STUDIES IN MILK SECRETION. V. ON THE VARIATIONS AND CORRELATIONS OF MILK SECRETION WITH AGE¹

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INTRODUCTION

A large body of exact scientific data on milk production has been accumulated since the time of the establishment of the ADVANCED REGISTRY SYSTEM of the HOLSTEIN-FRIESIAN ASSOCIATION OF AMERICA and its later adoption by the Associations of the other breeds of dairy cattle. These completed records are unique in several ways, chief among which

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is the fact that the records of the cows included among them must meet a certain standard of performance. This standard for entry has the effect of cutting out certain records of cows of the given breed. Thus the JERSEY REGISTRY OF MERIT (THE AMERICAN JERSEY CATTLE CLUB 1906) says that, "if the test is commenced the day the cow is two years old, or previous to that day, she must produce within one year from the date the test begins, 250.5 pounds of butter-fat. For each day the cow is over two years at the beginning of her year's test, the amount of butter-fat she must produce in the year is fixed by adding 0.1 (one-tenth) of a pound for each such day to the 250.5 pounds required when two years old. This ratio of increase applies until the cow is five years old at the beginning of her test, when the required amount will have reached 360 pounds, which will be the amount of butter-fat required of all cows five years old or over. These standards are based upon one complete year's record from the time of beginning, regardless of any time which may be lost by being dry or calving during the period." Three facts are obviously true of the JERSEY REGISTRY OF MERIT cows as compared with a true sample taken at random of the milking cows of the Jersey breed; (1) the cows making up the REGISTRY OF MERIT are a selected sample; (2) the scale of the selection is linear having its lower limit 250.5 pounds of butter-fat production at 2 years and its upper limit 360 pounds at 5 years and over; (3) this requirement means that for each day of age at test that frequency distributions of years production are cut off perpendicularly at the requirement and only those animals making greater yields than this are allowed to be entered into the REGISTRY OF MERIT. The data taken from this REGISTRY OF MERIT are not the true data for the Jersey breed and conclusions based on it cannot be considered as applying to the breed as a whole or to the general problems of milk secretion.

MATERIAL AND METHODS

To supply this need of exact data on the Jersey breed as a whole the MAINE AGRICULTURAL EXPERIMENT STATION has obtained the recorded data on one of the largest pure-bred Jersey herds known. The data are exceptional in the following ways: (1) The records extend back to the year 1897 when the herd was organized; (2) the animals are practically all straight Island stock; (3) they have been under the oversight and direction of one manager since 1901; (4) exact records are kept of the milk production, butter fat and butter-fat percent; (5) many of the individual animals have records for several different lactations. The elimi-

nation of variation of the milk production of cows or groups of cows caused by changes of any one or more of these five factors is important for the analysis of the causative mechanism of milk and butter-fat production. It is obvious that these records are free from such variables. They constitute a homogeneous group of data representing the Island Jersey under constant conditions of management and climate.

Such an accumulation of exact statistical data on the cows of the Jersey breed for problems of so much interest, both biologically and economically, warrants the application of adequate biometrical methods in their analysis. Such analysis is now well recognized by most investigators as of the utmost importance to our understanding of the fundamental principles of physiology which underlie the process of milk secretion itself.

The general problems attacked are those of the individual variation between the lactating functions of different dairy cows at a given age. What is the true type of the frequency distributions of milk production? What relation exists between the mean productions of the successive ages in a true random sample of the Jersey breed? An understanding of these and similar questions is necessary to the fuller utilization of the data found in the herd book of the registry association for as previously pointed out the requirement for entrance acts as a true truncating agent to the frequency distribution of a given age. Without a knowledge of the type of these frequencies no true correction factors may be determined to properly allow for the increase in age of the cows either for direct comparison of yields or for inheritance studies.

The biometrical methods used are in general those of any adequate analytical treatment of a quantitative subject. In each case reference to the source of the original publication where the methods may be found are given where the formula is first used. In each case the standard deviations have SHEPPARD'S correction applied to them as a glance at the data shows that high contact is present in the distributions.

The data used for study are all from normal, healthy cows. Two diseases have been present in the herd, tuberculosis and abortion. The tuberculous animals were all eliminated early in the herd's history by the use of the tuberculin test. All records from animals which were proven to be tubercular or which aborted were not used. Records from animals normally healthy but sick during a given lactation were not used. All of the cows have been kept in climatic conditions similar to those of western Virginia.

A word as to the method of keeping the data and its transfer to this

STATION is perhaps necessary. All records are made at the time of milking on the daily milk sheets for the given cow which are kept in the barn. The milking takes place twice a day, the records are for night and morning. The weekly production taken from these sheets is transferred to the herd ledger by a trained bookkeeper. The total production for a given month is found together with the yearly production by adding the weekly totals. All records are recorded to pounds and tenths. The cows are tested bi-monthly by the BABCOCK test and the percentage of butter-fat is recorded beside its corresponding monthly milk yield. All weighings and readings are recorded immediately after they are made so there is little chance of inaccuracy. From these records the author has extracted 1741 complete 8-months records of healthy cows for milk production. Of these 1741, 1713 have records for the butter-fat percent. The weighted monthly averages of the bi-monthly test have been used to obtain the weighted 8-months average for the 8-months lactation period chosen for study.

VARIAION OF JERSEY MILK PRODUCTION WITH AGE AT COMMENCEMENT OF TEST

The tabulation of these records in complete eight-months records has been done by the author. The choice of a proper interval of grouping for age was decided upon as one year, that is, grouping all of those records commenced while the cows were 2 years, no months and no days, to 3 years, no months and no days, and so on to include up to 10 years, no months and no days. The group 10 years-and-above has been included in one group as the numbers were quite small in the older age classes. The absolute and percentage frequencies determined for the different age classes are shown in table 1.

Table 1 together with figure 1 makes clear the following points in regard to frequency distributions of milk production.

(1) The frequency polygons show milk production to be quite symmetrical.

(2) The range of variation is quite considerable, extending from one cow which produced under 1500 pounds for the eight-months period to one which produced over 10,000 pounds.

(3) The polygons change their shape with increase in age at the commencement of the test, those of the early ages being higher at the mode and less scattered than those of the later years.

(4) There appears to be only one mode in all of the distributions. From this it seems probable that the polygons belong to the unimodal

TABLE I

Frequency distributions of variations for the eight-months lactation records of a homogeneous herd of Jersey cows at different ages. Absolute and percentage frequencies.

Pounds of milk produced	Age, 2 yrs., 0 mos., to 3 yrs., 0 mos.		Age, 3 yrs., 0 mos., to 4 yrs., 0 mos.		Age, 4 yrs., 0 mos., to 5 yrs., 0 mos.		Age, 5 yrs., 0 mos., to 6 yrs., 0 mos.		Age, 6 yrs., 0 mos., to 7 yrs., 0 mos.	
	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency
1000 to 1500	1	.3								
1500 to 2000										
2000 to 2500	5	1.6	3	1.2	2	.8			2	1.0
2500 to 3000	23	7.6	7	2.8	15	6.1	12	5.4	6	3.1
3000 to 3500	60	19.7	27	10.7	26	10.5	26	11.7	16	8.3
3500 to 4000	58	19.1	31	12.3	44	17.8	28	12.6	23	12.0
4000 to 4500	67	22.0	47	18.7	44	17.8	33	14.8	23	12.0
4500 to 5000	54	17.8	49	19.4	44	17.8	40	17.9	29	15.1
5000 to 5500	23	7.6	36	14.3	49	19.8	25	11.2	24	12.5
5500 to 6000	11	3.6	21	8.3	28	11.3	19	8.5	28	14.6
6000 to 6500	2	.7	15	6.0	15	6.1	16	7.2	13	6.8
6500 to 7000			6	2.4	11	4.5	13	5.8	13	6.8
7000 to 7500			7	2.8	6	2.4	5	2.2	7	3.6
7500 to 8000			2	.8	5	2.0	4	1.8	4	2.1
8000 to 8500			1	.4	1	.4	1	.4	2	1.0
8500 to 9000					1	.4			1	.5
9000 to 9500									1	.5
9500 to 10000										
10000 to 10500										
Total	304	100.0	252	100.1	247	99.9	223	99.9	192	99.9

TABLE I (continued)

Frequency distributions of variations for the eight-months lactation record of a homogeneous herd of Jersey cows at different ages. Absolute and percentage frequencies.

Pounds of milk produced	Age, 7 yrs., 0 mos., to 8 yrs., 0 mos.		Age, 8 yrs., 0 mos., to 9 yrs., 0 mos.		Age, 9 yrs., 0 mos., to 10 yrs., 0 mos.		Age, 10 yrs., 0 mos., to above		Total population	
	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency	Absolute frequency	Percentage frequency
1000 to 1500									1	.1
1500 to 2000									13	.7
2000 to 2500	2	1.2			2	2.4	1	.9	50	2.9
2500 to 3000	1	.6	1	.8	6	7.1	3	2.8	166	9.5
3000 to 3500	7	4.1	9	7.1	5	5.9	10	9.3	212	12.2
3500 to 4000	16	9.4	15	11.8	6	7.1	8	7.5	279	16.0
4000 to 4500	24	14.1	15	11.8	11	12.9	18	16.8	274	15.7
4500 to 5000	20	11.8	13	10.2	15	17.6	21	19.6	266	15.3
5000 to 5500	28	16.5	27	21.3	15	17.6	19	17.8	162	9.3
5500 to 6000	21	12.4	15	11.8	5	5.9	12	11.2	127	7.3
6000 to 6500	20	11.8	13	10.2	8	9.4	7	6.5	82	4.7
6500 to 7000	14	8.2	9	7.1	8	9.4	5	4.7	58	3.3
7000 to 7500	10	5.9	5	3.9	2	2.4	2	1.9	26	1.5
7500 to 8000	3	1.8	2	1.6	1	1.2	1	.9	15	.9
8000 to 8500	3	1.8	1	.8	1	1.2			3	.2
8500 to 9000	1	.6							2	.1
9000 to 9500			1	.8					2	.1
9500 to 10000			1	.8					3	.2
10000 to 10500										
Total	170	100.2	127	100.0	85	100.1	107	99.9	1741	100.0

type although this point cannot be accurately determined, as the histograms are quite flat-topped.

The accuracy of these conclusions is further brought out by a study of the chief physical constants of the distributions. Table 2 gives the four constants, mean, standard deviation, coefficient of variation, and skewness, together with their probable errors for each frequency of age.

TABLE 2
*Constants of variation of milk production for the successive ages at test in Jersey milk.
(Eight-months lactation period.)*

Age at test	Mean	Standard deviation	Coefficient of variation	Skewness
2 yrs., 0 mos., to 3 yrs., 0 mos.	4032.9 $\pm$ 31.7	818.9 $\pm$ 22.4	20.304 $\pm$.577	
3 yrs., 0 mos., to 4 yrs., 0 mos.	4686.5 $\pm$ 46.8	1101.5 $\pm$ 33.1	23.503 $\pm$.712	+0.2642 $\pm$.0548
4 yrs., 0 mos., to 5 yrs., 0 mos.	4992.9 $\pm$ 46.3	1079.4 $\pm$ 33.0	21.619 $\pm$.684	+0.3532 $\pm$.0635
5 yrs., 0 mos., to 6 yrs., 0 mos.	5281.4 $\pm$ 57.0	1262.8 $\pm$ 40.3	23.911 $\pm$.801	+0.3894 $\pm$.0714
6 yrs., 0 mos., to 7 yrs., 0 mos.	5536.5 $\pm$ 64.5	1325.3 $\pm$ 45.6	23.938 $\pm$.865	+0.2586 $\pm$.0628
7 yrs., 0 mos., to 8 yrs., 0 mos.	5314.7 $\pm$ 64.9	1255.0 $\pm$ 45.9	23.613 $\pm$.907	
8 yrs., 0 mos., to 9 yrs., 0 mos.	5226.4 $\pm$ 77.9	1302.0 $\pm$ 55.1	24.912 $\pm$ 1.111	+0.3141 $\pm$.1173
9 yrs., 0 mos., to 10 yrs., 0 mos.	4938.2 $\pm$ 95.3	1302.6 $\pm$ 67.4	26.377 $\pm$ 1.453	
10 yrs., 0 mos., and above	4838.8 $\pm$ 70.3	1077.3 $\pm$ 49.7	22.264 $\pm$ 1.028	
Total population	4887.6 $\pm$ 20.2	1249.7 $\pm$ 14.3	25.569 $\pm$.310	+0.3150 $\pm$.0222

It seems best to postpone the complete discussion of the facts brought out in this table until other data to be presented are available. A few general points of especial interest may be mentioned, however. The mean milk production for the eight-months period for those cows which are between 2 and 3 years of age is 4032.9 pounds. From this point the milk production rises rapidly at first, then more slowly to a maximum at about 7 years. From this maximum the decline in milk production is less rapid toward the higher ages.

The difference between the production of 2 years and 6 months and the maximum is significant, as the difference is about 20 times its probable error (1503.6 ± 71.9). The difference between the maximum production and that for the 10 years and above is also significant, as the difference is more than 6 times the probable error (697.7 ± 95.40). This rise and fall of milk production with age has already been shown, by the work of this laboratory (PEARL 1914), to conform to that group of curves described by a logarithmic function.

