DICE-CASTING AND PEDIGREE SELECTION. EXPERIMENTS WHICH PICTURE MATHEMATICALLY CLOSE ANALOGIES BETWEEN DICE-CASTING AND CERTAIN BREEDING PHENOMENA

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INTRODUCTION

There is a striking analogy between a situation which may be described on the several faces of a cube, and the actual potentiality of a given genotype to throw a number of phenotypes. By casting the dice according to given rules, the analogy may be carried further to the extent of illustrating the mathematical nature of filial regression, of rating pure lines, and of analyzing the nature of selection within them. In the improvement of domestic plants and animals, practical breeders, from time immemorial, have followed the empirical method of selection based upon somatic qualities. With the advent of modern genetics breeders are paying more attention to pedigree systems of improvement, and have often sought the aid of genetic analysis. The contrast between the outcome of selection based upon somatic qualities and of selection based upon attention to germinal constitution, may be shown clearly by dice-casting.

The several experiments which are here selected for description, including the recording, tabulation, and the analysis of data, may be completed in the course of a few hours. The only apparatus needed (figure 1) consists

of one dice cup and six blank dice, which latter may be labelled with pen or pencil to suit the particular experiment.

Figure 2 gives the surface patterns and lettering of the dice actually used in these demonstrations. The face values begin with 1 and continue up to 6 for die a; for die b they begin at 2 and continue up to 7, and so on through the series, die f beginning with 6 and ending with 11, the highest value.

FIGURE 1.—Apparatus needed. One dice cup, and six blank dice which may be labeled with pen or pencil.

FIGURE 2.—Surface patterns and lettering of dice used in the experiments here described.

BIOLOGICAL ANALOGIES

Table 1 presents a short statistical treatment of the possible somatic values, that is, the phenotypes, of the offspring of a given definite genotype. Each die of the whole series, a, b, c, d, e, and f, regardless of the values of its several faces, represents the definite germinal constitution, that is, the genotype, of a self-fertilized parent. Upon examination of the values

of the several faces of each die, the experimenter learns the potential range of value of each fraternity in the f_1 generation, derived from the several particular genotypes. If the demonstrator cares for a more elaborate system, he can use many-sided symmetrically or even irregularly shaped and weighted solids for the purpose of approaching, perhaps more nearly, the real biological situation. But in so doing, the directness and simplicity of the experiment would be somewhat sacrificed. It would probably not be worth making the change to the irregular or more complicated system, because the purpose of this demonstration is not to prove any biological fact, but, by analogy, to picture certain mathematical phenomena of biological facts and reactions already known.

TABLE 1

Description of the six dice used, and their biological analogies.

The Die is the (pedigree) level of some die with same letter.	The Die's Six Faces (i.e. the several—in this case 6—possible somatic variation-values—phenotypes—of the offspring of a given definite germinal constitution—genotype—of a self-fertilized parent.)											Sum of face values of same die	Mean face value of same die
	a	1	2	3	4	5	6					21	3.5
	b		2	3	4	5	6	7				37	4.5
	c			3	4	5	6	7	8			33	5.5
	d				4	5	6	7	8	9		29	6.5
	e					5	6	7	8	9	10	45	7.5
	f						6	7	8	9	10	51	8.5
	Face Value	1	2	3	4	5	6	7	8	9	10	For all 6 dice # faces = 36 # face values = 216 Mean face value = 6	
	Frequency	1	2	3	4	5	6	5	4	3	2		
	F x V	1	4	9	16	25	36	35	32	27	20		

— Pedigree selectionists generally, and pure line selectionists especially, view in this direction. —
— Potential values. —

By examining the several phenotypes (faces of the several dice), the experimenter may record their observed values, as was done in the preparation of table 1, which shows the distribution of face values among the several dice. He may then proceed to determine the sum of the several face values of the same die and the mean of such values. He may determine also the frequency of each face value as found in the whole collection of dice, and may proceed to calculate the mean value for the entire "potential population," that is for all of the face values of all of the dice.

