements has been found using diverse species, responses, and reinforcers (see de Villiers, 1977, for a review). Of particular relevance is the fact that the matching law has been confirmed with humans under a variety of circumstances (see Pierce & Epling, 1983, for a review). For example, matching has been reported with humans pressing buttons for money in a situation directly analogous to the typical animal experimental situation (Bradshaw, Szabadi, & Bevan, 1976), with subjects engaged in vigilance tasks (Baum, 1975; Schroeder & Holland, 1969), and with people conversing in small groups (Conger & Killeen, 1974). Although there is some dispute as to how precise a description the matching law provides (e.g., Baum, 1979; Myers & Myers, 1977; Wearden & Burgess, 1982), there is little doubt that the matching law provides a useful approximation of the relation between the distribution of reinforcements and the allocation of behavior.

Despite the fact that the matching law is a well-established, relevant principle of behavior, little attention has been given to its practical application. Consider the situation in which the applied behavior analyst attempts to decrease inappropriate behavior by reinforcing some competing, more appropriate response. This situation clearly involves concurrent schedules, but what guidance does the matching law give the applied behavior analyst? What does it tell the behavior analyst about the use of reinforcement that she or he does not already know? To answer these questions, we shall consider what happens when the behavior analyst uses different types of reinforcement schedules. It will become clear that the efficacy of the intervention depends not only on the schedule value but also on the type of schedule chosen by the behavior analyst, and, in addition, on the interaction of this

schedule with the schedule that maintains the inappropriate behavior.

One important aspect of the matching law is the way in which it predicts the interactions of different schedule types, a fact that has been relatively unappreciated but is of potentially great practical significance. This may be seen clearly in the first case to be considered: the case in which inappropriate behavior is maintained by a VR schedule of reinforcement. We shall compare the success in such an environment of interventions based on (a) VR schedules of reinforcement for more appropriate responses and (b) VI reinforcement schedules. Next, the success of different types of interventions in an environment in which inappropriate behavior is reinforced according to VI schedules will be examined. Finally, the generality of the principles derived from consideration of these cases will be discussed. It will be shown that the matching law leads to valuable suggestions concerning which reinforcement schedules will be successful in producing decreased levels of inappropriate behavior or in eliminating such behavior altogether, and furthermore, that these suggestions are likely to have broad applicability.

INAPPROPRIATE BEHAVIOR ON VR SCHEDULES

Consider a classroom situation in which a child engages in disruptive behavior which classmates reinforce. The schedule of reinforcement may be similar to a VR schedule. That is, not every incident of disruptive behavior will be reinforced, nor will reinforcement occur after some fixed number of incidents. However, the number of reinforcements obtained may increase in direct proportion to the number of disruptive incidents. When inappropriate behavior is being reinforced according to a VR schedule, what type of reinforcement schedule for competing responses is likely to be most effective?

VR Reinforcement for Competing Responses

In predicting the interaction of the reinforcement schedule implemented by the behavior analyst and the schedule maintaining the inappro-

appropriate behavior, it is important to remember that the numbers of reinforcements in the matching law (N_1 and N_2 in Equation 1) are the numbers of reinforcements actually obtained by an individual. Therefore, predicting the effect of an intervention requires estimating the obtained reinforcements for both alternatives, i.e., the inappropriate behavior and the competing responses in the applied situation. On a VR schedule, reinforcements follow, on the average, some fraction of the responses, i.e., on the average, 1 out of every 50 responses will be reinforced on a VR 50. Therefore the number of reinforcements (N) obtained is approximately equal to the number of responses (B) divided by the ratio value. For example, if 500 responses are emitted on a VR 50, then the number of reinforcements equals $500/50$ or 10. More generally, $N = B/VR$ where VR equals the value of the ratio schedule.