The standard deviation varies between 818.9 pounds at the age of 2 to 3 years to 1325.3 pounds at the age of 6 to 7 years. The rise of the

standard deviation is more direct than is the rise of the mean curve. From the maximum the curve remains parallel with the base line for some years when it again drops to a lower deviation at the highest age at test. This rise and fall is significant as judged by the probable errors of the difference (PEARL 1909). The difference of the maximum standard deviation at 6 to 7 years from that at 2 to 3 years is 506.4 pounds of milk where the probable error is 50.8 pounds, or the difference is practically 10 times the probable error and the probability that this difference did not come from random sampling is considerably greater than 1,000,000,000 to 1 (ELDERTON 1901, PEARSON 1900). The difference between the maximum standard deviation and that at 10 years and above is 248.0 pounds and the probable error is 67.5 pounds or 3.68 times the probable error and the probability that this difference did not come from random sampling is 75.7 to 1. The conclusion seems justified from the above facts that the standard deviation varies with age at test in such a way that the absolute amount of the standard deviation is least at the early age, increases rapidly to a maximum at about 5 years and 6 months, remains at this maximum for about four years then falls again toward the later ages.

The coefficient of variation for the milk production of the successive ages are all high as compared with those already known for other similar data. This large size in the coefficients is brought out by table 3.

TABLE 3
Comparison data for coefficients of variation of amount of secretion.

Organism	Secretion	Coefficient of variation	Source of data
Domestic fowl (Barred Plymouth Rock)	Annual egg production	34.21 $\pm$.37	(PEARL 1909)
Cattle, British Holstein	Gallons of milk	25.72 $\pm$.372*	(GAVIN 1912)
Cattle, Ayrshire	Gallons of milk	24.187 $\pm$.504†	(VIGOR 1913)
Cattle, Jersey	Pounds of milk	25.569 $\pm$.310	This paper

* Probable error calculated by author from totals given in GAVIN's paper.

† Calculated by the author from the means and standard deviation as given by VIGOR (1913).

Of the substances studied annual egg production is the most variable. The coefficients of variation for milk production correspond remarkably well considering the diversity of the sources from which they are taken. They all go to show that milk production varies around a mean of 25, or about 9 percent less variable than egg production. This difference is significant. The mechanism of the secretion of the sum of the egg

parts to form the egg is shown to be more variable than that for the secretion of the milk parts to form the milk. Taking this reasoning back to its ultimate source, such a significant difference shows that the secretory cells of the mammary glands work with greater precision than do the cells of the oviduct. Such a difference in the precision of action of the two sets of cells would seem to indicate a greater approach to perfection, in a mechanical sense, in the cells of the udder than is true of the cells of the oviduct.

The skewness of the two sets of data furnish another interesting contrast. The skewness for annual egg production for Barred Plymouth Rock hens is -0.205 whereas the skewness for Jersey milk production is $+0.315$. Not only is the sign different but the actual amount is different. This cannot be explained on the basis of any selection for high producers that may have taken place, as both sets of data are about equally subject to such selection. The data for the successive age groups all go to show that where the distributions for milk production are skew they are all plus. This lends further strength to the belief that the distribution for the yearly production of the two sets of glands is skew in opposite directions. One thing is common in the two cases, the skewness in each is small in amount. This is of especial importance as it shows an approach to the value where the typical Gaussian curve of error may describe these functions.

ANALYTICAL CONSTANTS DESCRIBING THE VARIATION OF MILK PRODUCTION AT DIFFERENT AGES

The general physical constants of milk production having been considered, the further analysis of its problems requires a knowledge of the types of frequency distributions which go to make up the lactation record of a cow during her life time. In the solution of this kind of data comes the answer to such problems as, does the type of milk-production distribution of cows milking at two years old differ from that of cows milking at three years old. This question is of great importance in considering the milking records of advanced-registry animals where it is necessary to form an opinion of the capacities of a breed from the milking abilities of a selected sample where the selection is not at random but removes those cows producing under a certain amount.

This feature of the problem is seen in the work of REITZ (1909) on the inheritance of butter-fat production in Holstein Friesian cattle.

The data for this problem came from the advanced registry of Holstein Friesian cattle. Since as a requirement to entry in this registry the cows must produce more than a certain amount of butter-fat, the correlation from such data measuring the strength of inheritance are subject to a double selection. RIETZ in correcting for this selection uses the method devised by PEARSON to determine the whole of a normal curve when a portion of it is known. The accuracy of the corrections depend then on the curves for milk production and for butter-fat percentage being normal.

The data on which this paper is based are excellent for an answer to these problems as to the types of the production curves for cows at different ages, as they represent a herd of pure-bred Jersey cows, accurately tested and recorded, constituting as near a random sample of the breed under uniform conditions as could well be found. The constants describing these distributions are given in Table 4.

TABLE 4
Analytical constants of variation in milk production for the successive ages at test for Jersey milk. (Lactation period, eight months.)

Constants	2 yrs., 0 mos., to 3 yrs., 0 mos.	3 yrs., 0 mos., to 4 yrs., 0 mos.	4 yrs., 0 mos., to 5 yrs., 0 mos.
Mean	4032.89 $\pm$ 31.67	4686.51 $\pm$ 46.80	4992.91 $\pm$ 46.33
Standard deviation	818.85 $\pm$ 22.40	1101.47 $\pm$ 33.09	1079.40 $\pm$ 32.96
Coefficient of variation	20.304 $\pm$.577	23.503 $\pm$.742	21.619 $\pm$.684
μ_2	+ 2.6821	+ 4.8529	+ 4.6604
μ_3	+ 0.2894	+ 5.0413	+ 6.2588
μ_4	+ 19.9393	+ 74.5962	+ 73.6275
β_1	0.0043 $\pm$.0023	0.2224 $\pm$.1003	0.3870 $\pm$.1391
β_2	2.7718 $\pm$.1362	3.1675 $\pm$.3244	3.3900 $\pm$.3811
$\beta_2 - 3$	- 0.2282	+ 0.1765	+ 0.3900
κ_1	- 0.4694 $\pm$.2719	- 0.3322 $\pm$.4483	- 0.3811 $\pm$.4631
κ_2	- 0.0070 $\pm$.0070	- 0.5303 $\pm$ 1.2280	- 0.8361 $\pm$ 2.9443
Skewness		+ 0.2642 $\pm$.0548	+ 0.3532 $\pm$.0635
Modal divergence		+ 291.06 $\pm$ 73.48	+ 381.219 $\pm$ 75.52
Mode		4395.45	4611.70
Type of curve	II	I	I
+ end of curve	8069.15	17695.15	18896.33
- end of curve	3.36	1314.12	2351.50
y_0	71.7407	45.9314	47.7073

TABLE 4 (continued)

Analytical constants of variation in milk production for the successive ages at test for Jersey milk. (Lactation period, eight months.)

Constants	5 yrs., 0 mos., to 6 yrs., 0 mos.	6 yrs., 0 mos., to 7 yrs., 0 mos.	7 yrs., 0 mos., to 8 yrs., 0 mos.
Mean	5281.39 $\pm$ 57.04	5536.46 $\pm$ 64.52	5314.71 $\pm$ 64.92
Standard deviation	1262.84 $\pm$ 40.34	1325.32 $\pm$ 45.62	1254.98 $\pm$ 45.91
Coefficient of variation	23.911 $\pm$.801	23.938 $\pm$.865	23.613 $\pm$.907
μ_2	+ 6.3791	+ 7.0259	+ 6.2999
μ_3	+ 10.1997	+ 7.5096	+ 3.0843
μ_4	+ 134.5412	+ 144.0314	+ 105.1169
β_1	0.4008 $\pm$.1436	0.1626 $\pm$.0833	0.0380 $\pm$.0300
β_2	3.3063 $\pm$.3745	2.9178 $\pm$.2565	2.6485 $\pm$.1655
$\beta_2 - 3$	+ 0.3063	- 0.0822	- 0.3515
κ_1	- 0.5898 $\pm$.4413	- 0.6523 $\pm$.3714	- 0.8171 $\pm$.3021
κ_2	- 0.5619 $\pm$.5240	- 0.1952 $\pm$.1928	- 0.0354 $\pm$.0347
Skewness	+ 0.3894 $\pm$.0714	+ 0.2586 $\pm$.0628	
Modal divergence	+ 491.70 $\pm$ 103.81	+ 342.77 $\pm$ 107.10	
Mode	4789.69	5193.69	
Type of curve	I	I	II
+ end of curve	16682.37	13960.06	10186.64
- end of curve	2430.70	1961.51	442.78
y_0	35.9420	28.3749	25.6501

TABLE 4 (continued)

Analytical constants of variation in milk production for the successive ages at test for Jersey milk. (Lactation period, eight months.)

Constants	8 yrs., 0 mos., to 9 yrs., 0 mos.	9 yrs., 0 mos., to 10 yrs., 0 mos.	10 yrs., 0 mos., and above
Mean	5226.38 $\pm$ 77.92	4938.24 $\pm$ 95.30	4838.79 $\pm$ 70.25
Standard deviation	1301.98 $\pm$ 55.10	1302.56 $\pm$ 67.38	1077.28 $\pm$ 49.67
Coefficient of variation	24.912 $\pm$ 1.111	26.377 $\pm$ 1.453	22.264 $\pm$ 1.028
μ_2	+ 6.7806	+ 6.7867	+ 4.6421
μ_3	+ 13.8706	+ 1.1758	+ 1.4117
μ_4	+ 201.7722	+ 118.7966	+ 60.7500
β_1	0.6171 $\pm$.4429	0.0044 $\pm$.0497	0.0199 $\pm$.0150
β_2	4.3886 $\pm$ 1.7775	2.5792 $\pm$.2166	2.8191 $\pm$.2517
$\beta_2 - 3$	+ 1.3883	- 0.4208	- 0.1809
κ_1	+ 0.9258 $\pm$ 2.4700	- 0.8548 $\pm$.3509	- 0.4216 $\pm$.4975
κ_2	+ 0.5794 $\pm$ 1.4184	- 0.0039 $\pm$.0051	- 0.0357 $\pm$.0424
Skewness	+ 0.3141 $\pm$.1173		
Modal divergence	+ 408.93 $\pm$ 156.62		
Mode	4817.45		
Type of curve	IV	II	II
+ end of curve	+ ∞	9498.86	10852.89
- end of curve	- ∞	377.61	1175.32
y_0	.0014	12.2016	19.3754

TABLE 4 (continued)
Analytical constants of variation in milk production for the successive ages at test for Jersey milk. (Lactation period, eight months.)

Constants	Total population
Mean	4887.56 $\pm$ 20.21
Standard deviation	1249.71 $\pm$ 14.28
Coefficient of variation	25.569 $\pm$.310
μ_2	+ 6.2471
μ_3	+ 9.6623
μ_4	+ 138.3606
β_1	0.3829 $\pm$.0613
β_2	3.5454 $\pm$.1979
$\beta_2 - 3$	+ 0.5454
κ_1	- 0.0581 $\pm$.2582
κ_2	- 5.4182 $\pm$ *
Skewness	+ 0.3150 $\pm$.0222
Modal divergence	+ 393.65 $\pm$ 29.91
Mode	4493.91
Type of curve	I
+ end of curve	95879.24
- end of curve	1029.4
y_0	288.674

* Probable error very large.

Attention may be directed first to the moments of the individual distributions of milk production for the groups of the successive yearly ages. The third moment is consistently plus and on the whole rather small. The moments lead to values of β_1 which are rather small. In no case is the value so great as three times its probable error. β_2 is likewise only slightly different from the value 3 demanded by the normal curve. In no case is this difference as great as three times its probable error. Out of the nine age groups into which the data have been divided $\beta_2 - 3$ is plus four times and minus five times. No consistency is shown in the sign and it seems safe to say its change of direction comes from random sampling. The sign of κ_1 is plus once and minus eight times; its value in no case exceeds three times its probable error. The sign of κ_2 is of course, governed by that of κ_1 . The absolute value varies between 0.0039 and 0.8361. The values of the probable errors are all large in comparison with the values of κ_2 .

These facts all lead to the conclusion that no great confidence can be placed in either the sign or the absolute magnitude of the criterion for the frequency type. Such big probable errors of κ_1 and κ_2 suggest that all of the curves are near the critical limit of more than one type of frequency curve. The problem now becomes: what is the chance that

a frequency distribution, other than that indicated by the criteria may be obtained to fit the data equally well. It is necessary therefore to turn to the probable errors of the frequency type itself to determine this question. The tables and diagrams necessary for the speedy determination of this have been calculated by RHIND (1909). Comparison of the values of $\Sigma_1 \Sigma_2$ with diagram for the frequency type contained in this paper together with the other necessary calculations shows the types indicated in table 4 should give a good fit.

The distributions for the successive ages are shown to be equally types I and II with one which is type IV. Five of the curves are skew and four symmetrical; eight are limited in range in both directions and one is unlimited in range. The equations to these curves are as follows.

Age at test 2 years to 3 years:

$$y = 71.7407 \left(1 - \frac{x^2}{16,291,354}\right)^{10.648}$$

Age at test 3 years to 4 years:

$$y = 45.9314 \left(1 + \frac{x}{3081.32}\right)^{6.2322} \left(1 - \frac{x}{13299.70}\right)^{26.8997}$$

Age at test 4 years to 5 years:

$$y = 47.7073 \left(1 + \frac{x}{2260.20}\right)^{4.0347} \left(1 - \frac{x}{14284.63}\right)^{25.4999}$$

Age at test 5 years to 6 years:

$$y = 35.9420 \left(1 + \frac{x}{2358.99}\right)^{2.8776} \left(1 - \frac{x}{11892.68}\right)^{14.5074}$$

Age at test 6 years to 7 years:

$$y = 28.3749 \left(1 + \frac{x}{3232.17}\right)^{3.8103} \left(1 - \frac{x}{8766.38}\right)^{10.3344}$$

Age at test 7 years to 8 years:

$$y = 25.6501 \left(1 - \frac{x^2}{23,735,702}\right)^{6.0352}$$

Age at test 8 years to 9 years:

$$y = .0014 \left(1 + \frac{x^2}{12,094,088}\right)^{-9.9809} e^{21.0802 \tan^{-1} \frac{x}{3477.66}}$$

Age at test 9 years to 10 years:

$$y = 12.2016 \left(1 - \frac{x^2}{20,799,307} \right)^{4.6294}$$

Age at test 10 years and above:

$$y = 19.3754 \left(1 - \frac{x^2}{36,169,519} \right)^{14.0831}$$

Total population:

$$y = 288.674 \left(1 + \frac{x}{3464.52} \right)^{8.086} \left(1 - \frac{x}{91385.33} \right)^{213.294}$$

The histograms and the fitted curves for the milk production of the individual ages are shown in figure 1. All of the areas have been reduced to a percentage base so that the ordinates and areas of each distribution could be compared directly.