It is not a complex picture of all that takes place in nature, but the barest and most direct analogy to the statistical essentials of the case, that we now desire. Analogies are useful for illustrating principles, but they all have their limitations in respect both to possible parallelism and to

usefulness. If all parallels were exact to the end, then, of course, the analogy would cease, and instead of having dice with numbers, we should have to deal with the genotype itself as found in actual living organisms. One advantage of dice as analogies is that an experimenter may pursue a number of "generations" in a few minutes, and may at the same time have "a peep behind the scenes," as it were, when he picks up the die, and examines and lists the whole range of "phenotypic" possibilities of the "offspring." The actual breeder at the beginning of his work does not know the nature of the genotype, that is to say, he at first, can see only the somatic aspect of the selected parent. With dice, however, the "genotype," or the potentiality for throwing a number of different values, can be determined by picking up the particular die and examining it. This insight is permissible because it makes the analysis of statistical results much clearer. A selected face value represents a parent, and the face values which come up from successive castings are possible f_1 offspring. Further one has the advantage of casting as many times as he chooses. In nature the greater the number of offspring, the earlier and more surely will the experimenter exhaust the phenotypic possibilities. Also, conversely, the smaller the number of such possibilities, the more quickly will they be found.

PARENTS AND OFFSPRING. REGRESSION

Table 2 presents the next step in logical analysis. In this table there is given a mathematical picture of a "breeding experiment." The experimenter examines all of the "potential parents," and first classifies them according to "face" values. In column 2 he records the number of "parents" within whose "genotypes" each of the several different values are found. In the next column he records the names of all the "genotypes" which carry the various selected values. He then, by examination or casting, finds the mean value of all of the "potential offspring" of each particular "genotype." The results are recorded in column 4. He then casts each particular group of "genotypes" a hundred times. In column 5 he records the number of "offspring" resulting from such castings; in column 6 is recorded the mean actual value of all of the "offspring" registered in column 5. A comparison between the values in columns 4 and 6 shows the difference between the mean absolute "genotypic" values and the values actually attained in the particular experiment.

Practically, table 2 was constructed as follows: First the dice were laid on the table and carefully turned until all faces (in this case

on one die only) with the face value of 1, were found. Then these were turned up for "parents" and cast 100 times; the mean face value was found to be 3.58. Next the six dice were again laid on the table and all of them with 2's were sorted out. The latter faces were then turned up for

TABLE 2
Data recording numbers and values of "parents" and their "offspring."

¹ *	2	3	4	5	** 6
Selected Face Values (i.e., Parents)	Number of dice on which selected values are found.	Name of dice, (i.e., Genotypes) on which selected values are found.	Absolute mean value of faces, (i.e., of possible phenotypes)	Number of faces cast up by 100 castings of group (i.e., number of offspring)	In an actual experiment mean cast up face value (i.e., mean value of offspring)
All 1's	1	a	3.5	100	3.58
All 2's	2	ab	4.0	200	3.75
All 3's	3	abc	4.5	300	4.64
All 4's	4	abc a	5.0	400	5.10
All 5's	5	abc de	5.5	500	5.67
All 6's	6	abc def	6.0	600	5.99
All 7's	5	bc def	6.5	500	6.54
All 8's	4	def ^c	7.0	400	7.04
All 9's	3	def	7.5	300	7.52
All 10's	2	ef	8.0	200	7.84
All 11's	1	f	8.5	100	8.37

"parents" and cast 100 times; the mean of their face values was found to be 3.75; and so on, thus selecting and casting through the series of 3's and 4's to the 11's.

Figure 3 uses the facts recorded in table 2 in such a manner as to demonstrate what is meant by such terms as "filial regression," "rever-

sion," "the drag of the race," and "the pull of mediocrity." The phenomena of dice-casting and of biological reproduction are treated statistically in the same manner. All of the data of table 2 were transferred to the regression plot. When the lines are duly connected up, we find

Mean values of cast up faces—column 6** table 2 (i.e., mean value of "offspring of same parental genotype.")

FIGURE 3.—Regression diagram, plotted from the data of table 2.

mR = mean of all face values (absolute) i.e., "mean of the race." (36 dice-faces.)

SP = line connecting values of selected face-value groups (i.e., "parental values.")

CO = regression line. (Dotted line theoretical, solid line actual, values found in 100 tossings of each group.)

a typical "filial regression" diagram. Now, by referring to table 1, the description of the dice used, we find that these regression values instead of running 3.58, 3.75, 4.64, 5.10, 5.67, 5.99, 6.54, 7.04, 7.52, 7.84, 8.37, actually run 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, making the theoretical regression line absolutely straight, and giving a "filial regres-

sion" of the value of 0.5. An indefinitely great number of tosses would of course, tend to convert the experimental values into the real values.

