A GRAPH FOR STANDARD ERRORS OF RATIOS

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The conventional technique for measuring the reliability of an observed ratio of one type within a population is comparing the ratio with its standard error. Each such reliability test, then, involves solving for the standard error, σ , in the equation

$$\sigma = \sqrt{p(1-p)/N}$$

where p is the ratio, and N the size of sample from which the ratio was obtained.

Frequent use of ratios, as in genetics and biometry, involves much laborious calculation, which may be shortened by the use of PEARSON'S (1924) or EMERSON'S tables. A graphic device for obtaining σ , where not too great accuracy is required, would have possibly an advantage over the former in carrying further the solution of the above equation, and over the latter, of including more values of p and N .

Such a device is here presented in one form of the nomogram or alignment chart as a suggestion that nomography may prove of some usefulness in genetics. The essential feature is the representation on three parallel lines of values for three related variables, so that any straight line, cutting all three, intersects them at values of the three variables which will satisfy the relationship between the variables represented by the nomogram.

BOOTHBY and SANDIFORD (1921) have used a group of nomograms for shortening greatly the time for obtaining human metabolic rate from gasometric data; FELDMAN and UMANSKI (1922) have presented a nomogram similar to one of the above for expressing the Dubois equation for surface area of the human body from its weight and height. L. J. HENDERSON (1928) has devised a more extensive, but similar, graph for representing simultaneous conditions of a number of components of human blood.

In figure 1 are included four independent nomograms for σ , each applicable to a certain range of p values and together extending from 0.5 percent to 99.5 percent. In each, the first scale covers a p range, the third, an N range; and the second, the corresponding σ range: thus scales 1, 11 and 12; 2, 9 and 10; 3, 7 and 8; and 4, 5 and 6 compose the four graphs.

In general, given a ratio and the size of the sample, these values can be located on matched p and N scales. A straight edge between the two points intersects the matched σ scale at the corresponding σ value.

FIGURE 1.—Four nomograms for obtaining the standard error of a ratio.

Thus, with a ratio 26.9 percent determined from 451 individuals, one can locate these values on scales 2 and 10, respectively, of figure 1. A straight edge between the two points intersects scale 9 at 2.1 percent, the standard error of the given ratio with a sample of 451 individuals.

Careful readings do not differ from calculated values of σ by more than ± 1 in the second figure, which falls in either the first or second decimal place when σ is expressed, as here, in percent. With practice one can make the two coincide.

The three-line nomogram can be used for graphic solution of any equation in which the dependent variable, or its logarithm, can be expressed as the sum, or difference of some linear functions of the two independent variables, or the logarithms of these functions.

Here the standard error equation has been expressed as

$$\log \sigma = \frac{1}{2} \log p(1-p) - \frac{1}{2} \log N$$

The heights on the p scales are actually proportional to the values of $\frac{1}{2} \log p(1-p)$ for the p values shown, and, on the N scales, to $\frac{1}{2} \log N$ for the values indicated. The σ scales then represent the difference of the two expressions, which is $\log \sigma$.

The construction of a three-line nomogram is a fairly simple matter since, once lengths for any two scales have been decided upon, the length of the other, size of units of all three, and distances between scales are thereby determined and obtainable from simple relationships given by FELDMAN (1923) in a lucid discussion of nomography.

In table 1 are included the data from which figure 1 was made (on two joined sheets of Schleicher and Schuell paper Number 916). Starting points of scales are given in terms of distance, in inches, from the left and lower borders, respectively; distances from these starting points are expressed in inches. The figure can be reduced any number of times, n , by dividing all dimensions of table 1 by $\sqrt{n}$.

LITERATURE CITED

- BOOTHBY, W. M., and SANDIFORD, R. B., 1921 Nomographic charts for the calculation of the metabolic rate by the gasometer method. *Boston Med. Surg. J.* 185: 337-354.
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- FELDMAN, W. M., 1923 *Biomathematics*. London: C. Griffin and Co.
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- HENDERSON, L. J., 1928 *Blood*. New Haven: Yale University Press.
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TABLE 1

Data for construction of nomograms shown in figure 1, on a 22 inch×36 inch sheet. Starting points of scales are given as distances, in inches, from left and lower borders, respectively, of sheet. The graph may be reduced any number of times, n , by dividing all given dimensions by $\sqrt{n}$.