It is clear to the eye that on the whole the graduation of the data by the fitted curves is rather good. Measured mathematically by the test of goodness of fit (ELDERTON 1901; PEARSON 1900) of the theoretical curve to the raw data the value of *P* ranges between 0.5864 and 0.9942 as shown by table 5.

TABLE 5
Values of P for the fit of the theoretical curves to the observational curves for milk production of Jersey cattle.

Age at test	P
2 years to 3 years	0.6680
3 years to 4 years	0.8453
4 years to 5 years	0.9114
5 years to 6 years	0.9424
6 years to 7 years	0.9942
7 years to 8 years	0.9145
8 years to 9 years	0.5864
9 years to 10 years	0.6873
10 years and above	0.9602

This table brings out the excellence of the graduation of the data by the frequency curves.

As previously stated eight of the nine curves are of limited range in both directions and one of unlimited range in either direction. The limits of these ranges are of especial interest since they indicate the highest and lowest milk production expected for a given age. It is evi-

FIGURE 1.—Histograms and fitted curves showing the variation of milk production for each year of age. The area of each curve is reduced to unity so that any distribution of a given age may be compared with any other at another age. The relative increase in the height of the model ordinates at the lower and higher ages of lactation as compared with the ages 6 to 10 years is quite evident. The fitted curves are all PEARSON type curves the constants of which are seen in table 4.

dent that in the symmetrical curves (type II) the plus end of the range is not as high as in the type I curves, in fact, the end of the range of high milk producers is rather underestimated than overestimated. The type I curves estimate the highest expected milk production at what we might consider a fair estimate. Thus, Jersey cows 3 years to 4 years old would not be expected to produce more than 17,695 pounds of milk in 8 months. This is borne out by the actual experience of the breed. The figure 10,186 for the 8-months production in the 7-year to 8-year class is low, however, compared with the production of certain REGISTRY-OF-MERIT cows; e.g., Passport is known to have produced 19,694 pounds in one year. Such a difference in the plus end of the range would seem to indicate a real difference of this group of Jerseys from those of the advanced registry group in the direction of an extended range for the REGISTRY-OF-MERIT animals.

The minus end of the range is equally interesting. The type II curves again do not give as good an estimate as the type I. They are in most cases too low. The low end of the production scale for this group of cattle is on the whole pretty well estimated, however. For the data themselves we have one observation as low as 1250-pound group. The estimate for this particular range is on the whole rather low (3.36 pounds of milk). In any case the author has in his note-book a record of milk production for one year of an Aberdeen Angus cow at 6 years 6 months of only 244.7 pounds which is not so far different from the above low-range estimate. Such records as these show us that the low margin of production is not on the whole overestimated. In all of this work it must also be remembered that the limited- and unlimited-range curves have only a small frequency at these points. Thus the fact that we have one curve whose range is not limited should not trouble us, as the frequencies of this range beyond the customary limits, become very small indeed.

All of the curves are of the mesokurtic type as the values of β_2-3 are small and in each case less than three times the probable error of β_2 . Furthermore the sign of the kurtosis is plus in four cases and minus in five showing more clearly if necessary the mesokurtic nature of all of the distributions.

The changes in the position of the modal ordinate with increasing age generates a curve rising almost linearly to its high point at a production of 5314 pounds for 7 years and 6 months. From this high point it falls toward a lower production for the maximum frequency of the older cows.

FIGURE 2.—Histogram and fitted curve showing the variation of milk production that may be expected to occur in a pure-bred Jersey herd without regard to age. It is evident that by and large the curve strikes through the observation well. The distribution of milk production is seen in general to be quite skew.

The variation of the total population of Jersey milk production is valuable both for itself and for comparison with other breeds. Table 4 gives the necessary constants for the calculation of the frequency curve suitable to fit the data. Figure 2 shows these data graphically.

The theoretical curve for these data is type I as the values of β_1 and β_2 are such as to lead to a κ_2 of -5.4182 calling for a type I curve. The curve is significantly skew in the plus direction. The range of this curve much more than covers the actual observations and goes a good deal beyond the probable value of the milk production of any cow in describing the highest milk production expected. This is no doubt due to the fact that toward the plus end of the range the observed frequency of high-producing cows trails off slowly. The minus end of the range is well described.

As shown by the above data the assumption previously indicated in the study of milk inheritance by RIETZ is open to the criticism that the distributions of milk production at a given age are not normal curves

8 Month's Milk Production (Pounds)

FIGURE 3.—Histograms and fitted curves showing the variation of milk production of each year of age. The fitted curves in this case are all normal curves calculated from table 4. The histograms are the same as those of figure 1. Each distribution is drawn on a basis of area equal to one, so that each curve is directly comparable with others. It is evident that the fit as a whole is not as good as in figure 1, especially at the ends of the range.

but belong in general to the type I and type II curves. The problem becomes that of how great an error is introduced by regarding the data as described by normal curves. Normal curves have been fitted to the above data to make the comparison. These curves together with their corresponding histograms are shown in figure 3.

In comparison with figure 1 the normal curves are seen in general to describe the tail frequencies poorly. The P for the fit of these curves are shown in table 6.

TABLE 6
Values of P for the fit of the theoretical normal curves to the observational curves for milk production of Jersey cattle.

Age at test	P
2 years to 3 years	0.4335
3 years to 4 years	0.4130
4 years to 5 years	0.0943
5 years to 6 years	0.7442
6 years to 7 years	0.7450
7 years to 8 years	0.8532
8 years to 9 years	0.5517
9 years to 10 years	0.5083
10 years and above	0.9333

These data (table 6) show that normal curves do not describe milk production at different ages as well as do the proper type curve. The difference in the fit of the type curves over the normal curve comes especially in the description of the tails and the skewness of these distributions. The fit of the normal curves is not bad, however, and in all probability no serious error would be made in the use of them for the determination of the corrections to be applied to the correlations from double-selected data as was done by RIETZ.

THE CORRELATION OF JERSEY 8-MONTHS MILK PRODUCTION WITH THE AGE AT TEST

Common knowledge among dairymen is that milk production and age at the commencement of the lactation are correlated in such a way that advancing age means increased production. The opinion is general that this increase is a linear one. This opinion has been shown to be erroneous by previous work of this laboratory (PEARL 1914) on the statistics of the 7-day records of American Jersey cattle found in "Jersey sires with their tested daughters," published by the AMERICAN JERSEY

CATTLE CLUB. In this work seven-day milk production is shown to be a logarithmic curve and not a straight line.

It remains to be shown what is the curve describing a true random sample of the Jersey breed for a longer milking period. Data of this sort is important for several reasons, chief among which, both practically and theoretically, is the necessity of having suitable correction factors for age to allow comparison of milk records at different periods in the lives of different cows. Toward the solution of this problem the following facts are necessary: What correlation exists between age and milk production? Is this correlation sufficient so that it must be taken into account in considering records of different cows at different ages? What is the equation of the regression line between these two variables? Table 7 furnishes the data necessary for this study.

The correlation and its accompanying constants for these two variables are shown in table 8.

TABLE 8
Constants measuring the association between amount of eight-months milk produced and age at test of Jersey cows.

r	η	$\eta - r$	$\eta^2 - r^2$
$0.2596 \pm .0151$	$0.4283 \pm .0132$	$0.1689 \pm .0201$	$0.1161 \pm .0108$

This table makes clear several facts concerning the influence of age on milk production. The correlation of $+0.2596 \pm .0151$ shows that age at test and milk production are significantly correlated variables. Taken in conjunction with the correlation ratio it shows clearly that age of the cow at commencement of test must be considered in comparing the records of different cows if the conclusion from the comparison is to be valid. The value of the correlation ratio $+0.4283 \pm .0132$ is considerably higher than the correlation coefficient. This difference is shown to be highly significant by the value of $\eta - r = 0.1689 \pm .0201$. It is altogether probable therefore that the regression of age on milk production is a skew regression. This is shown to be a fact by the constant to measure such skewness. $\eta^2 - r^2$, $0.1161 \pm .0108$, is about 11 times its probable error. The regression is therefore known to be skew. Since this is true the correlation ratio is a better measure of the true correlation than is the correlation coefficient. The relation of age at test is then doubly significant in any comparison of the records of two cows. The regression having been shown to be skew it becomes necessary to deal with it separately.

TABLE 7.

Correlation surface for pounds of milk and age at test for Jersey cattle. (Lactation period, eight months.)

	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	10500-11000	Total
1:6-2:0	1			5	15	10	2	2													34
2:0-2:6			4	19	54	51	56	42	19	7	2										255
2:6-3:0			1	4	6	7	11	12	4	4											45
3:0-3:6				4	21	22	33	30	20	13	8	3	2								156
3:6-4:0			3	3	6	9	14	19	16	8	7	3	5	2	1						96
4:0-4:6				2	8	15	31	24	25	15	7	8	1	2		1					139
4:6-5:0					7	11	13	20	24	13	8	3	5	3	1						108
5:0-5:6					7	13	16	14	23	10	5	7	9	2	1	1					108
5:6-6:0					5	13	12	19	17	15	14	9	4	3	3			1			115
6:0-6:6				2	2	10	14	8	14	12	12	5	6	3	3			1			92
6:6-7:0					4	6	9	15	15	12	16	8	7	4	1		2		1		100
7:0-7:6			2	1	4	5	16	11	19	7	6	5	5	2							87
7:6-8:0					3	11	8	9	9	14	14	9	5	1	3	1					87
8:0-8:6					7	7	7	10	15	8	7	4	2	1	1			1			71
8:6-9:0					2	8	8	3	12	7	6	5	3	1			1				56
9:0-9:6			1	3	4	3	7	9	7	3	3	5	1								43
9:6-10:0			1	3	1	3	4	6	8	5	5	3	1	1	1						42
10:0-10:6					1	1	3	9	6	1	1	4									26
10:6-11:0			1		3	3	2	2	4	3	3	1	1	1							24
11:0-11:6				1	3	2	6	5	3	4	1		1								26
11:6-12:0							2	1	1		1										5
12:0-12:6				1	3			2	1	2											5
12:6-13:0							1	2	1	2											7
13:0-13:6				1		1	1		1	1											5
13:6-14:0							1														2
14:0-14:6									1												2
14:6-15:0																					0
15:0-15:6																					1
15:6-16:6							1														
Total	1	0	13	50	166	212	279	274	266	162	127	82	58	26	15	3	2	2	3		1741

Age at Commencement of Test (Years and Months).

FIGURE 4.—Diagram showing the observed means and theoretical means for Jersey eight-months milk production at any given age. The observed means are calculated from table 7. The curve as a whole is evidently of the logarithmic type described well by the theoretical curve the constants of which are given below. The observations are represented as the small circles.

TYPES OF THE REGRESSION OF MILK PRODUCTION ON AGE
OF JERSEY CATTLE

The means for each array of age of table 6 have been calculated. These means are indicated by points within circles in figure 4.

Examination of the observational curve in figure 4 shows that it takes the general form of a logarithmic curve, agreeing in this respect with the other data from this laboratory (PEARL 1914). From these means the theoretical curve conforming to the general logarithmic type has been calculated by the method of least squares. The equation to this curve is

$$y = 3387.912 - 99.883x - .487x^2 + 2896.219 \log x$$

where x is taken in six-months intervals from an origin at 1 year and 3 months.

The observations at the higher ages vary a good deal as they are based on small numbers. The theoretical curve strikes through them quite accurately when the unevenness of the observed curve is considered. When we calculate the χ^2 by the method of SLUTSKY (1914) we find that 5 observations contribute a sum of 28.80 to the total of 45.41. These observations are at ages 2 years 9 months, 3 years 3 months, 6 years 9 months, 7 years 9 months and 9 years 3 months. If we measure the fit by the total χ^2 , 45.41, it is poor. Considering the above-mentioned five observations in connection with the other observations it is seen that two of them are plus and three are minus quantities. Not only that but they come at places in the curve so that they could practically counteract each other if the first smoothed curve were used as the observational. It seems altogether reasonable therefore to consider the fit of this curve measured by a χ^2 somewhat more than 17.00 or what would correspond to the P of a very good fit.

The equation of the curve has many practical uses aside from its interest in a scientific sense. By its use the records of cows at different ages may be brought to the same basis for comparison whether it be for milk-inheritance studies, analysis of judging experiments or the like. The time of the theoretical maximum of milk production may be easily calculated from it by differentiation. This maximum is shown to be 7 years 2.4 months, a figure considerably above the age customarily called mature form. Further the curve shows that the method used in advanced-registry work of determining the amount a Jersey cow should produce for the REGISTER OF MERIT is fallacious in that it is a linear method and does not recognize this logarithmic nature of milk production. In a previous paper (GOWEN 1919) the average fat percent of

Jersey cows is given as 5.12. Assuming this figure and dividing the pounds of butter-fat by it gives us the average requirement for milk production in one year. Supposing that $\frac{3}{4}$ of the year's records is made in the first 8 months of lactation (a figure reasonably close to the expected (PEARL 1915)) the required production is found to be 3600 pounds at 2 years and 5200 pounds at 5 years. From figure 3 it is evident therefore that if a cow is to be pushed for the advanced registry the best time to get her in, is to have her calve at about 3 years 3 months of age.