At this point the experimenter may, if he be interested, make a short clarifying excursion into the meaning of probable error. Here he knows the real values and by calculating the probable error for several series of experimental approximations which are determined from various numbers of samples, he sees with great clarity the relation between the number of samples and approximation to the truth.

The experimenter learns that with dice as well as with living organisms, the "offspring" of "genotypes" selected on the basis of extreme values will less nearly approximate their "parental" values and will more nearly approximate their "racial mean" than will the "offspring" of "genotypes" whose "somatic" values are found to lie nearer the mean value of the "race." By first casting the dice and preparing table 2, and then, in the regression diagram, plotting the data thus found, the experimenter receives a clear conception of what is meant by the term "filial regression," and he sees clearly several of the mathematical properties and the reasons for the existence of several associated phenomena. Thus, the regression line plotted from the experiment is straighter near the middle than at the extremities. This is due to the fact that while the number of tosses was the same for each "parental" value, the number of "parents" used varied from one on the extremes to six in the center, thus a regression value of the center group is based upon six times as many individual "offspring" as the regression values of the "parental" values 1 and 11.

Besides serving other purposes, regression is a sort of measure of the phenotypic range of a given genotype. If the regression be found very small, it may mean a number of things, for example, great homozygosity of the parents in reference to their own somatic standards, or a great "anti-mean" influence of environment upon the children. If the regression be very great, it may stand for considerable heterozygosity of the parents in reference to their own somatic measurements, or homozygosity in reference to the racial mean, possibly augmented in the children by a "racial mean-tending" influence of environment. This is nicely illustrated in DAVENPORT'S (1917) work on human stature. Among the children of tall parents he found but little regression, indicating relatively great constancy of germinal properties. Among children of shorter parents, however, he found a greater regression, indicating more complex and less constant hereditary natures.

It is obvious to the observer that figure 3, and the data from which it is constructed, could be used to good purpose for demonstrating the statistical nature of correlation.

RATING PURE LINES

If we turn now to table 3, "The determination and rating of pure lines," we may, by continuing the view behind the scenes (that is, the examination of the dice used for parents), come to an understanding of the theory of

TABLE 3

The determination and rating of pure lines. Letters describe genotypes and, in F_1 , locate members of same pure line.

P_1	Let actual chance casting represent P_1 Phenotypes.	Order → An actual chance casting	1	2	3	4	5	6
			4a	5c	6b	6e	8f	9d
F_1	Cast each die 10 times to find the approximate mean value of its several (6) faces, i.e., to find the approximate mean value of the several phenotypic aspects of each selected genotype.)	The 10 castings, (i.e., values of the 10 offspring.)	5a	4c	3b	6e	6f	4d
			1a	3c	5b	6e	10f	8d
			6a	5c	3b	10e	10f	7d
			2a	5c	7b	7e	7f	8d
			2a	6c	5b	10e	11f	9d
			5a	6c	3b	10e	7f	8d
			4a	7c	5b	9e	11f	5d
			5a	8c	6b	8e	11f	8d
			4a	4c	3b	9e	9f	9d
			6a	4c	7b	7e	7f	5d
Pure-Line Determination.	ΣF_1 values		40	52	47	82	89	71
	mean F_1 values (10 offspring)		4.0	5.2	4.7	8.2	8.9	7.1
	Order of mean values of F_1 (Compare with P_1 order.)		1 _a	3 _c	2 _b	5 _e	6 _f	4 _d
	Absolute mean value of 6 faces of die, (i.e. F_1 phenotypic range, or mean value of the several "Pure Lines.")		3.5 _a	5.5 _c	4.5 _b	7.5 _e	8.5 _f	6.5 _d
	Order of mean of absolute face values (i.e., mean of F_1 Phenotypes.) (Compare with above order-ranks.)		1 _a	3 _c	2 _b	5 _e	6 _f	4 _d