The matching law may now be rewritten for the specific case in which inappropriate behavior is reinforced according to a VR schedule, and the behavior analyst also elects to use a VR schedule for reinforcing a competing response. If the number of inappropriate and competing responses are represented by B_1 and B_2 , respectively, and the corresponding schedule values by VR_1 and VR_2 , then substituting B_1/VR_1 for N_1 and B_2/VR_2 for N_2 in the matching law (Equation 1 above) yields:

$$\frac{B_1}{B_2} = \frac{N_1}{N_2} = \frac{B_1/VR_1}{B_2/VR_2}$$

To understand the significance of this equation, one must ask: When will this relationship be true? The answer to this question may be more apparent if we cross multiply and rewrite the equation as:

$$B_1 \times (B_2/VR_2) = B_2 \times (B_1/VR_1)$$

Now, when will this equation be true? It is easy to see that if *either* B_1 or B_2 equals zero, then both sides equal zero. In fact, this is the only way in which this equation can be true if VR_1 does not equal VR_2 . In other words, when the schedules are unequal ($VR_1 \neq VR_2$), responding on one schedule must cease ($B_1 = 0$ or $B_2 = 0$). Not surprisingly, when unequal ratio schedules cause responding to cease on one schedule, responding

Figure 1. Reinforcement rate as a function of response rate for variable interval (VI) schedules.

on the richer schedule (lower VR) persists (Herrnstein & Loveland, 1975).

What does this mean for the applied behavior analyst? The good news is that if the behavior analyst reinforces the competing response according to a richer VR schedule than that maintaining the inappropriate behavior, then inappropriate behavior will be completely eliminated. The bad news, however, is that if the behavior analyst cannot offer a VR schedule as rich as that maintaining the inappropriate behavior, then his or her efforts will be completely ineffective.

VI Schedule for Competing Responses

Another option for eliminating inappropriate behavior is to reinforce competing responses according to a VI schedule. Again, in order to generate predictions from the matching law one must estimate the numbers of reinforcements obtained. To do so, it is necessary to determine how obtained reinforcement rates are affected by changes in either response rate or amount of time allocated to the VI schedule. The most generally accepted form of the relationship between response rate and reinforcement rate on VI schedules is hyperbolic (Baum, 1973; Prelec, 1982; Prelec & Herrnstein, 1978; Staddon & Motheral, 1978) and is shown in Figure 1. It may be seen that except at very low response rates, obtained reinforcement rates remain nearly constant and approximate the programmed reinforcement rates. Reinforcement rates obtained

on concurrently programmed VI schedules are also insensitive to changes in response rates, and obtained rates are very nearly equal to programmed reinforcement rates over a broad range of preference values (see Figure 3, Baum, 1981).

Myerson and Miezin (1980) successfully exploited this property of VI schedules in developing a kinetic model of choice behavior, and their approach is also useful here. That is, we can make the simplifying assumption that obtained reinforcement rates remain constant at approximately their programmed rates. Herrnstein (1970) made a similar assumption about the reinforcement rate for nonresponding in formulating his quantitative law of effect. The real test of such an assumption is, of course, the accuracy with which it enables us to predict behavior. As Myerson and Meizin (1980) have shown, the precise and detailed account of both transition-state and steady-state behavior which our assumption makes possible more than makes up for the slight loss of precision in describing reinforcement rates.

According to our simplifying assumption, the number of reinforcements obtained on a VI schedule (N_{vi}) is approximately equal to the duration of the observation period (T) divided by the interval value. For example, on a VI 120-s, over a period of 10 min or 600 s, approximately 600 divided by 120, or 5 reinforcements, will be obtained. More generally, $N_{vi} = T/VI$, where VI is the value of the interval schedule.

In the concurrent situation under consideration, the observation time period consists of time spent engaged in inappropriate behavior (maintained, in our example, by a VR schedule) and time spent engaged in making competing responses (maintained by a VI schedule). The cumulative durations of these two activities will be designated T_{vi} and T_{vr} , respectively. Note that total time (T) equals the sum of the time spent responding on the VR (T_{vr}) plus time spent on the VI (T_{vi}). The preceding equation may therefore be rewritten as:

$$N_{vi} = (T_{vi} + T_{vr})/VI.$$

Because our equation for the number of reinforce-

ments earned on a VI is based on time rather than number of responses, one more thing must be done before the matching law's prediction for the concurrent CONC VI VR situation can be obtained. The equation for the number of reinforcements earned on a VR schedule ($N_{vr} = B/VR$) must be rewritten so that it is expressed in terms of time. The number of VR responses is equal to the rate of responding on the VR, symbolized by r , multiplied by the time spent responding: $B = r \times T_{vr}$. Therefore, the equation for reinforcements obtained on the VR may be rewritten as:

$$N_{vr} = r \times T_{vr}/VR.$$

Note that whereas the number of VI reinforcements depends only on the passage of time, regardless of how the individual divides her or his time between the two concurrent schedules, the number of VR reinforcements depends specifically on the amount of time allocated to the VR schedule.