Causally considered the logarithmic nature of milk production is of a good deal of interest. The work of a number of students of growth, beginning with MINOT's (1891) notable studies on rabbits and guinea-pigs has shown that the phenomena of growth are also a logarithmic function of age. This law appears of wide general application as the work of LEWENZ and PEARSON (1904) have shown that it holds for growth in children; DONALDSON (1908, 1909, 1910, 1911), HATAI (1911) and JACKSON (1913) have shown it to be of general application to the growth of certain organs in the white rat; PEARL (1907) and PEARL and SURFACE (1915) have shown it to be true for *Ceratophyllum* and maize. It seems, therefore, altogether likely that the mammary glands of the cow also follow this rule. Should this prove true the increase of milk production with age seems of much significance in paralleling these growth phenomena. This paralleling of the two functions would in fact seem to indicate a causal relation between the two in that the increase in milk production may depend chiefly on the increase in actual mass of the mammary gland due to growth of this organ and not due to any relative increase in the ability of the cells to secrete more milk.

THE NATURE OF THE CURVE OF THE STANDARD DEVIATION OF MILK PRODUCTION

The curve found by plotting the standard deviations of 8-months milk production for each six months age intervals is shown in figure 4. The circles represent the actual standard deviations of the age arrays calculated from table 7. A glance at these suffices to show that the curve is not the customary linear one. It has rather the appearance of a parabola or logarithmic curve. The actual choice of the type of curve describing these data, other things being equal, depends on the fit of the theoretical curve to the observational data. Accordingly two curves

were tried, a cubic parabola having the form $y = a + bx + cx^2 + dx^3$, and a logarithmic curve $y = a + bx + cx^2 + d \log x$. These two curves were chosen as they had the same number of constants. The cubic fitted to the data by the method of MINER (1917) gave the equation to the data as

$$y = 399.46 + 188.82x - 11.664x^2 + 0.1919x^3.$$

The logarithmic curve was

$$y = 497.08 + 4.025x - 1.689x^2 + 909.13 \log x$$

The fit of these curves to the observational data was then compared by calculating for each the mean error and the root mean-square error. These values will of course be very large from the evident unevenness of the observation curve, they will however serve as a good comparative measure. The mean error for the parabola is 99.6, the root mean square error 132.0; for the logarithmic curve the mean error is 102.3 and the root mean square error 133.5. From this comparison the cubic parabola fits the data most accurately. Figure 5 shows the observation and

FIGURE 5.—Observational and theoretical curve for the standard deviations of milk production for the successive ages at which the lactation commenced. The theoretical curve is the cubic parabola given above. Considering the large variation of the observed standard deviations the cubic strikes through the observations well.

fitted cubic. The cubic parabola is seen by the figure to strike through the observation very well.

If we define with PEARSON (1905) the scedastic curve as the curve

formed by the ratios of standard deviations of the x arrays of β divided by the standard deviation of β plotted to x , then it is evident that the standard-deviation lines of milk production show that the value of η is will form, when divided by the standard deviation of milk production at all ages, a curve similar in shape to the curve plotted in figure 5. Milk production with age is therefore heteroscedastic. As the change in the scedastic curve follows a regular system the curve is nomic.

The facts brought forth in the discussion of the regression and standard-deviation lines of milk production show that the value of η is nearer the value of the true correlation of milk production with age than is the value of r . It therefore strengthens more strongly if need be the argument of age correction in all milk-production studies involving comparisons of milk production at different times in a cow's life.

THE CORRELATION BETWEEN THE MILK YIELD OF ONE LACTATION AND THAT OF SUCCEEDING LACTATIONS

Mean milk production of cows retained long or short periods in the herd

Up to this point the discussion has dealt entirely with the influence of age on amount of milk produced in a given lactation and the variability of the milk produced during it. In the present section the phase of the problem dealing with one lactation in its relation to another will be considered.

The functioning of the mammary glands may be considered dependent upon three main factors, which, taken in order of their natural sequence are: heredity; development through feeding, etc. (environmental circumstances), of these organs up to their commencing to secrete; and lastly environmental factors in their widest sense acting during the months when the gland is active. It is reasonably clear that on our ability to distinguish the relative influence of these three basic variables depends many of the common *a priori* dairy practices as well as furnishing a solid foundation for the analysis of the causal mechanism of milk production itself. The analysis is a complex one and needs to be attacked by many channels. The present investigation was undertaken in the hope that by an analysis of the intra-individual variation of milk secretion from lactation to lactation some light would be given on the relative merits of these three variables. The homogeneous nature of the material is such, however, that the investigation necessarily deals chiefly with the first of these variables.

Little work on milk secretion has been done that approaches the problem from this viewpoint. Of the available data those on the English herds analyzed by GAVIN (1913) are undoubtedly the best. This investigation on a mixed herd of British Holsteins and grade Short Horns furnishes data of value for the records of the first lactation in comparison with the yields of subsequent lactations. In all of this work the measure of the lactation used is what he designates as the "revised maximum," this term being defined as the maximum day yield of the lactation which is three times reached or exceeded. These results are considered largely for their strictly practical bearing. They are of little use to the American farmer in that he is accustomed to deal with records over a certain limit of time and not maximum productions. These facts make it evident that no repetition in either data or aim of the investigation occurs in these studies. The constants derived by GAVIN will be of a good deal of interest for comparison with those presented here as together they show the range of variation to be expected under the different conditions of England and the United States, a mixed herd and a pure-bred herd, and a difference in the measure of the lactating capacity from lactation to lactation.

The preliminary steps should include a study of the population of each frequency which is to be compared. The data for this comparison are taken from the correlation tables beginning with number 9 and ending with 40.

TABLE 9

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at three years old for pure-bred Jersey cows.

		3 years old													Total
		2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	
2 years old	2000-2500			I											I
	2500-3000			I	5	5									II
	3000-3500	I	I	8	5	II	5	I	I						33
	3500-4000			3	6	II	II	5	3						39
	4000-4500			2	2	4	II	7	4	3	I	I			35
	4500-5000		2	2	2	4	8	6	6	7	2				39
	5000-5500						2	5	2	2	I	3	I	I	17
	5500-6000			I		I	2			I	I	I			7
	6000-6500											I	I		2
Total		I	3	18	20	36	39	24	16	13	5	6	2	I	184

TABLE IO

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at four years old for pure-bred Jersey cows.

		4 years old												Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	
2 years old	2500-3000	1	4	4		2								11
	3000-3500	5	6	10	2	5	1							29
	3500-4000	12	3	7	7	8	3		1					41
	4000-4500	2	2	5	6	9	4	7	3	1	1	1		41
	4500-5000	1		5	6	8	7	1	2	3				33
	5000-5500			1	2	2	1	1	3	2	1			13
	5500-6000					1	1	2	1			1		6
Total		21	15	32	23	35	17	11	10	6	2	1	1	174

The tabled constants are presented in the following order, (1) the means of each of the compared populations (table 11), (2) the standard deviations of each of the compared populations (table 20), and the coefficients of variation of each of the compared populations (table 25). The vertical columns of each table present the means, standard deviations, etc., of the age indicated when correlated with each of the lactation records made at the age on the left of the table. Thus, the means of the 8-months' milk production of the two-year group that is correlated with the three-year group, is 4141.30 pounds of milk. The value of the mean production in this column should of course be the same if the population of milk production of the two-year group is the same for the cows kept to their three-year-old lactation as it is for the cows kept to their seven-year-old lactation. Table 11 presents the data for the comparisons of these mean milk productions.

TABLE 11

Mean 8-months milk production of a given age when correlated with other given ages.

Age	2-3	3-4	4-5	5-6
2 to 3		4796.20 $\pm$ 55.15	4062.64 $\pm$ 59.63	5542.86 $\pm$ 74.47
3 to 4	4141.30 $\pm$ 40.12		5031.05 $\pm$ 60.19	5344.41 $\pm$ 80.52
4 to 5	4091.95 $\pm$ 37.66	4642.16 $\pm$ 57.56		5190.55 $\pm$ 67.40
5 to 6	4178.57 $\pm$ 44.62	4739.51 $\pm$ 64.10	5033.22 $\pm$ 59.05	
6 to 7	4192.62 $\pm$ 48.71	4819.57 $\pm$ 72.78	5141.67 $\pm$ 67.93	5401.64 $\pm$ 76.40
7 to 8	4156.59 $\pm$ 58.81	4867.35 $\pm$ 77.53	5143.81 $\pm$ 65.17	5315.89 $\pm$ 65.87
8 to 10	4061.93 $\pm$ 48.09	4698.11 $\pm$ 75.14	5067.07 $\pm$ 62.08	5298.28 $\pm$ 69.64
10 and older	4162.50 $\pm$ 66.27	5171.05 $\pm$ 199.66	4994.90 $\pm$ 103.53	5468.18 $\pm$ 65.79

TABLE II (continued)

Mean 8-months milk production of a given age when correlated with other given ages.

Age	6-7	7-8	8-10	10 and older
2 to 3	5754.10 $\pm$ 88.77	5464.29 $\pm$ 86.47	5094.04 $\pm$ 81.11	4925.00 $\pm$ 119.59
3 to 4	5710.87 $\pm$ 89.16	5459.18 $\pm$ 87.31	5099.06 $\pm$ 91.12	4907.89 $\pm$ 144.32
4 to 5	5612.50 $\pm$ 104.63	5435.84 $\pm$ 80.10	5152.44 $\pm$ 78.20	4760.20 $\pm$ 101.39
5 to 6	5631.15 $\pm$ 83.22	5521.32 $\pm$ 73.79	5208.62 $\pm$ 75.90	4868.18 $\pm$ 108.99
6 to 7		5334.68 $\pm$ 78.04	5189.19 $\pm$ 71.69	4914.06 $\pm$ 84.65
7 to 8	5399.19 $\pm$ 115.94		5081.13 $\pm$ 74.40	4831.08 $\pm$ 85.76
8 to 10	5371.62 $\pm$ 74.90	5220.20 $\pm$ 63.66		4903.23 $\pm$ 69.14
10 and older	5382.81 $\pm$ 61.45	5195.94 $\pm$ 83.20	5185.48 $\pm$ 69.65	

The data presented in this table are graphically represented in figure 6. In this figure the mean 8-months milk production is given at the left. The line at the top of the figure represents the mean milk production of the two-year-olds that had lactations at the other ages shown on the lower margin of the figure. The next line down represents the 3-year mean production when compared with the other ages, etc. The ages with which these are compared are shown in the bottom margin of the figure. The lines generated by the mean productions of each of these age groups should, of course, be parallel with the base of the figure within the limits of random sampling if the population of 8-months milk production for the given age is the same irrespective of the age at which the subsequent lactation was made.

The curve for the mean eight-months milk production of the two-year-old group (the top curve of figure 6) is as close to a straight line parallel with the base of the figure as could well be expected. At no place is there more than 65-pound divergence from such a line. This divergence is seen in table II to be only slightly over the probable error of the mean production.

The curve for the three-year-old group is more variable than the two-year group due no doubt to the numbers on which it is based being less. The curve on the whole has a slight tendency upward as we progress from left to right. It is doubtful if this general trend is significant in comparison with the probable errors of the constants. Taken at its face value, however, it would indicate that the cows with a lactation record at 10 years of age would have had in general a slightly better record at three years old than would the general three-year-old group. That is, the cows of the three-year-old group were culled slightly so that the cows kept for milk production were somewhat better cows than the average run of three-year-olds.

FIGURE 6.—Curves showing the mean eight-months milk production of a given age at each successive age with which the given age is correlated. The ordinates are the pounds of milk produced and the abscissae are the ages of the groups with which the given age is correlated. Proceeding from top to bottom of the figure the first line represents the mean milk production of the 2-year to 3-year group; the second the 3-year to 4-year group; the third the 4-year to 5-year; the fourth the 5-year to 6-year; the fifth the 6-year to 7-year; the sixth the 7-year to 8-year; the seventh the 8-year to 10-year, and the last the 10-year-and-older group. The lines obviously are quite irregular but in general do not diverge greatly from a line parallel with the base of the figure.

TABLE 12

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at five years old for pure-bred Jersey cows.

		5 years old														Total	
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	
2 years old	2000-2500	1															1
	2500-3000		1	2			2										5
	3000-3500	1	5	6		2	2	1	1								27
	3500-4000	2	2	4	3	6	3		1		2						23
	4000-4500	1	2		8	9	4	5	3	1	1	2					36
	4500-5000	1			3	5	3	4	6	4	2					1	29
	5000-5500	1					1	1	3	2	4						12
	5500-6000					1		1	1	1	1						5
	6000-6500										1		1				2
Total		7	10	12	23	24	15	14	14	11	6	3	0	0	0	1	140

TABLE 13

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at six years old for pure-bred Jersey cows.

		6 years old																Total
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	Total
2 years old	2000-2500		I															I
	2500-3000		I	2	I		I				I							6
	3000-3500		I	5	5		5	I	2	2	I							22
	3500-4000		I	I	4		2	I	3	3	2							17
	4000-4500		I	I			I	7	7	5	2		5					32
	4500-5000					4	5	4	5	I	4	2				I		26
	5000-5500	I				I			2	I	3	I	I	I			I	13
	5500-6000									I		I						3
	6000-6500										I	I					I	2
Total		I	5	9	11	12	15	18	17	9	13	4	4	I	I	I	I	122

TABLE 14

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at seven years old for pure-bred Jersey cows.