the pure line, which appears to be a real biological principle of practical value to the breeder. Let us cast the six dice on the table. In the actual experiment recorded in table 3, they turned up as follows: 4a, 5c, 6b, 6e, 8f, and 9d. They are next arranged in order of values, regardless of the meaning of the letters. 4a was considered a p_1 parent, and cast 10 times for the 10 f_1 offspring. The mean value of these offspring was found to be 4.0. Similarly the remaining mean values ran: of 5c, 5.2; of 6b, 4.7; of 6c, 8.2; of 8f, 8.9; of 9d, 7.1. Then by examining the labels the mean values of the p_1 parents were found in the following ascending order; a, c, b, e, f, d. The mean value of the f_1 offspring of each parent gave the following order: a, b, c, d, e, f,—the actual ascending values which we found in earlier experiments. It is noticed that whereas the array of somatic or phenotypic values in the p_1 generation gave one order of values to the several letters, this order was a false one, so far as the potentialities of the line are concerned; the f_1 order is the true one.

We may say, then, that the self-fertilized "offspring" of face 4a constitute "pure line a," of face 5c "pure line c," and so on. Now the principal characteristic of the pure line is that, so far as hereditary constitution, that is, the genotype, is concerned, one individual member is the same as another, and consequently will produce the same sort of offspring. This is tested in tables 4 and 5.

SELECTION WITHIN PURE LINES

The next logical step is registered in experiment 1, table 4. The experimenter selects a low " p_1 " value, as face 1, die a. He casts the selected die twenty-five times in order to find the approximate mean value of the " f_1 offspring," and also to determine the range of possibility in variation among the offspring. He selects the lowest value of these twenty-five "offspring" as a "parent," and casts the die that bore it twenty-five times for the " f_2 generation," and continues the same process and records through the " f_3 generation," (or longer if he be skeptical). He finds that the mean of the third "generation" is 3.2, that the mean of the whole line is 3.41. Compared with this he selects face value 6 from the die of "pure line a," and selects for high values. He finds the mean value of the third "filial generation" to be 4.28, and of the whole line to be 3.88. While he began on the one hand, with 1, and the other with 6, he has at the end of 3 generations mean values of 3.41 and 3.88 respectively. In real life, a selectionist might believe that he was making some progress, but our knowledge of the dice in this particular analogy shows us that difference in the direction of selection is a mere chance. This slight

temporary difference in favor of the direction of selection is one of the characteristic phenomena of pure-line manipulation. If we should continue it through many generations, the "line 1a" would many times fluctuate

TABLE 4

The nature of selection within the pure line. Each figure represents one actual casting of the die, which in turn represents one offspring.

Actual Experiment No.1. Showing the Futility of Selection within the Pure Line.			Actual Experiment No.2. Showing that A Small Value from a Superior Line Constitutes a Better Parent than an even Higher Value from an Inferior Line.		
Selected P_i	Face, (i.e., Somatic) value = 1 Member of Pure Line a. (Select for low values)	Face, (i.e., Somatic) value = 6 Member of Pure Line a. (Select for high values)	Selected P_i	Face, (i.e., Somatic) value = 4 Member of Pure Line d. (Select for high values)	Face, (i.e., Somatic) value = 7 Member of Pure Line b. (Select for high values)
F_1	4 2 4 *1 3	5 5 4 1 5	F_1	7 4 6 6 8	3 5 5 4 5
	4 2 3 1 4	2 5 1 4 3		6 7 5 4 *9	2 5 3 7 6
	5 3 4 5 6	2 6 5 1 1		6 8 5 8 8	5 2 3 6 7
	3 2 5 1 4	6 2 6 *6 5		6 5 7 9 4	4 4 5 3 6
	6 1 1 6 5	1 2 6 3 5		5 9 7 6 7	4 4 *7 6 5
	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 3.4	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 3.68		25 Casting values i.e. "Offspring" Range = 1 to 9 Mean = 6.48	25 Casting values. i.e. "Offspring" Range = 1 to 7 Mean = 4.6
F_2	4 *1 2 4 5	4 5 3 1 *6	F_2	7 7 4 7 *9	7 5 4 5 *7
	4 5 3 6 2	3 6 1 1 6		6 6 7 5 6	3 6 4 5 6
	3 5 3 1 5	3 3 1 2 5		7 4 8 7 8	4 5 5 4 3
	2 3 4 6 1	3 5 5 6 3		9 4 4 7 7	2 4 4 5 7
	6 4 5 5 2	1 5 5 5 4		6 5 6 7 8	3 3 2 4 5
	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 3.64	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 3.68		25 Casting values i.e. "Offspring" Range = 1 to 9 Mean = 6.44	25 Casting values i.e. "Offspring" Range = 1 to 7 Mean = 4.6
F_3	*1 4 1 2 6	6 6 6 2 2	F_3	7 8 9 6 5	3 3 3 4 4
	5 2 4 5 3	6 6 4 5 6		5 6 6 7 4	7 6 5 4 4
	3 5 1 1 1	4 4 6 5 3		5 6 7 6 6	5 4 3 5 3
	4 3 2 3 5	5 5 1 1 5		7 9 9 *9 8	2 6 4 6 7
	4 3 5 5 2	4 5 *6 2 2		7 6 6 7 7	3 5 *7 3 3
	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 3.2	25 Casting values i.e. "Offspring" Range = 1 to 6 Mean = 4.28		25 Casting values i.e. "Offspring" Range = 1 to 9 Mean = 6.72	25 Casting values i.e. "Offspring" Range = 1 to 9 Mean = 4.32
	Range of Line 1 to 6 Mean of Line 3.41	Range of Line 1 to 6 Mean of Line 3.88		Range of Line 1 to 9 Mean of Line 6.54	Range of Line 1 to 7 Mean of Line 3.50