It is now possible to substitute the expressions for the numbers of reinforcements obtained on the VI and VR schedules into the matching law. Because these expressions are formulated in terms of time, the matching law for time allocation, $T_1/T_2 = N_1/N_2$, will be used. It has been argued (e.g., Baum & Rachlin, 1969; Brownstein & Pliskoff, 1968; Myerson & Miezin, 1980; Rachlin, 1978) that time matching is a more fundamental relationship than the matching of response allocation. If T_1 (or T_{vr}) represents time on the VR and T_2 (or T_{vi}) time on the VI, then substituting for N_1 and N_2 , yields:

$$\frac{T_{vr}}{T_{vi}} = \frac{r \times T_{vr}/VR}{(T_{vi} + T_{vr})/VI}.$$

This equation may be rearranged to produce one that is easier to interpret. First, multiplying both sides by T_{vi}/T_{vr} yields:

$$1 = \frac{T_{vi}}{T_{vi} + T_{vr}} \times \frac{r/VR}{1/VI}.$$

Finally, multiplying both sides by $(1/VI)/(r/VR)$ yields equation 3:

$$\frac{T_{vi}}{T_{vi} + T_{vr}} = \frac{1/VI}{r/VR} \quad (3)$$

Equation 3 has a very simple interpretation. The left-hand side represents the proportion of time allocated to the VI schedule, $T_{vi}/(T_{vi} + T_{vr})$. The right-hand side is the ratio of two expressions: $1/VI$ which is the programmed rate of reinforcement for the VI, and r/VR which is the rate at which VR responses are reinforced when the individual is actually engaged in responding on the VR schedule, i.e., the local VR reinforcement rate. The right-hand side of Equation 3 may thus be thought of as the ratio of the reinforcement rates that would be obtained on the VI and VR schedules if they were presented singly rather than concurrently. This ratio of single schedule reinforcement rates, which we shall term the relative richness ratio, will prove useful below for comparing schedules of reinforcement, especially when these schedules are of different types and interact when presented concurrently. Note that in order to compare the richness of a VR to that of a VI, it is necessary to know the rate of responding on the VR (but not that on the VI) in addition to the schedule values. Therefore for a given schedule pair, the relative richness ratio will be different for individuals with different VR response rates.

In relating Equation 3 to the applied problem under consideration, it is assumed that the VR schedule for inappropriate behavior is not subject to manipulation by the behavior analyst. It is also assumed that the value of the VI schedule for competing responses can be varied by the behavior analyst. Figure 2 shows a graph of changes in the proportion of time allocated to the VI schedule, i.e., increases in appropriate behavior, as the VI schedule is made richer while the VR schedule is held constant. If the schedule for inappropriate behavior remains constant, as is assumed in our analysis, then the relative richness ratio is directly proportional to the richness of the schedule of reinforcement for the competing response introduced by the behavior analyst. For purposes of comparison, Figure 2 also plots the change in appropriate behavior as the schedule of reinforcement for com-

peting responses is made richer when both appropriate and inappropriate behaviors are maintained by VR schedules.

What insight into the efficacy of VI schedules can the behavior analyst obtain from looking at Figure 2 and the preceding equation? When $1/VI$ equals r/VR , the relative richness ratio equals one, and the proportion of time allocated to the VI also equals one, i.e., inappropriate behavior is completely eliminated. Thus the good news for the behavior analyst who uses VI schedules is similar to that for the user of VR schedules when inappropriate behavior is reinforced according to a VR schedule: If the behavior analyst can offer a richer schedule, he or she can completely replace undesirable behavior with more appropriate, competing responses. But the really good news is that unlike VR schedules which tend to operate in an all-or-none manner in concurrent situations, either eliminating inappropriate behavior completely or failing to produce any decrease at all, VI schedules are effective in decreasing the level of inappropriate behavior even when they are not rich enough to completely abolish it. Even when the VI is less rich than the VR ($1/VI < r/VR$), there will be a decrease in inappropriate behavior directly proportional to the richness of the VI schedule for competing responses.