7 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	Total
2 years old			I												I
2000-2500				I											4
2500-3000				I	I	2									19
3000-3500	I		I	5	2	4	4			2					13
3500-4000				I	I	3	3	2	I	2					26
4000-4500				I	I	4	9	3	4	2		I			14
4500-5000					I			3	3	5			I	I	8
5000-5500							2	2	I		3				4
5500-6000							2		I				I		2
6000-6500										I	I				91
Total	I	0	3	8	6	13	20	10	10	12	4	I	2	I	

TABLE 15

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

8 and 9 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	10000-10500	Total
2 years old																	
2000-2500		I		I													2
2500-3000				4	I		2										7
3000-3500	I	I	I	6	4		I	2		I							17
3500-4000		I	I	I	2	3	6	3	4								21
4000-4500			I	4	5	3	6	5	2	4	2						32
4500-5000	I					7	3		3	3	2						19
5000-5500							4	2	I	I					I		9
5500-6000									I		I						2
Total	2	3	3	16	12	13	22	12	11	9	5	0	0	0	0	1	100

The four-year- and five-year-old records are practically linear and parallel with the base. No selection has taken place in these cows to weed out the poor milkers.

The six- and seven-year curves both appear linear but have a downward tendency which looks significant when compared with its prob-

able errors. Thus in the six-year group the difference is 371.3 ± 107.9 or 3.4 times its probable error. Furthermore the drop in the value of the milk production of the six-year group is quite sudden, occurring with those cows which lactate in their seventh year and older. Selection can-

TABLE 16

Correlation surface for amount of eight-months milk production at two years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

		10 years old and older											
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	Total
2 years old	2500-3000	I				I							2
	3000-3500		I		2	I	I						5
	3500-4000		I		I		I						3
	4000-4500		2	2	5	5	3	3		I		I	22
	4500-5000						3		2				5
	5000-5500									2			2
	5500-6000									I			I
Total		I	4	2	8	7	8	3	2	4	0	I	40

TABLE 17

Correlation surface for amount of eight-months milk production at three years old and amount of eight-months milk production at four years old for pure-bred Jersey cows.

		4 years old												Total	
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	
3 years old	2000-2500		I												I
	2500-3000	I		I	I			I							4
	3000-3500		3	4	3	2	I								13
	3500-4000		2	3	9	5	4	I							24
	4000-4500		4	2	7	9	10	I	I		I			I	36
	4500-5000			I	6	4	7	5	2	2					27
	5000-5500		I		I	5	6	2	2	2					19
	5500-6000					3	2	2	2	I	I				11
	6000-6500					I	2	2		2	I				8
	6500-7000								3	I	I				5
	7000-7500				I					I		I	I		4
	7500-8000											I			I
Total		I	11	11	28	29	32	14	10	9	4	2	I	I	153

not account for this drop for it goes directly opposite to the direction in which selection would be made unless the highly unlikely assumption is made that the cows with high six-year-old records die before their seventh lactation. This assumption is known not to be true as the records of death are accurately kept. The history of the herd is accurately known and while perhaps a half a dozen old cows purchased at its beginning their production was fair and they were of ages 6 and older with no centering on any age. No other possible reason for the drop is known to the author. The reason for this drop consequently remains obscure.

The curves for the eight-and-nine-year and the ten-and-above groups all appear linear and parallel with the base, indicating that no selection had been practiced in choosing these animals.

To sum up the results of this study, no clear evidence of selection of animals to be kept as further milk producers is evidenced by the mean milk production of any age in comparison with the other lactations that the cow was known to be milking.

TABLE 18

Correlation surface for amount of eight-months milk production at three years old and amount of eight-months milk production at five years old for pure-bred Jersey cows.

		5 years old															Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	
3 years old	2000-2500	I															I
	2500-3000	2				I	I			I							5
	3000-3500	3	3	5	3	2	I										17
	3500-4000	I	4	2	6	I		I				I					16
	4000-4500		5	4	6	5	I		I								22
	4500-5000	I	I	4	5	5	6	3		I	I						27
	5000-5500	I		I	2	6	I	2	4	3						I	21
	5500-6000	I			I	I	4	I	I	2	I						12
	6000-6500				I	I	3		5	2	I						13
	6500-7000						I	I		I							3
	7000-7500							I	I	2		I					5
7500-8000											I					I	
Total		10	13	16	24	22	18	9	12	12	4	2	0	0	0	I	143

TABLE 19
Correlation surface for amount of eight-months milk production at three years old
and amount of eight-months milk production at six years old
for pure-bred Jersey cows.

		6 years old																Total
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	
3 years old	2000-2500						1											1
	2500-3000	1				1		1										3
	3000-3500		2	2	5	1	3	1										14
	3500-4000		1	4	1	2	2			1	1							12
	4000-4500				2	1	3	3	1								1	14
	4500-5000					2	1	2	3	7	3	2		1				21
	5000-5500	1					2	4	5	4	1	3	2					22
	5500-6000				1	1		2	2			1				1	1	9
	6000-6500					1	2	1	2	2				2				10
	6500-7000						1				1	1						3
	7000-7500								1		1	2						5
	7500-8000												1					1
Total		2	3	8	10	12	18	17	16	8	9	6	3	0	1	1	1	115

The standard deviations of milk production as influenced by the length of time the cow is retained in the herd

A similar analysis for the effect of selection of cows at a given age on the basis of their milk production to be retained as milkers in after years is given in table 20. The arrangement of the data is entirely similar to that of table 11. The vertical columns give the standard deviations (without SHEPPARD'S correction) for the age of the cows indicated; the values are, from top to bottom, those of the given year when the same-age cows also have other records at the years indicated on the

TABLE 20
Standard deviations of milk production of a given year when correlated with other given years.

Age	2 years	3 years	4 years	5 years
2 to 3		1109.14 $\pm$ 39.00	1166.17 $\pm$ 42.17	1306.56 $\pm$ 52.67
3 to 4	806.99 $\pm$ 28.37		1103.81 $\pm$ 42.56	1427.69 $\pm$ 56.94
4 to 5	736.58 $\pm$ 26.63	1055.55 $\pm$ 40.70		1195.11 $\pm$ 47.66
5 to 6	782.79 $\pm$ 31.55	1136.60 $\pm$ 45.33	1047.07 $\pm$ 41.76	
6 to 7	797.53 $\pm$ 34.44	1157.00 $\pm$ 51.45	1103.37 $\pm$ 48.04	1251.05 $\pm$ 54.02
7 to 8	831.76 $\pm$ 41.59	1137.94 $\pm$ 54.83	1027.18 $\pm$ 46.09	1109.12 $\pm$ 46.57
8 to 10	744.36 $\pm$ 34.00	1147.04 $\pm$ 53.13	1020.66 $\pm$ 43.89	1243.36 $\pm$ 49.25
10 and older	621.36 $\pm$ 46.86	1290.28 $\pm$ 141.18	1074.39 $\pm$ 73.20	1174.59 $\pm$ 75.54

TABLE 20 (continued)

Standard deviations of milk production of a given year when correlated with other given years.

Age	6 years	7 years	8 and 9 years	10 and older
2 to 3	1453.51 $\pm$ 62.76	1222.82 $\pm$ 61.14	1255.63 $\pm$ 81.12	1121.10 $\pm$ 84.54
3 to 4	1417.51 $\pm$ 63.04	1281.58 $\pm$ 61.75	1390.96 $\pm$ 64.43	932.63 $\pm$ 102.05
4 to 5	1699.38 $\pm$ 73.99	1262.45 $\pm$ 56.65	1285.72 $\pm$ 55.29	1052.16 $\pm$ 71.68
5 to 6	1362.64 $\pm$ 58.84	1235.78 $\pm$ 51.89	1355.07 $\pm$ 53.67	1198.34 $\pm$ 77.07
6 to 7		1288.48 $\pm$ 55.19	1293.65 $\pm$ 50.69	1004.11 $\pm$ 59.87
7 to 8	1274.72 $\pm$ 54.60		1355.50 $\pm$ 52.61	1093.68 $\pm$ 60.63
8 to 10	1236.58 $\pm$ 48.47	1159.80 $\pm$ 45.01		1141.55 $\pm$ 48.89
10 and older	1108.36 $\pm$ 66.08	1061.13 $\pm$ 58.83	1149.98 $\pm$ 49.25	

left margin of the table. The data for the calculation of these constants are taken from correlation tables 9 to 40.

These data are graphically represented in figure 7, the values of the standard deviations being represented as ordinates and the ages at which the cows had another lactation as the abscissae. The curves progressing from top to bottom of the figure are for the lactation at the 2, 3, 4, 5, 6, 7, 8-and-9 and 10-and-older groups. The lines generated by those curves should, of course, be parallel with the base if there has been no selection practiced. If selection has been at any given place it should be expected to curtail the low-producing cows and thus reduce the standard deviation of the population as a whole.

TABLE 21

Correlation surface for amount of eight-months milk production at three years old and amount of eight-months milk production at seven years old for pure-bred Jersey cows.

		7 years old															Total
		2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000		
3 years old	2500-3000		I													I	
	3000-3500	I		2	I	I	2	3								10	
	3500-4000			2	3	4	2	I	I		I				I	15	
	4000-4500				3	I	3	I	2	I	I	I				13	
	4500-5000					2	4	4	I	4	3			I		19	
	5000-5500					2	2	2	2	2	2	2		I	I	15	
	5500-6000							I		2	I		I			5	
	6000-6500							4	I	3	3					11	
	6500-7000							I	2		I					4	
7000-7500									I		3				4		
7500-8000										I					I		
Total		I	I	4	7	10	13	17	9	13	13	6	I	2	I	98	

TABLE 22

Correlation surface for amount of eight-months milk production at three years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

8 and 9 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	Total
3 years old																		
2500-3000		1	1			2			1	1								6
3000-3500	1	2	3	3		1	2	1	2									15
3500-4000		1	2	3	3	2	2											13
4000-4500				3	1	1	2	1	1		1							10
4500-5000			1	2	2	5	5	2	1									18
5000-5500				1	2	2	6	2	5	1		1						20
5500-6000					2	1		2									1	6
6000-6500					2	1	2	2	2	1	2							12
6500-7000							1			1	2							4
7000-7500												1				1		2
Total	1	4	7	12	12	15	20	8	14	4	5	2	0	0	0	1	1	106

The curves for the two, three, four, five, seven, eight-and-nine and ten-and-older age groups are obviously linear and follow a general course parallel with the base. The standard deviations for each of these given ages are therefore constant to all intents and purposes. Since one effect of selection is to reduce the standard deviation of the selected race in

TABLE 23

Correlation surface for amount of eight-months milk production at three years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

3 years old

	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	Total
10-and-above years old											
3000-3500	1	1									2
3500-4000											0
4000-4500			1			1	1	1	1		5
4500-5000		1	1			1					3
5000-5500				1		2	1	1			5
5500-6000						1		1			2
6000-6500											0
6500-7000								1	1		2
Total	1	2	2	1	0	5	2	3	2	1	19

FIGURE 7.—Curves showing the standard deviations for eight-months milk production of a given age in the cow's life at the successive ages with which the given age is correlated. The ordinates are the standard deviations for the pounds of milk produced. The abscissae are the ages of the groups with which the given age had the standard deviation plotted. Proceeding from top to bottom the curves show the standard deviation for the 2, 3, 4, 5, 6, 7, 8- and-9 and 10-year-and-older age groups. The lines are quite regular and in general are parallel with the base as would be expected if no selection were practiced.

comparison with that of the unselected race, it follows that no selection is evidenced in these age groups. The case for the six-year group is not so clear. Here the general trend of the curve seems downward. This downward trend is only slight as compared with its probable error. The reason for it is as obscure as in the similar trend of the mean-8-months-production curve.

TABLE 24

Correlation surface for amount of eight-months milk production at four years old and amount of eight-months milk production at five years old for pure-bred Jersey cows.

		4 years old											Total	
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000		8000-8500
5 years old	3000-3500	I	I	I	2	I	I	I						8
	3500-4000		I	6	5	I	3		I					17
	4000-4500		3	5	2	3	2	I				I		17
	4500-5000			4	8	6	6	2	I	2				29
	5000-5500			I	3	6	9	4		I				24
	5500-6000			I	2		4	2	2					11
	6000-6500				2	2	2	3	3		2			14
	6500-7000					2	2	2		3	I			10
	7000-7500				I	I	I		I	2		I		7
	7500-8000						I	2		I				4
8000-8500								I				I	2	
Total		I	5	18	25	22	31	18	8	9	3	2	I	143

The general conclusion indicated by these data is that no selection of the high-producing cows to retain only the high milker in the older ages based on their earlier-recorded production has been practiced.

The coefficients of variation of milk production as influenced by the time the cow is retained in the herd

The relative measure of the effect of any change which may have occurred is given by the coefficients of variation for the milk production of any given age. The constants for this comparison are given in table 25. The arrangement of them in this table is identical with that for tables 11 and 20. The age for which the coefficients of variation are determined is given above the columns. The ages at which the cows have records at other ages are given on the left. The data for these are calculated from correlation tables 9 to 40.

TABLE 25
Coefficients of variation of milk production of a given year when correlated
with other given years.

Age	2 years	3 years	4 years	5 years
2 to 3		23.13 $\pm$.85	23.50 $\pm$.87	23.57 $\pm$ 1.02
3 to 4	19.49 $\pm$.69		21.94 $\pm$.89	26.71 $\pm$ 1.15
4 to 5	18.00 $\pm$.67	22.74 $\pm$.93		23.02 $\pm$.96
5 to 6	18.73 $\pm$.79	23.98 $\pm$ 1.01	20.80 $\pm$.87	
6 to 7	19.02 $\pm$.85	24.01 $\pm$ 1.13	21.46 $\pm$.95	23.16 $\pm$ 1.04
7 to 8	20.01 $\pm$ 1.04	23.38 $\pm$ 1.17	19.97 $\pm$.93	20.86 $\pm$.92
8 to 10	18.33 $\pm$.85	24.42 $\pm$ 1.17	20.14 $\pm$.89	23.39 $\pm$.96
10 and older	14.93 $\pm$ 1.16	24.95 $\pm$ 2.90	21.51 $\pm$ 1.57	21.48 $\pm$ 1.41

TABLE 25 (continued)
Coefficients of variation of milk production of a given year when correlated
with other given years.

