III. ON THE POSITION OF THE OPTIMUM TEMPERATURE OF THE ACTION OF A FERMENT OR LYSIN.

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As the question of the optimum temperature of the action of a ferment has not, so far as I have seen, been mathematically treated, the following solution is offered.

Generally it is found that the coefficient of increase in the rate of a chemical action with temperature is given to a first approximation by Arrhenius' expression

$$e^{\mu\left(\frac{1}{T_0} - \frac{1}{T}\right)},$$

where T is the absolute temperature and T_0 the absolute temperature chosen as zero in the experiments. For a short range of temperature, however, this can be sufficiently approximately expressed by a coefficient of the form

$$e^{\mu(T-T_0)},$$

which lends itself more easily to mathematical manipulation.

Let then the rate at which the ferment acts at T_0 be λ and the temperature factor $e^{\nu T}$, so that the rate of action of the ferment at any temperature is

$$\lambda e^{\nu T}.$$

Further, assume that the ferment at a temperature T_0 is destroyed at a rate K , that the temperature factor is

$$e^{\mu T}$$

and the rate of destruction at any time

$$K e^{\mu T}.$$

Let f denote the amount of the ferment present at any time. Then

$$\frac{df}{dt} = -K e^{\mu T} f,$$

whence

$$f = a e^{-K t e^{\mu T}}, \quad \dots\dots\dots(1)$$

where a is the amount of ferment when $t = 0$.

The fermentable substrate may be considered as altered according to the law of a unimolecular reaction. If the initial amount be A and the amount

fermented at any time t be x and if the rate of action of the ferment is proportional to its amount, it follows from equation (1) that

$$\frac{dx}{dt} = a\lambda e^{\nu T} e^{-tK e^{\mu T}}, \quad \dots\dots\dots(2)$$

which on integration becomes

$$-\log(A - x) = C - \frac{a\lambda}{K} e^{T(\nu - \mu)} e^{-tK e^{\mu T}}.$$

If $x = 0$ when $t = 0$ we have

$$C = -\log A + \frac{a\lambda}{K} e^{T(\nu - \mu)},$$

so that

$$\log(A - x) = \log A - \frac{a\lambda}{K} e^{T(\nu - \mu)} + \frac{a\lambda}{K} e^{T(\nu - \mu)} e^{-Kte^{\mu T}}. \quad \dots\dots\dots(3)$$

If an optimum action exist for a given temperature, $A - x$ must be a minimum, i.e. $\frac{d \log(A - x)}{dT}$ is zero.

It is obvious that equation (3) involves the time as an essential constituent so that the solution obtained is not the optimum temperature for action of the ferment but the optimum temperature for a specified duration of time.

The differentiation gives

$$-\frac{a\lambda}{K} (\nu - \mu) e^{T(\nu - \mu)} - a\lambda \mu t e^{T(\nu - \mu)} e^{-Kte^{\mu T}} + \frac{a\lambda}{K} (\nu - \mu) e^{T(\nu - \mu)} = 0$$

$$\text{or} \quad (\nu - \mu) \{1 - e^{-Kte^{\mu T}}\} + K\mu t e^{\mu T} = 0. \quad \dots\dots\dots(4)$$

If $Kte^{\mu T}$ be noted by ξ this becomes

$$(1 - e^{-\xi}) = \frac{\mu}{\mu - \nu} \xi e^{-\xi}.$$

As ξ is essentially positive there is no optimum unless $\mu > \nu$.

The equation has only one real positive root, say r . Thus

$$Kte^{\mu T} = r$$

$$\text{or} \quad T = \frac{1}{\mu} \log \frac{r}{K} - \frac{1}{\mu} \log t. \quad \dots\dots\dots(5)$$

Having started with four constants, it is to be observed that only two are found in the final equation, which thus becomes an empirical relationship and provides the means of determining only one of the specific values of the constants of reaction on which the reasoning is based. From equation (4) it is obvious that the value of r depends on μ , ν and K , and that from equation (5) only $\frac{r}{K}$ can be determined. Thus μ is the only one of the original constants which survives.

As an example of this formula compared with the facts, the only observations I have found which are sufficiently extensive to compare are those given by Walbum [1915] with reference to the optimum temperature of an *Epeira lysin*. The observations are reproduced in the accompanying table. In order that the optimum temperature with each time of action should be as far as possible independent of personal equation a parabola of the third degree

was fitted to each set of observations. The fitting was done to the observations between 10° and 40° for the experiments which lasted from 10 to 20 minutes and between 0.2° and 32.6° for the experiments which lasted for one hour or more. These observations having been fitted, the maximum was determined in the usual way and a fitting made to equation (5). The results obtained from the experiment are shown in the accompanying table with the theoretical fitting obtained by least squares in parallel columns. Considering the probable error of the optimum obtained from the observations, this must be considered a very good fit.

Table showing the percentages of haemolysis in a series of experiments of different duration and at different temperatures.

Time	0.2°	10.3°	19.0°	27.4°	32.6°	40.4°	45.3°
10 minutes	0	2	12	15	17	10	0
20 "	0	8	19	23	20	11	2
1 hour	7	25	35	30	25	17	9
2 hours	17	35	40	35	30	19	11
5 "	27	48	50	42	35	25	15

Optimum temperature.

Duration of experiment	Actual	Theoretical
10 minutes	29.8	29.3
20 "	26.6	26.4
1 hour	20.5	21.7
2 hours	18.6	18.8
5 "	15.7	15.0

REFERENCE.

Walbum (1915). *Z. Immunitätsforsch.* Orig 23, 623.