INAPPROPRIATE BEHAVIOR ON VI SCHEDULES

Although the matching law provides clear guidance for the applied behavior analyst choosing between interventions based on VI and VR schedules when inappropriate behavior is reinforced according to a VR schedule, the reinforcement schedule for inappropriate behavior may not always be a VR. In fact, the nature of this schedule may not even be known. How general is the superiority of VI over VR schedules? Are they more effective, for example, when reinforcement maintaining inappropriate behavior is delivered according to a VI schedule?

Such a situation is not difficult to imagine. Consider again the problem of disruptive incidents in

Figure 2. Predicted levels of appropriate behavior that will result from variable interval reinforcement and from variable ratio reinforcement are shown for situations in which inappropriate behavior is maintained by a variable-ratio reinforcement schedule. Appropriate behavior (as a percentage of all behavior) is plotted as a function of the relative richness of the reinforcement schedules for appropriate and inappropriate behavior.

the classroom. In the previous example, there was some probability that disruptive behavior would be reinforced and that probability was independent of the passage of time. However, if the disruptive child's behavior is reinforced by classmates only when they are bored, and the amount of time that passes before they are sufficiently bored with their schoolwork varies from one instance to the next, then the reinforcement schedule for the disruptive child is formally analogous to a VI.

VR Reinforcement for Competing Responses

First, consider the use of a VR schedule by the behavior analyst. Because the CONC VI VR situation has already been analyzed when the roles of the schedules were reversed, almost all the mathematical development is done. Equation 3 gives the proportion of time allocated to the VI. Because the proportion of time devoted to the VI

and the proportion allocated to the VR must sum to one, the proportion of time allocated to the VR is one minus that for the VI:

$$\frac{T_{vr}}{T_{vi} + T_{vr}} = 1 - \frac{1/VI}{r/VR}.$$

The preceding equation shows that the proportion of time allocated to the VR depends on the relative richness of the VI and VR schedules. However, unlike the relative richness ratios used previously, this one involves the richness of the schedule for inappropriate behavior (which is the VI in this case) divided by the richness of the schedule for the more appropriate response (the VR). It is more useful, however, to always use a relative richness ratio in which the value of the ratio increases in proportion to the richness of the schedule introduced by the behavior analyst. Therefore, the preceding equation will be reformulated in terms of

the ratio of the richness of the VR to the richness of the VI rather than vice versa. This may be accomplished by dividing both the numerator and the denominator of the rightmost expression by $1/VI$ (the richness of the VI schedule), and then canceling in the numerator, yielding:

$$\frac{T_{vr}}{T_{vi} + T_{vr}} = 1 - \frac{1}{(r/VR)/(1/VI)} \quad (4)$$

In interpreting Equation 4, the relative richness ratio again plays a key role: When it equals one ($r/VR = 1/VI$), the proportion of time spent on the VR equals zero. (When the relative richness ratio is less than one, Equation 4 does not apply; if it did, the proportion of time on the VR would have a negative value. As the proportion of time can only take values between zero and one, the proportion of time on the VR is assumed to be zero when the richness of the VR is less than that on the VI). Only when the richness of the VR actually exceeds that of the VI does any time get allocated to the VR schedule. Therefore, unless the VR schedule introduced by the behavior analyst is richer than the VI maintaining the inappropriate behavior, it will be completely ineffective in decreasing the level of that behavior.