Age	6 years	7 years	8 and 9 years	10 years and older
2 to 3	25.26 $\pm$ 1.14	22.38 $\pm$ 1.15	24.65 $\pm$ 1.21	22.76 $\pm$ 1.82
3 to 4	24.82 $\pm$ 1.18	23.48 $\pm$ 1.17	27.28 $\pm$ 1.34	19.00 $\pm$ 2.15
4 to 5	30.28 $\pm$ 1.42	23.72 $\pm$ 1.09	24.95 $\pm$ 1.14	22.10 $\pm$ 1.57
5 to 6	24.20 $\pm$ 1.09	22.38 $\pm$.97	26.02 $\pm$ 1.10	24.62 $\pm$ 1.71
6 to 7		24.15 $\pm$ 1.09	24.02 $\pm$ 1.04	20.43 $\pm$ 1.24
7 to 8	23.61 $\pm$ 1.09		26.68 $\pm$ 1.12	22.64 $\pm$ 1.34
8 to 10	23.02 $\pm$.95	22.22 $\pm$.89		23.28 $\pm$ 1.04
10 and older	20.59 $\pm$ 1.31	20.42 $\pm$ 1.15	22.18 $\pm$.99	

To facilitate comparison of the constants for one age with those of another age the values of the coefficients are represented as curves in figure 8. The arrangement of these curves is entirely similar to that of figures 6 and 7.

The values of the coefficients of variation are represented as ordinates and the ages at which the cows had another lactation as abscissae. The curves progressing from top to bottom of the figure are for ages 2, 3, 4, 5, 6, 7, 8 and 9, and 10 and above. The same reasoning, that if no selection of animals based on their records of previous years has taken place the curves generated by these coefficients of variation should be straight lines parallel with the base, still holds.

The curves in figure 8 are individually somewhat more irregular than the curves for the mean milk production of a given age (figure 6) and the standard deviations of a given age (figure 7). No definite trend of the coefficients either toward a higher value or toward a lower value is seen in the coefficients except those for the six-year period. This six-

FIGURE 8.—Curves showing the coefficients of variation for the eight-months production of a given age at each successive age with which the milk yield of the given age is correlated. The ordinates are the coefficients of variation and the abscissae are the ages of the groups with which the given age is correlated. The significance of the curves proceeding from top to bottom is the same as in figures 6 and 7. The lines generated by the curves should, of course, as in the preceding case be parallel with the base and linear if no effect of selection of cattle based on their previous records has taken place.

TABLE 26

Correlation surface for amount of eight-months milk production at four years old and amount of eight-months milk production at six years old for pure-bred Jersey cows.

		4 years old													Total
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	
6 years old	2500-3000	I							I						2
	3000-3500		I		2		I								4
	3500-4000		I	4	3		I								9
	4000-4500			3	4	I	2								10
	4500-5000			2	3	6	2	I		I					15
	5000-5500		2	I	3	2	4	2	I		I				16
	5500-6000		I		2	4	4	2	I	I	I	I			17
	6000-6500				2	4	6	6	3	I	I				23
	6500-7000				I	I	I	2		I	I	I			8
	7000-7500				I	2	I	I		I					6
	7500-8000						I			3					4
	8000-8500							I	I	I					3
	8500-9000														0
	9000-9500						I		I						2
	9500-10000														0
	10000-10500													I	I
Total		I	5	10	21	20	24	15	8	9	4	2	0	I	120

TABLE 27

Correlation surface for amount of eight-months milk production at four years old and amount of eight-months milk production at seven years old for pure-bred Jersey cows.

		4 years old											Total
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	
7 years old	2000-2500		I										I
	2500-3000	I											I
	3000-3500			I	2	I							4
	3500-4000		I	2	4	2	I						10
	4000-4500				5	2	2	I					10
	4500-5000				5	4	3	3					15
	5000-6000			2	2	2	6	3	3	I			19
	5500-6000				I	7	4	2	2		I		17
	6000-6500				2	3	I	3	3	I	I		14
	6500-7000					2	3			3	2		10
	7000-7500							I	I	I	2	I	6
	7500-8000								I	I			2
	8000-8500						2	I					3
	8500-9000							I					I
Total		I	2	10	20	22	22	13	9	7	6	I	113

year curve is quite irregular. The general tendency is slightly downward indicating if it proves significant some selection on the basis of the

TABLE 28

Correlation surface for amount of eight-months milk production at four years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

8 and 9 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	Total
4 years old																		
2500-3000		1	1															2
3000-3500				1														1
3500-4000					2	3	1	2	3	1								12
4000-4500	2		2	2	5	3	6		4	2								26
4500-5000			1	1	5	3	8	1	1	3	2							25
5000-5500			4	2		2	4	1	2	1	1							17
5500-6000					2	4	4	2		4		1	1					18
6000-6500						2	3	3		1	1							10
6500-7000					2		1	2									1	6
7000-7500							2			1	1							4
7500-8000								1			1							2
Total	2	1	10	9	15	16	31	11	7	12	6	1	1	0	0	0	1	123

TABLE 29

Correlation surface for amount of eight-months milk production at four years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

4 years old

	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	Total
10-and-above years old											
2000-2500				1							1
2500-3000						1					1
3000-3500	1	1	1	2							5
3500-4000				1		1	1				3
4000-4500			1	3	3	1	1	1		1	11
4500-5000			1	1		2			2		6
5000-5500		1		1	3	3	2		1		11
5500-6000					2		1		1	1	5
6000-6500				1	1	2					4
6500-7000				1							1
7000-7500				1							1
Total	1	2	3	12	9	10	5	1	1	5	49

six-year milk production. Neglecting the rise at four years as obviously extraneous to the general trend of the curve, the difference between the six-year coefficients of variation for cows which were milked at 2 years and the six-year coefficients of variation for cows which were milked at 10 years is 4.67 ± 1.74 , or the divergence is 2.7 times its probable error. The difference is therefore doubtfully significant, in fact probably not significant.

The conclusion which may be derived from these data is that no effect of selection of cows on the basis of their milk records is manifested by the coefficients of variation for any given age taken in conjunction with the length of time the cows remain as milkers in the herd.

TABLE 30

Correlation surface for amount of eight-months milk production at five years old and amount of eight-months milk production at six years old for pure-bred Jersey cows.

		5 years old												Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	Total
6 years old	2500-3000	I	I											2
	3000-3500	2												2
	3500-4000		4	3	2									9
	4000-4500	2	4	2	5	I								14
	4500-5000	I		I	4	5	3							14
	5000-5500	I	2	I	4	5	4		I	I				19
	5500-6000		2	I	I	7	I	I	3	I				17
	6000-6500			I	I	3	2	5	3	I				16
	6500-7000					I	I	4	2					8
	7000-7500				I	I	I		2	2	I	I	I	10
	7500-8000								2	3		I		6
	8000-8500					I				I				2
	8500-9000													0
9000-9500						I			I				2	
9500-10000									I				I	
Total		7	13	9	18	24	13	10	13	11	I	2	I	122

Among the important general conclusions coming out of this study of the mean 8-months milk production, the standard deviation and coefficients of variation of this milk production within the different age groups determined separately for the other ages at which they had lactations in the herd, perhaps the most important is that the data for which the curve of mean milk production with age (figure 4) was obtained are free

TABLE 31

Correlation surface for amount of eight-months milk production at five years old and amount of eight-months milk production at seven years old for pure-bred Jersey cows.

		5 years old											Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	
7 years old	2000-2500	1											1
	2500-3000	1											1
	3000-3500	1	1										2
	3500-4000	1	1	1	2		3	1					9
	4000-4500	2	2	1	4	4							13
	4500-5000		2	6	4	5	3		1				21
	5000-5500	2	2	2	4	5	4	1	2	1			23
	5500-6000			1	2	4	2	2					11
	6000-6500				3	5	3	4	5	1			21
	6500-7000			1	2	2	1	2	2			1	11
	7000-7500							3	1	3		1	8
	7500-8000		1			2	1						4
	8000-8500						1	1		1			3
	8500-9000										1		1
Total		7	10	12	21	27	18	14	11	6	1	2	129

from any error due to selection of cows that were to remain in the herd on the basis of the records they made in their first, second or later lactations. From this it follows that the rise and fall of the curve for mean milk production with advancing age is due strictly to the physiological changes brought about in the mammary functions of the cow by age. The general equation to this physiological change expresses the law by which it is governed in the same way that MINOT (1891, 1907), LEWENZ and PEARSON (1904), DONALDSON (1908, 1909, 1910, 1911), HATAI (1911), and PEARL with the assistance of PEPPER (1907), have expressed the similar law for the manner in which the metabolic functions producing growth change with increasing age.

Added significance is given by this analysis to the conclusions previously indicated concerning the variation of milk within a given age, the correlation of age and milk production and the change of the variability of milk production with age.

TABLE 32

Correlation surface for amount of eight-months milk production at five years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

		5 years old												Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	
8 and 9 years old	2000-2500		I						I					2
	2500-3000	2	I				I							4
	3000-3500	3	I	I		I								6
	3500-4000	3	5	I	I	3		2	I					16
	4000-4500	2	2	4	4	4	3	I						20
	4500-5000	I		2	2	7	3	I	I					17
	5000-5500			4	4	4	4	2	I	I				20
	5500-6000		2	3	2	4	3	2		2				18
	6000-6500		I	I	I	6	4	I	2	2	I	I		20
	6500-7000			I	I	I	I	3	4			2		13
	7000-7500							I		2	I			4
	7500-8000				I									I
	8000-8500											I	I	2
	8500-9000													0
	9000-9500													0
	9500-10000							I						I
	10000-10500						I							I
Total		11	13	17	16	30	20	14	10	7	2	4	I	145

TABLE 33

Correlation surface for amount of eight-months milk production at five years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

		5 years old										Total
		3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	
10 years and older	2000-2500		I									I
	2500-3000		I		I							2
	3000-3500	2	I		I	I						5
	3500-4000	I				I			3			5
	4000-4500		I		2	I	I	I		I		7
	4500-5000		I	2	I	I	2	2	I			10
	5000-5500			3		I	4	2	I			11
	5500-6000			I			I	2	I			5
	6000-6500					I		I				2
	6500-7000						I			3	I	5
	7000-7500							I				I
	7500-8000							I				I
Total		3	5	6	5	6	9	10	6	4	I	55

The correlation of eight-months milk production at a given age with the eight-months milk production at any other given age

For a firm foundation of our practical agriculture, particularly dairying, knowledge of the inter-relationships of the workings of the mammary functions at one age in comparison with its precision of action at another age can scarcely be too exact. The existing practice is, as already pointed out, largely empirical in its nature, often leading to questionable results. The reflection of the questionable nature of these practices is seen in the not uncommon practice of dairymen neglecting the records of the first lactation as a measure of the cow's possibilities for future production.

TABLE 34
Correlation surface for amount of eight-months milk production at six years old and amount of eight-months milk production at seven years old for pure-bred Jersey cows.

		6 years old													Total	
		2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	Total
7 years old	2000-2500			I	I											2
	2500-3000	I														I
	3000-3500		I	2	2											5
	3500-4000		2	4	2	I	I		2							12
	4000-4500			I	4	2	3		I	I						12
	4500-5000			3	2	6	5	2								18
	5000-5500	I		I	4	I	5	3	2	3			2			22
	5500-6000			I	I	I	2	3	4	I	I					14
	6000-6500				I		3	2	7		I	I				15
	6500-7000					I	I	3	3		2	I				11
	7000-7500					I		I	I	I	I	I				6
	7500-8000							I					I			2
	8000-8500								I			I			I	3
	8500-9000										I					I
Total		2	3	13	17	13	20	15	21	6	6	4	3	0	I	124

The correlations between the milk production of various lactations with one another have material importance as well as theoretical interest for these problems. If the correlation is high it is evident from the practical side that the culling of the herd through the use of dependable criteria will result in the increase of profits to its owners. From the biological side, a high correlation means that the animals composing the herd are innately differentiated (presumably due to their inherited complex) in their mammary capacities.

TABLE 35

Correlation surface for amount of eight-months milk production at six years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

8 and 9 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	Total
2500-3000		I	I				I	I									4
3000-3500		I	2	2													5
3500-4000			I	2	4	3	I										11
4000-4500	I		2	2	I		2	I	3	I	I						14
4500-5000		2	4	I	3	6	6	I	3	2							28
5000-5500		I	I	2	I	4	4		3	2	I	I					20
5500-6000					3	2	5	3	5	I	I						20
6000-6500	I				3	2	6	4		3	I						20
6500-7000						I	3	I		3						I	9
7000-7500						I	2	2	4	I				I			11
7500-8000										I	I						2
8000-8500							I	I		I	I						4
Total	2	5	11	9	15	19	31	14	18	15	6	I	I	0	0	I	148

The determination of the correlations for these data is of especial interest, for, as previously shown in the earlier part of this paper, the data are of exceptional value in that they are on a pure-bred Jersey herd kept intact for many years; the lactation records for several lactations are recorded on a number of the animals; and the herd has been subject to no detectable culling based on the production of a given lactation. From these data there have been extracted lactation records to the number of 3178 pairs having a full eight-months lactation free from any disease or sickness or other trouble known to influence the records. These records have been formed into twenty-eight correlation tables the most of which are of considerable size.

After careful consideration it was decided that since these tables have much of scientific and practical value to the practice of dairy husbandry they had better be printed *in toto*. These tables are reproduced in the text tables numbers 9 to 40.