P ₁		P ₂		P ₃		P ₄	
0 POINT AT 0.32		1.24		2.19		3.21	
SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT
40.0	0.00	25.0	0.00	10.0	0.00	0.5	0.00
40.5	0.35	25.5	0.64	10.5	0.94	1.0	2.99
41.0	0.68	26.0	1.26	11.0	1.83	1.5	4.73
41.5	1.00	26.5	1.86	11.5	2.67	2.0	5.95
42.0	1.29	27.0	2.44	12.0	3.47	2.5	6.90
42.5	1.57	27.5	3.00	12.5	4.23	3.0	7.67
43.0	1.82	28.0	3.54	13.0	4.96	3.5	8.32
43.5	2.06	28.5	4.07	13.5	5.66	4.0	8.87
44.0	2.28	29.0	4.57	14.0	6.32	4.5	9.36
44.5	2.49	29.5	5.06	14.5	6.95	5.0	9.80
45.0	2.67	30.0	5.54	15.5	8.15	5.5	10.19
45.5	2.84	30.5	6.00	16.0	8.71	6.0	10.54
46.0	2.99	31.0	6.44	16.5	9.25	6.5	10.87
46.5	3.12	31.5	6.86	17.0	9.76	7.0	11.17
47.0	3.23	32.0	7.27	17.5	10.26	7.5	11.44
48.0	3.40	32.5	7.67	18.0	10.74	8.0	11.70
49.0	3.51	33.0	8.06	18.5	11.20	8.5	11.94
50.0	3.55	33.5	8.42	19.0	11.65	9.0	12.16
		34.0	8.78	19.5	12.08	9.5	12.38
		34.5	9.12	20.0	12.49	10.0	12.57
		35.0	9.45	20.5	12.89		
		35.5	9.74	21.0	13.28		
		36.0	10.07	21.5	13.65		
		36.5	10.36	22.0	14.01		
		37.0	10.63	22.5	14.36		
		37.5	10.90	23.0	14.70		
		38.0	11.16	23.5	15.02		
		38.5	11.40	24.0	15.34		
		39.0	11.63	24.5	15.64		
		39.5	11.85	25.0	15.94		

TABLE 1 (continued)