This result is reminiscent of the CONC VR VR case in which the schedule for competing responses also had to be richer than that for inappropriate behavior or else it was without effect. However, a VR schedule is even less effective against a VI than it is against a VR. Even if the behavior analyst introduces a VR schedule that is richer than the VI schedule maintaining the inappropriate behavior, such inappropriate behavior will not be completely eliminated as it would be in the CONC VR VR case. Instead, as may be seen in Figure 3, increasing the richness of the VR only gradually increases the level of appropriate responding. Moreover, each successive increase in the richness of the VR produces a successively smaller increase in appropriate responding.

VI Reinforcement for Competing Responses

Does the behavior analyst fare any better if he or she reinforces competing responses according to

a VI instead of a VR schedule? Again, the numbers of obtained reinforcements must be estimated. The general expression for reinforcements obtained on a VI schedule, $N = T/VI$, may be rewritten for the CONC VI VI situation as $N = (T_1 + T_2)/VI$ where T_1 and T_2 are the times allocated to two VI schedules: Summed together, these two times constitute the total observation period of duration T . Substituting for N in the matching law (Equation 2), gives:

$$\frac{T_1}{T_1 + T_2} = \frac{(T_1 + T_2)/VI_1}{(T_1 + T_2)/VI_1 + (T_1 + T_2)/VI_2}$$

and canceling yields:

$$\frac{T_1}{T_1 + T_2} = \frac{1/VI_1}{1/VI_1 + 1/VI_2}$$

for the proportion of time allocated to the VI_1 . The reinforcement schedule for competing responses shall be designated VI_1 . This equation can now be rewritten so that, like the others plotted in Figures 2 and 3, the proportion of time allocated (or the decrease in inappropriate behavior) is a function of the relative richness ratio. Dividing both the numerator and the denominator of the right-hand side by $1/VI_2$ gives:

$$\frac{T_1}{T_1 + T_2} = \frac{(1/VI_1)/(1/VI_2)}{1 + (1/VI_1)/(1/VI_2)} \quad (5)$$

Equation 5 indicates that in CONC VI VI situations, the proportion of time allocated to appropriate responding equals the relative richness ratio divided by one plus that ratio.

Figure 3 shows the predicted increase in appropriate behavior as a function of the relative richness ratio. It may be seen that the shape of the curve for a VI intervention, when displaced by one unit, is identical to that for a VR intervention. Thus, if the behavior analyst offers a VI schedule of reinforcement for competing responses two times as rich as the VI schedule for inappropriate behavior, the result will be the same as would be obtained

Figure 3. Predicted levels of appropriate behavior that will result from variable interval reinforcement and from variable ratio reinforcement are shown for situations in which inappropriate behavior is maintained by a variable interval reinforcement schedule. Appropriate behavior is plotted as a function of the relative richness of the reinforcement schedules for appropriate and inappropriate behavior.

with a VR schedule three times as rich as the schedule for inappropriate behavior. Likewise, a VI schedule three times as rich results in the same decrease as a VR four times as rich as the reinforcement schedule for inappropriate behavior. Thus, the behavior analyst will always maintain more competing responding (thereby producing a greater decrease in inappropriate behavior) with a VI schedule of reinforcement than with an equally rich VR.

Looking at both Figures 2 and 3, it can be seen that if the behavior analyst can offer a very rich reinforcement schedule relative to that maintaining inappropriate behavior, then it does not matter much what schedule type she or he selects regardless of the schedule type maintaining inappropriate behavior. However, if the reinforcement schedule for inappropriate behavior is relatively rich, as is

likely to be true in difficult cases, then the behavior analyst will be much more effective with an intervention based on a VI schedule than with one based on a VR schedule. This conclusion holds regardless of whether VI or VR reinforcement maintains the inappropriate response.

THE GENERALITY OF INTERVAL SCHEDULE SUPERIORITY

We have shown that the matching law predicts that VI schedules are likely to be more effective in maintaining responding that competes with inappropriate behavior regardless of whether VI or VR reinforcement is available for inappropriate behavior. This independence of the reinforcing environment is important because of the generality it gives our conclusions. We have, however, postponed to

this point discussion of the ways in which applied settings may differ from the situation in the standard operant choice experiment. Almost all of the research supporting the matching law involves two responses of identical topography which produce the same reinforcing consequence and differ only in the schedule of reinforcement. Concurrent schedules in applied settings may not fit these specifications, and to establish the generality of our conclusions, we will consider three ways in which the real world is likely to differ from the ideal cases described by our equations. Specifically, these three ways are: (a) topographic differences between inappropriate and competing responses; (b) qualitatively different reinforcers for inappropriate and competing responses; and (c) reinforcement schedules for inappropriate and competing responses that differ from the probabilistic schedules considered above.