The means, standard deviations and coefficients of variation for these correlation tables have been presented in tables 11, 20 and 25. Table 32 gives the correlations and their probable errors for all ages at which the lactation records were divided. The vertical columns give the correlations of the milk production at the age heading the column with the

TABLE 36

Correlation surface for amount of eight-months milk production at six years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

6 years old

											Total
	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	
10-and-above years old											
2000-2500			1								1
2500-3500						1					1
3000-3500	1	1		1		1					4
3500-4000			2			1		2			5
4000-4500			1		2		3	1	1		8
4500-5000			2	1	4	2	2	3		1	15
5000-5500		1	1		2	1	3	5	1		14
5500-6000					4		1	2			7
6000-6500				1				3	1		5
6500-7000										3	3
7000-7500								1			1
Total	1	2	7	3	12	6	9	17	2	5	64

TABLE 37

Correlation surface for amount of eight-months milk production at seven years old and amount of eight-months milk production at eight and nine years old for pure-bred Jersey cows.

8 and 9 years old

	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	7500-8000	8000-8500	8500-9000	9000-9500	9500-10000	10000-10500	Total
7 years old																		
2500-3000		1	1															2
3000-3500	1	1	1	2	2		1											8
3500-4000			3	3	5	3	3											17
4000-4500	1	1		3	1	2	4	1		1								14
4500-5000		1	3	5	2	6	5	2	2	1								27
5000-5500		1		2	3	2	5	3	3	2	2							23
5500-6000				1	1	2	6	1	3	1	3							18
6000-6500	1				4	4	2	1	3	2								17
6500-7000					2	1	3	4	2	2	1							15
7000-7500							1	1	1	1	1	1	1			1		8
7500-8000										1							1	2
Total	3	5	8	16	20	20	30	13	14	11	7	1	1	0	0	1	1	151

milk production at the ages indicated at the left-hand margin of the table.

As will be noted the correlations necessary to give the complete set of correlations for any given age are repeated, e.g., the correlation of milk production at 2 years with that at 3 years is $+0.5764 \pm .0332$ and appears in the 2-year column. The correlation of the 3-years milk production with that of 2 years will, of course, be the same ($0.5764 \pm .0332$) and is repeated in the three-year column. In this manner a complete picture of the relationship of the lactations of the 2-year groups with the other lactations of the other groups is given in the column.

TABLE 38
Coefficients of correlation of milk production of a given age with the milk production of another given age.

Age	2-3 years	3-4 years	4-5 years	5-6 years
2 to 3		$+0.5764 \pm .0332$	$+0.5426 \pm .0361$	$+0.5373 \pm .0406$
3 to 4	$+0.5764 \pm .0332$		$+0.6206 \pm .0335$	$+0.5479 \pm .0395$
4 to 5	$+0.5426 \pm .0361$	$+0.6206 \pm .0335$		$+0.5541 \pm .0391$
5 to 6	$+0.5373 \pm .0406$	$+0.5479 \pm .0395$	$+0.5541 \pm .0391$	
6 to 7	$+0.5500 \pm .0426$	$+0.5305 \pm .0452$	$+0.5624 \pm .0421$	$+0.7306 \pm .0284$
7 to 8	$+0.5815 \pm .0468$	$+0.5394 \pm .0483$	$+0.6328 \pm .0380$	$+0.5667 \pm .0403$
8 to 10	$+0.4938 \pm .0488$	$+0.5376 \pm .0465$	$+0.4154 \pm .0503$	$+0.5405 \pm .0397$
10 and older	$+0.5603 \pm .0732$	$+0.6336 \pm .0925$	$+0.2144 \pm .0919$	$+0.5371 \pm .0697$

TABLE 38 (continued)
Coefficients of correlation of milk production of a given age with the milk production of another given age.

Age	6-7 years	7-8 years	8-10 years	10 years and older
2 to 3	$+0.5500 \pm .0426$	$+0.5815 \pm .0468$	$+0.4938 \pm .0488$	$+0.5603 \pm .0732$
3 to 4	$+0.5305 \pm .0452$	$+0.5394 \pm .0483$	$+0.5376 \pm .0465$	$+0.6336 \pm .0925$
4 to 5	$+0.5624 \pm .0421$	$+0.6328 \pm .0380$	$+0.4154 \pm .0503$	$+0.2144 \pm .0919$
5 to 6	$+0.7306 \pm .0284$	$+0.5667 \pm .0403$	$+0.5405 \pm .0397$	$+0.5371 \pm .0697$
6 to 7		$+0.6515 \pm .0349$	$+0.4800 \pm .0427$	$+0.4578 \pm .0666$
7 to 8	$+0.6515 \pm .0349$		$+0.5750 \pm .0367$	$+0.3036 \pm .0712$
8 to 10	$+0.4800 \pm .0427$	$+0.5750 \pm .0367$		$+0.5113 \pm .0448$
10 and older	$+0.4578 \pm .0666$	$+0.3036 \pm .0712$	$+0.5113 \pm .0448$	

The order of magnitude of these correlations is from $+0.7306$ for the correlation of 8-months milk productions during the ages 5 to 6 and 6 to 7 years, and $+0.2144$ for the correlation of the productions during the ages 4 to 5 and 10 years and older. Such correlations indicate that the milk production of one lactation may be predicted with relatively little inaccuracy from the milk production of another lactation. The graphs showing these correlations for each year are seen in figure 9.

The arrangement of this figure is similar to that of figure 6. The magnitudes of the correlations are represented as ordinates. The ages of the cows whose productions are correlated with those of the given age are represented as abscissae. The curves progressing from top to bottom of the figure represent the correlations of the 8-months milk productions at the ages two, three, four, five, six, seven, eight and nine, and over ten years with the eight-months productions occurring at the other ages shown on the lower margin of the figure.

The graph for the correlation coefficients of the milk production at two years with the milk production of the subsequent lactations of the same cows shows little deviation from a straight line. The values of these correlations range from $+0.4938 \pm .0488$ to $+0.5815 \pm .0468$. The values for the three-year correlations with the other years range somewhat higher, $+0.5305 \pm .0452$ to $+0.6336 \pm .0925$. These correlations would be quite accurately described by a linear function. The values of the correlation of the four-year-olds' production with those of other years range from $+0.2144 \pm .0919$ to $+0.6328 \pm .0380$. On the whole these values are slightly lower than are those for other years. The values for the five-year-olds range from $+0.5371 \pm .0697$ to $+0.7306 \pm .0284$. These values are the highest in their range of any of the ages. The correlations for the six-year productions with those of other ages range from $0.4578 \pm .0666$ to $+0.7306 \pm .0284$. The correlations for the seven-year productions range from $0.3036 \pm .0712$ to $+0.6515 \pm .0349$. The correlations for the 8-and-9-year period range from $+0.4154 \pm .0503$ to $+0.5750 \pm .0367$ and the correlations for the productions at 10 years and older with those of the younger years range from $+0.2144 \pm .0919$ to $+0.6336 \pm .0925$. From this brief resumé of the tabled results it is seen that all of the results have the plus sign and have a rather large value for data of this kind. The values of the coefficients are in general higher for the younger ages than they are for the older ages. This is a fortunate circumstance from a practical standpoint for the breeder is more desirous of selecting the animals to remain in the herd from their two-year records than he is to select his animals at ten years old. The values of the correlations are fortunately such that a gain in accuracy results by predicting from the two-year-olds as over predictions at later ages.

From the practical side of culling the poor milkers out of the herd these results are highly satisfactory. The data are equally interesting when considered in their biological aspects. The correlations show that the cows composing the herd are innately differentiated in their milk-

FIGURE 9.—Curves showing the coefficients of correlation of milk production of a given age and the milk production occurring at any other given age. The ordinates are the values of the correlation coefficients. The abscissae are the ages at which the lactation record compared with the record at the given age, was made. Thus the top curve of the figure shows the correlations of records made at two years old with the records made at subsequent ages. From this curve the correlation between two-years milk production and three-years milk production is found on the three-year line of the two-year curve (top curve of figure) to be 0.5764. The correlation of the two-year and four-year record is found on the four-year line where the two-year line crosses it (top curve of figure) to be 0.5426, etc. The dotted lines serve to connect the correlations immediately preceding and following the age for which the line is drawn.

TABLE 39

Correlation surface for amount of eight-months milk production at seven years old and amount of eight-months milk production at ten years and older for pure-bred Jersey cows.

7 years old

	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	Total
2000-2500					1						1
2500-3000				1			1				2
3000-3500	1		1		1	3		1			7
3500-4000		1		1	1	2					5
4000-4500			1	1	2	2	4	1			11
4500-5000		1	2	4	3	1	3	1	1	1	17
5000-5500		1	4		2		3	2	1	1	14
5500-6000					1		2	1	1	2	7
6000-6500						1	4				5
6500-7000						1	1				2
7000-7500				1				1			2
7500-8000									1		1
Total	1	3	8	8	11	10	18	7	4	4	74

TABLE 40

Correlation surface for amount of eight-months milk production at eight and nine years old and amount of eight-months milk production at ten years old and older for pure-bred Jersey cows.

8 and 9 years old

	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000	7000-7500	Total
2000-2500		1		1							2
2500-3000			1		1	1					3
3000-3500	2	2	3	2		1		1			11
3500-4000			2	2	2				2		8
4000-4500		1	3	2	3	5		2	1	1	18
4500-5000	1	1	5	2	4	10	2		2		27
5000-5500			2	3	2	6		5	4	1	23
5500-6000				1		3	2	2	5		13
6000-6500						1		1	1	2	5
6500-7000						5	1	2		1	9
7000-7500						1		1	1		3
7500-8000							1		1		2
Total	3	5	16	13	12	33	6	14	17	5	124

producing abilities. The plane of production once established the cow tends to maintain this relative plane from lactation to lactation. The values of the correlations indicate clearly that there is a fairly accurately working mechanism behind this function.

From this point of view it is of a good deal of interest to compare the only other statistics available for cattle, with those derived here. GAVIN (1913) using his measure of the lactation as the "revised maximum" found the correlation between the milk productions of the various lactations and that of their maximum "revised maximum" GAVIN (1912). Table 4I is a copy of GAVIN's table showing these correlations.

TABLE 4I

Lactation	<i>r</i> with maximum "revised maximum"	Probable error
First	+ .394	± 0.031
Second	+ .452	± 0.030
Third	+ .506	± 0.028
Fourth	+ .605	± 0.024
Fifth	+ .762	± 0.016

This table shows in general a lower value for the correlations measuring the consistency of milking performance from lactation to lactation than is shown by our data. This may in part be due to the fact that GAVIN's material was somewhat heterogeneous, including data from British Holsteins and Shorthorns. In all events, the correlations confirm our general conclusions that milk production of one lactation is quite closely correlated with that of the other lactations. This reasoning transferred to the individuals of the race of dairy cattle, appears to prove beyond any shade of doubt that the individuals of the race are innately differentiated as regards the capacity for milk production.

Of the quantitative data on other species of practical interest perhaps the most complete is that of HARRIS and BLAKESLEE (1918) on the White Leghorn. In this work they determined the correlations between the monthly egg production and the other-eleven-months production of the same bird. The correlations for these monthly ovulations with the other-eleven-months ovulation take values ranging from $+0.240 \pm .033$ to $+0.573 \pm .023$. The range is there quite similar to those obtained in this although lower in value. The knowledge of these sets of constants gives criteria for the fairly accurate prediction of the records that may be expected at a subsequent date in the life of these two species of extreme economic importance.

THE REGRESSION OF THE MILK PRODUCTIONS OF THE DIFFERENT LACTATIONS ON EACH OTHER

The constants necessary to determine the approach of the regression to linearity are given in table 42. The values of the correlation coefficients have been repeated for greater ease in drawing conclusions from the values of the correlation ratios.

In this table the values of the correlation ratios range from 0.4382 to 0.8189. The conclusions drawn from a study of the correlation coefficients is thus confirmed by these correlation ratios.

As is generally well recognized, the coefficients of correlation depend for their accuracy on the regression lines being closely described by a linear function. The values of the correlation ratio on the other hand do not depend on this fitting of linear functions to the rough regression lines but do give a measure of the association from the regression line itself. If this rough regression is linear the two constants should agree within the limits of their probable errors. In table 42 columns $\eta-r$ and η^2-r^2 give constants to measure this agreement. The last column gives the multiple number of times η^2-r^2 is to the probable error. This column gives us in a form easily appreciated the measure of what significance can be attached to these constants. The values range from 0.33 to 4.46, from an insignificant deviation to one probably significant. Of the 28 values only six are less than 2 times their probable error. The average of the 28 constants shows that the mean deviation is 2.42 times the probable error. On the face of this it would seem that there is a slight deviation of the regression lines from the linear. This deviation may be due to the duration of life covered by the age units within which the lactation records were correlated. These units were in general of one year in length. During this time, as was shown earlier in this paper, there is a non-linear rise in milk production. One year appears to be of sufficient length of time to have produced this effect in the milk yield. To this effect the appearance of a regression differing slightly from the linear may be attributed. It would appear that should a shorter unit of time be chosen, the value of the correlation coefficients would agree more closely with those of the correlation ratios.

This reasoning, while apparently probable, does not prove true, as a later section of the paper dealing with regressions over five lactations shows. This slight lack of linearity must therefore be considered due either to chance or to some obscure causes. In any case it will not influence the results materially where the correlations are used as constants for calculation.

TABLE 42
Analytical constants for intra-individual variation of cow's milk from lactation to lactation.