NOTE:—* indicates parents selected for next generation.

in mean value, past "line 6a," but of course, the two real mean values are infallibly fixed at identity. In the case of dice-casting a little reflection shows the reason for this apparent progress. Possibly analogous causes exist in other selection phenomena within living species.

Now let the experimenter cast the six dice representing six p_1 individuals (experiment 2, table 4). If he finds "die d" showing a face value of 4, and die b a face value of 7 (as actually came up for the demonstration here recorded), he knows that he has a situation which he desires, because "line b" is of lower average value than "line d." He then repeats the experiment just described in selecting for high values through three or more generations. At the end of the experiment he finds that, regardless of the values of the original "parents," the true mean value of the line is soon apparent. Indeed here again the principal story is told in the " f_1 generation," and indeed the " f_1 generation" would tell it completely if there were enough castings made. Carrying the selection through further "generations" only serves to verify and emphasize this phenomenon. In the inferior "line b" the selection started with the chance high value 7, and ended with the mean value 4.32, approximating the true mean 4.5 of the line. In the "better line d," a chance low value 4 ended in a mean value of 6.54, true mean 6.5. Thus the principal practical corollaries from selection read as follows:

a. A small value from a superior line constitutes a better parent than an even higher value from an inferior line.

b. Selection within a pure line is futile. In both cases selection only uncovers the mean potential value of the line; it does not create. We must cross-breed, that is, introduce new blood, or hope for mutations, if we would change or improve it further.

If the experimenter cares to make the pure-line demonstration a little more vivid historically, he may parallel JOHANNSEN'S famous "19-bean experiment," by labelling nineteen dice as follows: 1, 2, 3, 4, 5, 6; 2, 3, 4, 5, 6, 7; and so on to 19, 20, 21, 22, 23, 24. The processes described in tables 3, 4, and 5, may then be followed out.

SELECTION: SOMATIC BASIS

Proceed now to table 5, "Selection based upon somatic qualities." This table was built upon four actual dice-casting experiments. In each case the selection was begun with six "unknown" dice ("self-fertilized parents"), and was then continued, seeking to establish (a) a strain of lowest average value, and (b) one of highest average value. To begin each experiment, first the six dice were cast; the upturned faces represented the six different parental values. The lowest of these "parental values," face 3 of die a, was taken for the "ancestry" of the " f_1 generation" in seeking to establish a low "strain;" for the high "strain" face 7 of die e was selected. Ten castings of each of the selected " p_1 parents" gave

the ten " f_1 offspring." Then in the low "strain" the lowest of these ten " f_1 offspring" was selected for the "parent of the f_2 generation;" and the highest of the "offspring" of the high "strain" was selected for the "parent" of the " f_2 high strain." This type of casting and selection was continued through four generations.

The comments which are written in the last column under each experiment are those which the practical breeder would be apt to make in seeking to interpret what he had actually found. We must remember that without pedigree analysis, the actual breeder sees only somatic values; the genotypic constitutions (represented by letters and figures on the dice) which we have added to guide us through these experiments, are unknown to him.