The question of differences in response topography is clearly of considerable practical significance. Although some responses are inappropriate simply because of the circumstances under which they are emitted and would be quite acceptable at another time or in another place, other responses are by the nature of their topography inappropriate. Does the matching law, and thus the conclusions that follow from it, hold when an individual chooses between operant responses that are topographically different? Although most laboratory confirmations of the matching law are based on topographically similar responses, recent results indicate that matching may occur even when concurrently available schedules reinforce topographically different responses (Hanson & Green, 1982).

The question of whether qualitative differences in reinforcers invalidate our conclusions is certainly as important as that of the role of differences in responses. The answer to the reinforcer question, however, is somewhat more complex. As those behavior analysts exploring the relevance of operant theory to economic behavior have pointed out (e.g., Green, Kagel, & Battalio, 1982; Rachlin, Green, Kagel, & Battalio, 1976), qualitatively different reinforcers raise important issues that have generally been neglected by operant researchers. In the

laboratory, biased matching (Baum, 1974) may occur when there is a choice between two different reinforcers, e.g., between different types of food reinforcement (Miller, 1976) and even between food and reinforcing brain stimulation (Hollard & Davison, 1971). The term *biased matching* refers to a relation between behavior allocation and reinforcement distribution described by the equation:

$$\frac{B_1}{B_2} = \frac{b \times N_1}{N_2}$$

where b is a bias factor that may be thought of as specifying how many units of the first reinforcer are equivalent to one unit of the second reinforcer. The bias factor allows one to take into account the value (Rachlin, 1971) as well as the number of obtained reinforcements when reinforcers are qualitatively different. The inclusion of a bias factor rescales the x-axes in Figures 2 and 3 but, most importantly, does not change the shape of the functions. As our conclusions concerning the superiority of interval schedules are based on the shapes of these functions, inclusion of a bias factor in no way affects the validity of these conclusions.

There exists, however, an important class of exceptions to the biased matching relation for choice between qualitatively different reinforcers. Behavior analysts exploring the analogies between operant theory and microeconomic theory suggest that subjects responding on ratio schedules may be thought of as purchasing reinforcers for a price specified in terms of the number of responses required (Green et al., 1982; Rachlin et al., 1976). They note, however, that although subjects on CONC VR VR cease responding on the leaner schedule, people do not always cease buying one type of goods simply because another type is a better deal. People may cease buying one brand of breakfast cereal and only buy another brand, or even give up breakfast cereal for eggs if egg prices drop, but they cannot give up buying food and spend all their money on shoes no matter how low shoe prices go.

When two reinforcers may be thought of as satisfying different needs, microeconomic theorists refer to them as being non-substitutable. The

matching law, however, describes choice between substitutable reinforcers only, as exemplified by its prediction of exclusive preference for the richer schedule on CONC VR VR (Rachlin et al., 1976). The important implication for the applied behavior analyst is that one will do better offering a reinforcer that is substitutable for the reinforcer maintaining inappropriate behavior. When the reinforcer for appropriate responding is substitutable for the reinforcer for inappropriate behavior and the latter is available on a VR schedule, for example, offering a richer schedule for appropriate responding will completely eliminate inappropriate behavior. If the reinforcers are not substitutable, however, complete elimination will not result.

As the laboratory results cited earlier demonstrate, such different reinforcers as brain stimulation and food reinforcement turn out to be substitutable. The clear implication is that a qualitative difference between reinforcers does not necessarily imply nonsubstitutability; substitutability of reinforcers is thus an empirical question, and one deserving serious investigation by analysts of human behavior. The results of such investigation would provide valuable guidance for the applied behavior analyst in choosing the reinforcers for more appropriate, competing responses.