σ_1		N_1		σ_2		N_2	
6.5	11.48	10	12	20	3.32	14	19
SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT
0.157	0.00	25	0.00	0.69	0.00	25	0.00
0.160	0.18	30	0.79	0.70	0.12	30	0.79
0.170	0.34	35	1.46	0.80	1.09	35	1.46
0.180	0.59	40	2.04	0.90	1.92	40	2.04
0.190	0.83	45	2.55	1.00	2.70	45	2.55
0.200	1.05	50	3.01	1.10	3.40	50	3.01
0.220	1.46	55	3.42	1.20	4.02	55	3.42
0.240	1.84	60	3.80	1.30	4.60	60	3.80
0.260	2.19	65	4.15	1.40	5.14	65	4.15
0.280	2.51	70	4.47	1.50	5.64	70	4.47
0.300	2.81	75	4.77	1.60	6.11	75	4.77
0.320	3.09	80	5.05	1.70	6.54	80	5.05
0.340	3.35	85	5.31	1.80	6.96	85	5.31
0.360	3.60	90	5.56	1.90	7.35	90	5.56
0.380	3.84	95	5.80	2.00	7.72	95	5.80
0.400	4.06	100	6.02	2.10	8.07	100	6.02
0.420	4.27	110	6.43	2.20	8.41	110	6.43
0.440	4.47	120	6.81	2.30	8.73	120	6.81
0.460	4.67	130	7.16	2.40	9.04	130	7.16
0.480	4.85	140	7.48	2.50	9.34	140	7.48
0.500	5.03	150	7.78	2.60	9.62	150	7.78
0.550	5.44	160	8.06	2.70	9.90	160	8.06
0.600	5.82	170	8.32	2.80	10.16	170	8.32
0.650	6.17	180	8.57	2.90	10.41	180	8.57
0.700	6.49	190	8.81	3.00	10.66	190	8.81
0.750	6.79	200	9.03	3.20	11.12	200	9.03
0.800	7.07	210	9.24	3.40	11.56	210	9.24
0.850	7.33	220	9.44	3.60	11.97	220	9.44
0.900	7.58	230	9.64	3.80	12.37	230	9.64
0.950	7.82	240	9.82	4.00	12.74	240	9.82
1.000	8.04	250	10.00	4.20	13.09	250	10.00
1.100	8.45	260	10.17	4.40	13.43	260	10.17
1.200	8.83	270	10.33	4.60	13.76	270	10.33
1.300	9.18	280	10.49	4.80	14.06	280	10.49
1.400	9.50	290	10.64	5.00	14.35	290	10.64
1.500	9.80	300	10.79	5.20	14.64	300	10.79
1.600	10.08	320	11.07	5.40	14.91	320	11.07
1.700	10.34	340	11.33	5.60	15.17	340	11.33
1.800	10.59	360	11.58	5.80	15.43	360	11.58
1.900	10.83	380	11.82	6.00	15.67	380	11.82
2.000	11.05	400	12.04	6.20	15.91	400	12.04
2.100	11.26	420	12.25	6.40	16.14	420	12.25
2.200	11.46	440	12.45	6.60	16.36	440	12.45
2.300	11.65	460	12.65	6.80	16.58	460	12.65
2.400	11.84	480	12.83	7.00	16.79	480	12.83
2.500	12.02	500	13.01	7.20	16.99	500	13.01
2.600	12.19	550	13.42	7.40	17.19	550	13.42
2.700	12.35	600	13.80	7.60	17.38	600	13.80
2.800	12.51	650	14.14	7.80	17.57	650	14.14
2.900	12.66	700	14.47	8.00	17.75	700	14.47
3.000	12.81	750	14.77	8.20	17.93	750	14.77
3.200	13.09	800	15.05	8.40	18.11	800	15.05
3.400	13.35	850	15.31	8.60	18.28	850	15.31

TABLE 1 (continued)

σ_4		N_4		σ_3		N_3	
6.5	11.48	10	21	12	3.32	14	19
SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT
3.600	13.60	900	15.56			900	15.56
3.800	13.84	950	15.80			950	15.80
4.000	14.06	1000	16.02			1000	16.02
4.200	14.27	1100	16.43			1100	16.43
4.400	14.47	1200	16.81			1200	16.81
4.600	14.67	1300	17.16			1300	17.16
4.800	14.85	1400	17.48			1400	17.48
5.000	15.03	1500	17.78			1500	17.78
5.200	15.20	1600	18.06			1600	18.06
5.400	15.36	1700	18.32			1700	18.32
5.600	15.52	1800	18.57			1800	18.57
5.800	15.67	1900	18.81			1900	18.81
6.000	15.82	2000	19.03				