The dice-caster sees at first-hand that his selection had only sorted out and brought to light the highest values at first hidden, and that it had not created anything that did not previously exist. He learns also that a single experiment of this type does not suffice to reach the limits of possibilities of selection; that the weakness in this system consists principally in its dependence upon chance in the " p_1 generation" to isolate the highest and lowest "genotypic" values. Thus in the four experiments here tabulated, in seeking the lowest "strain," genotype a, which in fact constituted the lowest "strain," was segregated three times, and the next lowest, "genotype b," once. In seeking the highest strain, genotype f, the actual highest was isolated twice, genotype e, the next-highest, once, and genotype d, the next highest after e, once. In this system there is, then, a relation between the number of potential strains and the number of experiments necessary for their determination.

It would seem that the practical breeder, by following his usual somatic methods of selection, works largely in the dark. He, nevertheless, as the splendid results in plant and animal breeding show, has made substantial progress. A closer attention to genotypic analysis would doubtless result in more rapid progress, as we have demonstrated in tables 3 and 4.

The tables and diagrams here presented consider principally the case of pure lines, that is, of the descendants of self-fertilized individuals. Originally JOHANNSEN (1903) defined a pure line as "the progeny of a homozygous, self-fertilized individual." Later JENNINGS modified, or rather extended the definition to mean "all of the descendants of a single self-fertilized individual." In leaving out the requisite "homozygous," the change was not so great as might at first be expected, because continued self-fertilization makes rapidly for homozygosity (see EAST and JONES 1919, chapter 5; JENNINGS 1916, 1917; PEARL 1917).

CONSTRUCTION OF TABLE 5

Selection based upon somatic qualities

- A. Purpose: In each experiment beginning with the same 6 unknown dice, (i.e., self-fertilized parents), and continuing selection, without knowing number of sides or their labels (i.e., the range or number of potential types of offspring, save as they appear in the tossing, i.e., breeding), seek to establish (a) a strain of *lowest* average value, and (b) one of *highest* average value.
- B. Instructions: 1. In p_1 generation cast all dice and arrange in somatic (i.e., face) order.
2. Select as parents the individual soma, (i.e., face) with the lowest value for founding the (—) strain, and with the highest value for founding the (+) strain.
3. Continue this system. In each succeeding generation cast the selected parent 10 times to represent the 10 offspring.
4. The soma, (i.e., face) bearing most strongly in the desired direction (marked *) is saved for breeding, (i.e., for casting the next generation); the remainder are measured to find the mean of the generation, and are then discarded.

TABLE 5

Four actual dice-casting experiments illustrating the nature of selection based upon somatic qualities.