This brings us to the final question concerning the generality of our conclusions. Clearly, a VI schedule is the schedule of choice for the behavior analyst when the schedule maintaining inappropriate behavior is either a VI or a VR. In applied settings, however, reinforcement schedules may vary from probabilistic ones like VI and VR schedules to more periodic ones that pay off at regular intervals or for fixed amounts of work, i.e., schedules that are more like the fixed interval (FI) and fixed ratio (FR) schedules studied in the laboratory. Is an interval schedule, and more specifically a VI schedule, still the most effective intervention when inappropriate behavior is maintained by a periodic schedule of reinforcement?

If an FR is pitted against another FR in a concurrent situation, the result is similar to that obtained with two VR schedules; exclusive preference develops because responding ceases on the leaner

schedule (Herrnstein, 1961). As in the CONC VR VR case, a behavior analyst would have to offer a richer reinforcement schedule or the intervention would be without effect. However, when an FI is presented concurrently with an FR, more than 90% of behavior is allocated to an FI alternative which is one-half as rich as the FR (LaBounty & Reynolds, 1973). VI schedules are even more effective. When a VI schedule is presented concurrently with an FR, more than 90% of behavior will be allocated to a VI which is one-fourth as rich as the FR alternative (Bacotti, 1977). Looking at Figure 2, it is clear that if a VI were presented concurrently with a VR, the VI would have to be nearly as rich as the VR ($9/10$ as rich, in fact) to produce this same effect. Thus a VI is an even better intervention strategy if inappropriate behavior is maintained by a periodic rather than a probabilistic ratio schedule. Finally, on CONC VI FI, more behavior is usually allocated to a VI schedule than to a concurrently available FI schedule offering the same rate of reinforcement (Nevin, 1971; Trevitt, Davison, & Williams, 1972). These results strongly support the use of interval reinforcement schedules rather than ratio schedules by applied behavior analysts, and in addition, argue for the use of probabilistic interval schedules as opposed to periodic ones, regardless of the type of schedule maintaining the inappropriate behavior.

Although our conclusions concerning the superiority of interval schedules are derived from the matching law, our results testify not only to the generality but also to the robustness of these conclusions. Periodic schedules may result in undermatching in concurrent situations, i.e., differential allocation of behavior less extreme than that predicted by the matching law (see Baum's, 1974, analysis of LaBounty & Reynolds', 1973, data). Nevertheless, interval schedules are demonstrably superior to ratio schedules even when the assumption of matching (in the sense of equations 1 and 2) is violated and pronounced undermatching is observed (e.g., LaBounty & Reynolds, 1973).

It should be noted that our conclusions are not really about interval and ratio schedules, but about procedures characterized by *feedback functions*

(Baum, 1973) similar to those of interval and ratio schedules. Feedback functions are mathematical statements of the relations between responding and reinforcements obtained on different schedules. In other words, any procedure that provides response-contingent reinforcement in such a way that the reinforcement rate will be relatively constant despite large changes in response rate (like an interval schedule) is superior to any procedure for which the reinforcement rate increases and decreases in direct proportion to changes in response rate (like a ratio schedule).

The same principle applies to schedules of reinforcement for inappropriate behavior: How they will interact with the procedure introduced by the behavior analyst depends on the way in which the obtained reinforcement rate depends on response rate, i.e., on the feedback function. Although an intervallike procedure remains the intervention of choice regardless of the reinforcement schedule for inappropriate behavior, the feedback function governing reinforcement of such behavior determines the likelihood of completely eliminating inappropriate responses. For practical purposes, it is certainly not necessary to specify feedback functions mathematically, but a characterization of procedures (and environments) in terms of intervallike or ratiolike feedback functions may prove invaluable to the applied behavior analyst. Many failures of reinforcement to maintain responding in applied settings, like some of those reported in the experimental literature (e.g., Herrnstein & Heyman, 1979; Herrnstein & Loveland, 1975; LaBounty & Reynolds, 1973), may prove to be consequences of the schedule interactions in concurrent situations. These results are predicted by the matching law when the feedback functions for the alternative schedules are incorporated into the analysis, and may be avoided by the selection of appropriate intervallike schedules of reinforcement.