σ_2		N_2		σ_1		N_1	
16	3.09	18	24	20	3.07	22	32
SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT
1.4	0.00	25	0.00	1.58	0.00	25	0.00
1.5	0.78	30	1.19	1.6	0.38	30	1.56
1.6	1.52	35	2.19	1.7	1.34	35	2.92
1.7	2.22	40	3.06	1.8	2.24	40	4.08
1.8	2.87	45	3.83	1.9	3.09	45	5.10
1.9	3.49	50	4.52	2.0	3.90	50	6.02
2.0	4.08	55	5.14	2.1	4.67	55	6.85
2.1	4.64	60	5.70	2.2	5.41	60	7.60
2.2	5.17	65	6.22	2.3	6.11	65	8.30
2.3	5.68	70	6.70	2.4	6.78	70	8.94
2.4	6.16	75	7.16	2.5	7.43	75	9.54
2.5	6.63	80	7.57	2.6	8.05	80	10.01
2.6	7.08	85	7.97	2.7	8.64	85	10.63
2.7	7.51	90	8.34	2.8	9.22	90	11.13
2.8	7.93	95	8.70	2.9	9.77	95	11.59
2.9	8.33	100	9.03	3.0	10.31	100	12.04
3.0	8.72	105	9.35	3.1	10.82	105	12.46
3.1	9.09	110	9.65	3.2	11.32	110	12.87
3.2	9.46	115	9.94	3.3	11.81	115	13.25
3.3	9.81	120	10.22	3.4	12.28	120	13.62
3.4	10.15	125	10.48	3.5	12.74	125	13.98
3.5	10.48	130	10.74	3.6	13.18	130	14.32
3.6	10.80	135	10.98	3.7	13.62	135	14.65
3.7	11.12	140	11.22	3.8	14.04	140	14.96
3.8	11.42	145	11.45	3.9	14.45	145	15.27
3.9	11.72	150	11.67	4.0	14.85	150	15.56
4.0	12.01	155	11.88	4.1	15.24	155	15.85
4.1	12.29	160	12.09	4.2	15.62	160	16.12
4.2	12.57	165	12.29	4.3	15.99	165	16.39
4.3	12.84	170	12.49	4.4	16.35	170	16.65
4.4	13.10	175	12.68	4.5	16.71	175	16.90

TABLE 1 (continued)

σ_2		N_2		σ_1		N_1	
16	3.09	18	24	20	3.07	22	32
SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT	SCALE READING	DISTANCE ABOVE ZERO POINT	SCALE READING	DISTANCE BELOW ZERO POINT
4.5	13.36	180	12.86	4.6	17.06	180	17.15
4.6	13.61	185	13.04	4.7	17.40	185	17.38
4.7	13.85	190	13.21	4.8	17.73	190	17.62
4.8	14.10	195	13.38	4.9	18.05	195	17.84
4.9	14.33	200	13.55	5.0	18.37	200	18.06
5.0	14.56	210	13.86	5.1	18.69	210	18.49
5.2	15.01	220	14.17	5.2	18.99	220	18.89
5.4	15.44	230	14.46	5.3	19.29	230	19.28
5.6	15.86	240	14.73	5.4	19.59	240	19.64
5.8	16.26	250	15.00	5.5	19.88	250	20.00
6.0	16.65	260	15.26	5.6	20.16	260	20.34
6.2	17.02	270	15.50	5.7	20.44	270	20.67
6.4	17.39	280	15.74	5.8	20.72	280	20.98
6.6	17.74	290	15.97	5.9	20.99	290	21.29
6.8	18.08	300	16.18	6.0	21.25	300	21.58
7.0	18.41	320	16.61	6.2	21.77	310	21.87
7.2	18.74	340	17.00	6.4	22.27	320	22.14
7.4	19.05	360	17.37	6.6	22.76	330	22.41
7.6	19.35	380	17.73	6.8	23.23	340	22.67
7.8	19.65	400	18.06	7.0	23.69	350	22.92
8.0	19.94	420	18.38	7.2	24.13	360	23.17
8.2	20.22	440	18.68	7.4	24.56	370	23.40
8.4	20.50	460	18.97	7.6	24.98	380	23.64
8.6	20.77	480	19.25	7.8	25.40	390	23.86
8.8	21.03	500	19.52	8.0	25.80	400	24.08
9.0	21.29	550	20.14	8.2	26.18	410	24.30
9.2	21.54	600	20.70	8.4	26.57	420	24.51
9.4	21.79	650	21.22	8.6	26.94	430	24.71
9.6	22.03	700	21.71	8.8	27.30	440	24.91
9.8	22.26	750	22.16	9.0	27.66	450	25.11
		800	22.58	9.2	28.00	460	25.30
		850	22.97	9.4	28.34	470	25.48
		900	23.34	9.6	28.67	480	25.66
		950	23.70	9.8	29.00	490	25.84
				10.0	29.32	500	26.02
						525	26.44
						550	26.85
						575	27.23
						600	27.60
						625	27.96
						650	28.30
						675	28.63
						700	28.94
						725	29.25
						750	29.54
						775	29.83
						800	30.10
						825	30.37
						850	30.63
						875	30.88
						900	31.13
						925	31.36
						950	31.59
						975	31.82