Actual Experiment No.1										Actual Experiment No.2										Actual Experiment No.3										Actual Experiment No.4									
strain										strain										strain										strain									
P ₁	*3a	4c	4d	5b	6f	*7e	Mean of P ₁ = 4.83	Notes		P ₁	*2a	4c	6f	7b	*8d	Mean of P ₁ = 5.66	Notes		P ₁	*1a	2b	4d	4c	9e	*9f	Mean of P ₁ = 4.63	P ₁	3b	3c	3a	7d	8e	11f	Mean of P ₁ = 4.83					
F ₁	*1a					10e	Notes Save for breeding, then discard the remainder of each generation. Mean = 2.2	F ₁	3a					5d	Notes + Strain Parent of F ₂ - Strain Parent of F ₂ Mean = 3.2	F ₁	4a					6f	Notes - strain about as in Experiment No.2. + strain good if appears for the first time. Save it for seed. Mean = 8.0	F ₁	4b					11f	Notes Poorest results with - strain, but best with +. Evidently a thrifty season. Mean = 9.6								
	2a					7e			5a				6d	5a							7f	6b										11f							
	1a					10e			6a				5d	4a							7f	7b										11f							
	4a					7e			2a				9d	1a							10f	4b										7f							
	3a					9e			6a				9d	3a							*11f	4b										11f							
	2a					8e			2a				8d	6a							7f	3b										11f							
	2a					8e			2a				*9d	3a							11f	5b										9f							
	3a					7e			4a				8d	*1a							8f	7b										8f							
	2a					5e			*1a				9d	2a							7f	4b										*11f							
	Mean	2.2							Mean = 7.9	Real progress.		Mean = 3.2							Mean = 7.2	Real progress but not so marked as in Experiment No.1		Mean = 3.2										Mean = 8.0		Mean = 4.6					
F ₂	6a					6e	Notes A little discouraging. May be a bad season. Mean = 6.9	F ₂	2a					4d	Notes - strain Stationary. + strain degenerates a little. Mean = 6.5	F ₂	3a					11f	Notes - strain looses some of its smallness. + strain improves. Mean = 8.5	F ₂	3b					11f	Notes Both strains move in the undesired direction. Mean = 9.1								
	6a					7e			2a				9d	1a							6f	6b										9f							
	1a					7e			4a				5d	2a							*11f	6b										7f							
	5a					8e			1a				9d	5a							10f	3b										10f							
	*1a					7e			4a				7d	*1a							8f	6b										10f							
	2a					6e			3a				4d	6a							6f	7b										*11f							
	6a					5e			4a				9d	4a							10f	4b										8f							
	4a					8e			*1a				5d	5a							8f	*3b										10f							
	2a					5e			5a				*9d	2a							7f	6b										7f							
	1a					*10e			6a				4d	5a							8f	mean										mean							
Mean	3.4					Mean = 6.9			Mean = 3.2					Mean = 6.5			Mean = 3.4							Mean = 8.5		Mean = 5.1						Mean = 9.1							
F ₃	3a					8e	Notes No sign of progress. Wait and see. Mean = 6.8	F ₃	6a					5d	Notes - strain looses some of its smallness. + strain constant. Mean = 6.5	F ₃	4a					*11f	Notes Continued gain in - strain. Loss in + strain, but still greater than anything in Experiments Nos. 1 or 2. Mean = 8.1	F ₃	6b					10f	Notes - strain constant. + strain shows a little recovery. Mean = 9.4								
	3a					6e			2a				5d	*1a							7f	7b										7f							
	2a					7e			6a				7d	6a							6f	7b										10f							
	*1a					*10e			*1a				7d	6a							9f	6b										10f							
	1a					5e			3a				*9d	4a							7f	3b										11f							
	5a					7e			5a				4d	4a							6f	5b										9f							
	3a					5e			6a				7d	5a							10f	2b										*11f							
	6a					8e			1a				6d	4a							10f	*2b										7f							
	6a					7e			3a				8d	6a							6f	6b										11f							
	6a					5e			1a				7d	1a							9f	7b										8f							
Mean	3.6					Mean = 6.8			Mean = 3.4					Mean = 6.5			Mean = 4.1							Mean = 8.1		Mean = 5.1						Mean = 9.4							
F ₄	4a					8e	Notes Some improvement in + strain, but continued debacle in - strain. But it is apparent that all of the progress was made in the F ₁ generation. Mean = 7.3	F ₄	*1a					5d	Notes Encouraging drop in value of - strain. Discouraging loss in + strain. Mean = 6.3	F ₄	1a					10f	Notes Original F ₁ value in - strain. Greatest value yet in + strain. The most successful generation. Possibly continuing along these lines will result in the sought for values. Mean = 9.1	F ₄	7b					9f	Notes Both strains less desirable than in F ₁ . Mean = 9.0								
	*1a					7e			3a				5d	3a							10f	4b										8f							
	5a					8e			5a				6d	1a							9f	3b										7f							
	3a					6e			3a				8d	6a							8f	7b										6f							
	5a					8e			4a				5d	4a							*11f	5b										*11f							
	3a					5e			3a				9d	3a							9f	*2b										10f							
	5a					*9e			4a				*9d	*1a							8f	7b										10f							
	3a					8e			1a				4d	5a							7f	4b										11f							
	5a					6e			6a				8d	2a							10f	3b										8f							
	Mean	3.9							Mean = 7.3			Mean = 3.1							Mean = 6.3			Mean = 3.2										Mean = 9.1		Mean = 4.9					
Mean of line	3.28					Mean of line 7.22			Mean of line 3.22					Mean of line 6.62			Mean of line 3.47								Mean of line 8.22		Mean of line 4.92					Mean of line 7.25							

