XXII. AN ARITHMETICAL TEST OF THE VALIDITY OF THE THEORY OF BAYLISS REGARDING FERMENTATION AND ADSORPTION.

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(Received January 9th, 1925.)

ON page 315 or 316 in the various editions of Bayliss' *Principles of General Physiology*, a table is given, which is reproduced below, in which the values of the "specific activity" of the ferment trypsin when added in different concentrations to a solution containing equal quantities of caseinogen are shown, as determined by experiment.

Relative concentration of enzyme (1)	Time taken for equal change in minutes (2)	Mean rate × 1000 (3)	"Specific Activity" (4)	Theoretical values (5)
8	41	24	3	3
5	48	20.8	4.16	3.9
4	55	18.2	4.55	4.55
2	81	12.4	6.2	6.1
1	144	7	7	7

Bayliss remarks upon this table:

"The reader will probably notice at once that these results are precisely what we should expect if the velocity were controlled by adsorption; it was, in fact, this relationship which first led me to suggest the hypothesis of adsorption as applying to the case."

Neither Bayliss, however, nor his present editors seem to have thought it worth while to test arithmetically whether the values obtained by the experiments are consistent with the usually adopted formula of adsorption.

Mathematically this is a piece of elementary algebra though the arithmetic is lengthy. The adsorption formula is usually written in the form,

$$x = \frac{1}{\alpha C^n} \dots\dots\dots(1),$$

where x is the amount of a specific substance adsorbed by a surface or surfaces, α a constant dependent on the units of measurement adopted and C the concentration of the specific substance remaining unabsorbed after equilibrium has been attained.

If the concentration of the substance, in the present instance trypsin, be originally m , this may be written in the form

$$x = \alpha (m - x)^{\frac{1}{n}} \dots\dots\dots(2).$$

It is convenient, as $\frac{1}{n}$ is a fraction, to raise both sides to the power n . This is due to the fact that the unknown is contained in the expression on the right-hand side of the equation, not by itself but combined with a known but varying quantity m . The form of the equation used in calculation is thus

$$x^n = \alpha^n (m - x) \dots\dots\dots(3).$$

If the amount of adsorption, as is assumed in Bayliss' theory, be taken as proportional to the rate of action, column (3) in the table, the equation can be easily solved. It is convenient for purposes of calculation to divide the figures in the third column of the table by seven making the lowest figure unity. With this notation, taking the first, third and fifth terms of column (3) and comparing with the corresponding amount of enzyme, column (1), we have the three equations

$$\left(\frac{24}{7} x\right)^n = \alpha^n \left(8 - \frac{24}{7} x\right) \dots\dots\dots(4),$$

$$\left(\frac{18.2}{7} x\right)^n = \alpha^n \left(4 - \frac{18.2}{7} x\right) \dots\dots\dots(5),$$

$$x^n = \alpha^n (1 - x) \dots\dots\dots(6).$$

The solution is obtained by dividing equations (4) and (5) by equation (6) and then eliminating x . The eliminant equation after reduction has the form

$$68.6 \left(\frac{24}{7}\right)^n - 224 \left(\frac{18.2}{7}\right)^n + 347.2 = 0 \dots\dots\dots(7).$$

From this equation n has to be determined. This was solved by taking a series of values of n , namely, 2, 3, 4, and 5. The root having been located between $n = 4$ and $n = 5$, a series of approximations resulted in the final value of 4.1723. This gives $\frac{1}{n} = .240$, a number in accord with all the values experimentally found to hold in adsorption experiments.

Using the values of n it was directly calculated that the value of α^n was equal to 22.84 and of x equal to .973. From these the values of the "specific activity" for $m = 5$ and $m = 2$ were evaluated and are given in the fifth column of the Table. The chief difference which exists is that in the second line between the values 3.9 and 4.16. A second approximation distributing the error over all the observations would give a closer fit but as the labour of solving such equations is considerable, and as the fit is all that could be desired from the experiments, it does not seem worth the trouble. The calculation clearly demonstrates that with trypsin and caseinogen, the experiments are consistent with the theory that the action of a ferment is in accordance with the adsorption formula as generally accepted.