As a final caveat, we recommend that the behavior analyst consider the role of verbal behavior. For example, verbal instructions have been shown to influence the effect of single schedules on human operant behavior (see Baron & Galizio, 1983, for a review), often by controlling the degree to which

subjects' behavior actually contacts the reinforcement contingencies. It is unlikely that such influences are restricted to single schedules. Human subjects match their time allocation to the distribution of reinforcers in the absence of verbal instructions concerning how behavior affects reinforcement (e.g., Conger & Killeen, 1974). Thus, although further research on the instructional control of human choice behavior is needed (Pierce & Epling, 1983), our conclusions should apply whenever verbal instructions do not interfere with the individual making contact with the contingencies of the intervention schedule.

CONCLUSIONS

In recent years, there has been growing concern over the widening gap between the basic and applied areas of behavior analysis (e.g., Cullen, 1981; Dietz, 1978; Edelstein, 1982; Hayes, 1978; Michael, 1980; Pierce & Epling, 1980). The lack of communication between behavior analysts working in different areas is due in part to the technical drift of the applied area (Hayes, Rincover, & Solnick, 1980), i.e., an increased emphasis on effective techniques coupled with a decreased concern for analysis in terms of fundamental principles. The problem has been exacerbated, however, by the theoretical drift of basic research, e.g., the proliferation of mathematical models aimed at elucidating the behavioral processes which lead to matching in operant choice situations (e.g., Baum, 1974; 1981; Herrnstein & Vaughan, 1980; Killeen, 1982; McDowell & Kessel, 1979; Myerson & Miezin, 1980; Rachlin & Burkhard, 1978; Staddon, 1977).

Some behavior analysts have reacted with dismay to the apparent preoccupation with matching and choice behavior on the part of many basic researchers and to the mathematical language that pervades the recent literature on concurrent reinforcement schedules. It is easy to understand why many applied behavior analysts may have chosen to ignore this relatively inaccessible literature. Why should applied behavior analysts go to a great deal of effort to try and discern potentially useful new

principles (Cullen, 1981) when they may be doing rather well implementing established principles (Baer, 1981)? Moreover, until very recently there had been almost nothing written concerning what might be the practical implications of this line of research. Basic researchers have been seemingly unwilling to suggest to applied behavior analysts what the practical relevance of their investigations and theorizing might be (Edelstein, 1982).

As we and the recent contributions of McDowell (1981, 1982) have shown, basic researchers can suggest applications. We have attempted to demonstrate the relevance of operant choice research and the utility of the mathematical language used to describe the results of this research. We have done this not by reviewing the literature and translating findings into terms more familiar to those in the applied area, but rather by using the matching law and its mathematical implications as the basis for a fresh attack on problems of importance to applied behavior analysts. It should be noted that in the cases of both our suggestions and those of McDowell, the applications are suggested not directly by experimental data but by mathematical descriptions of those data. These equations represent the basic principles that emerge from many different experiments, and, in that respect, learning to understand the mathematical generalizations may be a more efficient use of the applied behavior analyst's time than trying to read and digest each new experimental report. Accordingly, we have attempted not only to point out an important implication of the matching law, the superiority of interventions based on interval schedules of reinforcement, but also to show how this conclusion follows from the logic of mathematical argument.

We hope that applied behavior analysts attend to the important role of mathematics in reaching these conclusions, and we believe that we have presented an important message for basic researchers and behavior theorists. As in the case of the matching law, the practical implications of research findings or theoretical ideas are often not immediately obvious. Instead, as in this case, one may see potential applications only if one specifically formulates questions based on applied problems:

What happens if some inappropriate response is reinforced according to a particular schedule which is beyond your control, but you can introduce a schedule of reinforcement for a second response? How will the results be affected not only by the schedule parameters, but also by the type of schedule that is introduced? Our suggestions obviously require empirical testing in applied settings. Regardless of their ultimate evaluation, however, we hope that we have contributed to reviving the view that applied behavior analysis and basic behavioral research are interdependent parts of the same enterprise. Our basic principles must meet the empirical tests of the applied setting and our applied problems should provide inspiration for our research as well as for some of our deepest thinking.

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