But these experiments with dice need not be confined to pure lines. While pure-line phenomena are especially easy to parallel in dice-casting, bisexual heredity, even in the widest crosses, may be experimentally paralleled by appropriate dice-casting. Within bisexual species, let a single die represent the range of the combined genotypes of the two parents. In some respects the two genotypes will overlap; this narrows the total range; in others they will not; this extends it. If a solid with more than six faces be used, the several faces should be labelled with figures representing the range of possible values of offspring of a given pair of parents. This range will naturally vary greatly with different matings, but in each case will be complicated by such factors as Mendelian heterozygosity, intra-zygotic chemical reaction, due to the contact in the fertilized zygote of normally non-compatible elements, lethal factors, mutations regardless of size, crossing over, non-disjunction, deficiency, contamination, and other mechanical chromosomal phenomena, difference in environment furnished to offspring, and difference in response to environment of differently organized offspring. The whole range of value in the offspring will, of course, be limited by the possibilities of the physiological complex which heredity and environment furnish them, but, regardless of the cause of the range of phenotype in the offspring, we have simply to record these limits on the faces of the dice and proceed with the selection process exactly as in the case of pure lines. It would indeed be a complicated process, and would probably lose in clarity what it would gain in parallelism. Nor is the present scheme of dice-casting especially well adapted to demonstrating the statistical proportion of Mendelian behavior; coin-tossing is the scheme *par excellence* for this.

SUMMARY

The statistical analogy between dice-casting and certain breeding phenomena is in many respects very close, and may be pushed as far as one may care to devise appropriate parallel mathematical contrivances.

The demonstrations described in this paper are mathematical—not physiological—pictures of only a few of the many facts and processes with which genetics deals. Practically, the value of these experiments consists in presenting concretely a situation in which the experimenter may, by handling the dice, casting them at will and recording and analyzing the data thus found, go to the root of the causes of the statistical phenomena with which he is dealing. This familiarity is denied in statistical studies dealing with the complexities of actual life; for the statistical meaning of the latter we are confined to logical deductions based upon similarity of

behavior to processes like dice-casting which we can follow throughout with the eye.

There are, of course, many differences between dice-casting and breeding phenomena, but they have this in common, in both dice-casting and organic reproduction the experimenter is permitted to see, at first, only a single value (the phenotype in life; the upturned face of a die) in a given individual, but if he continues the processes (dice-casting or breeding) long enough, he can test the range of variability possible to the "offspring" of the original "parental" individual. All of the essential external phenomena of schemes of selection and the mathematical treatment of their counts and measures may be clearly shown with dice. The parallel is especially clear-cut if the analogy be to pure lines.

Tables 1 and 2 and figure 3 present, in systematic form, from dice-casting data, the schemes usually followed in recording measurements for studies in variation, regression, and correlation in living organisms. One experiment (table 3) tests the "breeding properties" of a given individual here represented by the several face values of a single die. The demonstration described in table 4 shows how the selective processes, without creation, secures the highest values potential within a particular line or strain. Table 4 demonstrates also that in mongrel stock there is no essential relation between a particular somatic value and the breeding qualities of its possessor, and that for breeding purposes a low value from a good strain is better than a high value from a poor strain. We thus see that selection, as its name implies, only sorts; it does not necessarily create anything, much less does it create in the direction of its own wish. Table 5 shows the nature of selection based upon somatic qualities as contrasted with selection based upon a knowledge of the germ-plasm.

LITERATURE CITED

- DARBISHIRE, A. D., 1907 Some tables for illustrating statistical correlation. Manchester Memoirs, Vol. 51, No. 16.
- DAVENPORT, C. B., 1917 Inheritance of stature. *Genetics* 2: 313-389.
- EAST, E. M., AND JONES, D. F., 1919 Inbreeding and outbreeding. 285 pp. Philadelphia: J. B. Lippincott & Co.
- JENNINGS, H. S., 1916 The numerical results of diverse systems of breeding. *Genetics* 1: 53-89.
- 1917 The numerical results of diverse systems of breeding with respect to two pairs of characters, linked or independent, with special relation to the effects of linkage. *Genetics* 2: 97-154.
- JOHANNSEN, W. J., 1903 Über Erbllichkeit in Populationen und in "reinen Linien." 68 pp. Jena: Gustav Fischer.
- PEARL, RAYMOND, 1917 A single numerical measure of the total amount of inbreeding. *Amer. Nat.* 51: 636-639.
- WELDON, W. F. R., 1906 Inheritance in plants and animals. pp. 81-109 in a book entitled *Lectures on methods of science*, edited by T. B. Strong. Oxford: Clarendon Press.