Character	r	η	$\eta - r$	$\eta^2 - r^2$	$\frac{\eta^2 - r^2}{E_{\eta^2 - r^2}}$
2 years to 3 years	$+0.5764 \pm .0332$	$+0.6275 \pm .0301$	$0.0512 \pm .0448$	$0.0615 \pm .0239$	2.58
" " 4 "	$.5426 \pm .0360$	$.5922 \pm .0332$	$.0496 \pm .0490$	$.0563 \pm .0236$	2.39
" " 5 "	$.5373 \pm .0406$	$.6348 \pm .0340$	$.0976 \pm .0530$	$.1143 \pm .0363$	3.15
" " 6 "	$.5500 \pm .0426$	$.6206 \pm .0375$	$.0706 \pm .0568$	$.0826 \pm .0336$	2.46
" " 7 "	$.5815 \pm .0468$	$.6607 \pm .0398$	$.0792 \pm .0614$	$.0984 \pm .0421$	2.34
" " 8 and 9 "	$.4938 \pm .0488$	$.6033 \pm .0411$	$.1095 \pm .0638$	$.1201 \pm .0420$	2.86
" " 10 and older	$.5603 \pm .0732$	$.7081 \pm .0532$	$.1478 \pm .0905$	$.1875 \pm .0832$	2.25
3 years to 4 years	$.6206 \pm .0335$	$.7592 \pm .0231$	$.1386 \pm .0407$	$.1912 \pm .0429$	4.46
" " 5 "	$.5479 \pm .0395$	$.6648 \pm .0315$	$.1169 \pm .0505$	$.1418 \pm .0393$	3.60
" " 6 "	$.5305 \pm .0452$	$.6120 \pm .0393$	$.0823 \pm .0599$	$.0942 \pm .0368$	2.56
" " 7 "	$.5394 \pm .0483$	$.6381 \pm .0404$	$.0987 \pm .0630$	$.1162 \pm .0437$	2.66
" " 8 and 9 "	$.5376 \pm .0465$	$.6083 \pm .0413$	$.0707 \pm .0622$	$.0810 \pm .0358$	2.27
" " 10 and older	$.6336 \pm .0925$	$.8189 \pm .0509$	$.1853 \pm .1056$	$.2691 \pm .1409$	1.91
4 years to 5 years	$.5541 \pm .0301$	$.5661 \pm .0383$	$.0120 \pm .0547$	$.0134 \pm .0411$	.33
" " 6 "	$.5624 \pm .0421$	$.6186 \pm .0380$	$.0562 \pm .0567$	$.0664 \pm .0306$	2.17
" " 7 "	$.6328 \pm .0380$	$.7163 \pm .0309$	$.0835 \pm .0400$	$.1127 \pm .0401$	2.81
" " 8 and 9 "	$.4154 \pm .0503$	$.5359 \pm .0433$	$.1205 \pm .0664$	$.1146 \pm .0387$	2.96
" " 10 and older	$.2144 \pm .0919$	$.5353 \pm .0588$	$.3208 \pm .1148$	$.2406 \pm .0824$	2.92
5 years to 6 years	$.7306 \pm .0284$	$.7552 \pm .0263$	$.0246 \pm .0387$	$.0366 \pm .0229$	1.59
" " 7 "	$.5667 \pm .0403$	$.6638 \pm .0332$	$.0971 \pm .0522$	$.1195 \pm .0385$	3.10
" " 8 and 9 "	$.5405 \pm .0397$	$.6617 \pm .0315$	$.1212 \pm .0507$	$.1457 \pm .0395$	3.69
" " 10 and older	$.5371 \pm .0647$	$.6816 \pm .0486$	$.1445 \pm .0809$	$.1761 \pm .0693$	2.54
6 years to 7 years	$.6515 \pm .0349$	$.6712 \pm .0333$	$.0198 \pm .0482$	$.0261 \pm .0193$	1.35
" " 8 and 9 "	$.4800 \pm .0427$	$.5385 \pm .0393$	$.0585 \pm .0580$	$.0596 \pm .0262$	2.27
" " 10 and older	$.4578 \pm .0666$	$.5722 \pm .0507$	$.1145 \pm .0875$	$.1178 \pm .0544$	2.17
7 years to 8 and 9 years	$.5750 \pm .0367$	$.5827 \pm .0352$	$.0077 \pm .0515$	$.0089 \pm .0103$	.86
" " 10 years and older	$.3036 \pm .0712$	$.4382 \pm .0634$	$.1346 \pm .0953$	$.0998 \pm .0470$	2.12
8 and 9 years to 10 years and older	$.5113 \pm .0448$	$.5356 \pm .0432$	$.0243 \pm .0622$	$.0253 \pm .0100$	1.33

From these correlation coefficients in tables 41 and 42 it is possible to form the straight-line prediction of the milk yield of any age from the yield of any other age. As emphasized repeatedly in the various sections of this paper dealing with the separate subjects, the predictions may be criticised on the following grounds: that the mean milk yield rises with age in a line described by a logarithmic function; that the standard deviation of this milk yield rises in line described by a cubic parabola; and that the values of the correlation coefficients differ from the values of the correlation ratios by 2.42 times the probable error of $\eta^2 - r^2$. Such criticisms are recognized and admitted at once. It is believed that even admitting these there are a number of important points which may be elucidated by these equations.

The general equation for these regressions is given by

$$Y = (\text{Mean}_Y - r_{Yy} \frac{\sigma_Y}{\sigma_y} \text{Mean}_y) + r_{Yy} \frac{\sigma_Y}{\sigma_y} y$$

where y = 8-months milk yield in a given year and Y the expected milk yield for the year desired. The results of the substitution of the values of tables 11 to 42 in this equation are given in table 43.

As the milk production is given in pounds the second term of each of these equations gives the gain in expected milk yield for the given age, if one pound increase in actual production is made during the test. The calculation of an expected yield is, therefore, a simple matter of direct substitution.

Before discussing the relative practical and theoretical merits of these equations for the study of milk records it may be well to take up the equations individually. The coefficients of the y 's (actual yields) represent the reliance that may be put on these actual yields for subsequent productions. The information desired by those who use these coefficients for actual records may be divided into two heads: (1) At what age should the cow make her record to give the best index of her potential producing abilities? (2) To what age should the records, made at different periods in the life of the cow, be corrected so that the maximum reliance may be placed on the corrected record?

The first of these questions entails a study of the coefficients of the y 's for the ages at which the record was made. The data for this are given in table 44. The equations in this table are of no value save for this comparison, for if any record is to be corrected the equations in table 43 should be used.

TABLE 43

Regression equations of the milk yield for any age from that of any lactation record at another age.

Age at which milk yield was made	Age at which expected milk yield is desired			
	2 years to 3 years	3 years to 4 years	4 years to 5 years	5 years to 6 years
	Regression equation	Regression equation	Regression equation	Regression equation
2 years to 3 years y_2	$Y = 2130.0 + .4194y_3$	$Y = 1515.5 + .7922y_2$	$Y = 1447.4 + .8591y_2$	$Y = 1795.7 + .8968y_2$
3 years to 4 years y_3	$Y = 2391.2 + .3427y_4$	$Y = 1056.2 + .5935y_4$	$Y = 2013.7 + .6500y_3$	$Y = 2082.5 + .6882y_3$
4 years to 5 years y_4	$Y = 2394.4 + .3219y_5$	$Y = 2408.3 + .4362y_5$	$Y = 2513.3 + .4855y_5$	$Y = 2007.2 + .6325y_4$
5 years to 6 years y_5	$Y = 2456.2 + .3018y_6$	$Y = 2346.6 + .4330y_6$	$Y = 3092.2 + .3652y_6$	$Y = 1624.4 + .6708y_6$
6 years to 7 years y_6	$Y = 1995.3 + .3955y_7$	$Y = 2252.6 + .4790y_7$	$Y = 2345.0 + .5149y_7$	$Y = 2597.8 + .5086y_7$
7 years to 8 years y_7	$Y = 2570.8 + .2927y_8$	$Y = 2437.6 + .4433y_8$	$Y = 3367.9 + .3298y_8$	$Y = 2715.2 + .4959y_8$
8 years to 10 years y_8	$Y = 2633.1 + .3105y_{10}$	$Y = 868.8 + .8766y_{10}$	$Y = 3952.6 + .2190y_{10}$	$Y = 2905.2 + .5265y_{10}$
10 years and above y_{10}				

TABLE 43 (continued)

Regression equations of the milk yield for any age from that of any lactation record at another age.

Age at which milk yield was made	Age at which expected milk yield is desired			
	6 years to 7 years	7 years to 8 years	8 years to 10 years	10 years and above
	Regression equation	Regression equation	Regression equation	Regression equation
2 years to 3 years y_2	$Y = 1551.7 + 1.0023y_2$	$Y = 1910.8 + .8549y_2$	$Y = 1710.8 + .8329y_2$	$Y = 717.0 + 1.010y_2$
3 years to 4 years y_3	$Y = 2578.2 + .6500y_3$	$Y = 2502.2 + .6075y_3$	$Y = 2036.3 + .6519y_3$	$Y = 2539.6 + .4580y_3$
4 years to 5 years y_4	$Y = 1138.8 + .8662y_4$	$Y = 1435.1 + .7778y_4$	$Y = 2500.7 + .5233y_4$	$Y = 3711.3 + .2100y_4$
5 years to 6 years y_5	$Y = 1332.7 + .7958y_5$	$Y = 2105.0 + .6314y_5$	$Y = 2087.7 + .5890y_5$	$Y = 1871.7 + .5480y_5$
6 years to 7 years y_6	$Y = 1960.8 + .6445y_7$	$Y = 1779.2 + .6585y_6$	$Y = 2493.0 + .5019y_6$	$Y = 2681.7 + .4147y_6$
7 years to 8 years y_7	$Y = 2989.5 + .4591y_8$	$Y = 2720.5 + .4920y_8$	$Y = 1573.3 + .6720y_7$	$Y = 3205.1 + .3120y_7$
8 years to 10 years y_8	$Y = 2899.8 + .5053y_{10}$	$Y = 3772.8 + .2946y_{10}$	$Y = 2660.1 + .5150y_{10}$	$Y = 2271.5 + .5075y_{10}$
10 years and above y_{10}				

TABLE 44

Average regression equations of milk records at the same age for the expected records made throughout the life of the cow. (Summarized from table 43.)

Age at which milk record was made	Regression equation
2 years to 3 years	$Y = 1521.3 + .8927y_2$
3 " to 4 "	$Y = 2268.9 + .5893y_3$
4 " to 5 "	$Y = 2122.9 + .5637y_4$
5 " to 6 "	$Y = 2110.4 + .5440y_5$
6 " to 7 "	$Y = 2353.3 + .4780y_6$
7 " to 8 "	$Y = 2262.8 + .5039y_7$
8 " to 10 "	$Y = 2724.7 + .4315y_8$
10 years and over	$Y = 2813.2 + .4639y_{10}$

These data in table 44 show the average values of the regression equations for each age. The coefficients of the y 's are the largest at the early years of the cow's life. While quite irregular the values tend to decline as the age when the lactation is made increases. Practically considered this manner of decline is very fortunate. The first lactation being the one most accurately measuring the expectation of the cow in subsequent years enables the farmer or dairymen to determine with greatest certainty from this lactation what the cow is most likely to produce. On this basis the herd may be culled of the poor producers after the first lactation in full knowledge that they would always remain relatively low milkers throughout their life time. Theoretically considered the trend of this decline is of equal interest. Most published records on all breeds of cattle have the greatest number of cattle tested when two years old. It is of considerable consequence, therefore to have the records of the two-year-olds a reasonably accurate measure of the cow's milking capacity. It is of further interest to know that these two-year-olds are the most dependable index of this producing ability.

The data in table 45 summarize the information necessary to deal with the second question. As for table 44 the equations are of no value save for the comparison in hand. If any record is to be corrected the equations of table 43 should be used.

This table shows that the highest values of the coefficients of the y 's occurs for the ages 6 years to 7 years. On each side of these ages the values decline toward both the lower and higher years. Practically considered this maximum of the curve shows that the relative potentialities of the cows within the herd is most accurately determined by correcting the records of these animals to the age basis of 6 years to 7 years. This as determined previously in the paper is near the age when the maximum production of this group of Jerseys may be expected.

TABLE 45

Average regression equations of expected milk records at a given age for records at other ages throughout the life of the cow. (Summarized from table 43.)

Age for expected milk yield	Regression equation
2 years to 3 years	$Y = 2367.3 + .3406y_{x-2}$
3 " to 4 "	$Y = 1926.5 + .5791y_{x-3}$
4 " to 5 "	$Y = 2676.0 + .4891y_{x-4}$
5 " to 6 "	$Y = 2234.0 + .6313y_{x-5}$
6 " to 7 "	$Y = 2067.4 + .7033y_{x-6}$
7 " to 8 "	$Y = 2326.5 + .6167y_{x-7}$
8 " to 10 "	$Y = 2151.7 + .6123y_{x-8}$
10 years and over	$Y = 2428.3 + .4946y_{x-9}$

Before turning to other phases of the subject it may be well to discuss the practical and theoretical errors that in some measure vitiate the use of these equations given in table 43. In the practical application the equations will only give values approximately correct. The reason for this is found in an earlier section of the paper where it was shown that milk production is a highly variable character. This variation is sufficient to cause a not inconsiderable difference in the records of certain cows from lactation to lactation. The difference in these records is also expressed in the corrected expected records as calculated from these equations.

The equations are in slight error since they do not take account of the lack of linearity of regression. This difference is such as to make only a very slight error as may be judged by the fact that $\eta^2 - r^2$ is on an average 2.42 times its probable error (page 164).

None of the above errors are very serious drawbacks to the actual use of their equations in practical dairying. The reason for this is simple. The main object for which these equations are of use in a practical sense (the culling the herd of the poor milkers) is little influenced by small errors as the dairyman is not going to select the animals he is going to keep on a difference of one pound. His selections will be as widely separated as possible.

In dealing with milk records from the view-point of inheritance another question should be considered, namely, what makes the regression of the different lactations toward the mean of the general population? Is it an expression of a genotype or is it variation impressed by environment on the genotype in its phenotypic expression? From the general physiological side of milk-production studies the facts all point to the explanation of this regression as principally due to the conditions not

being ideal for the full expression of a cow's potentialities. A few illustrations will make this clear. The work of WING and FOORD (1904) in New York and ECKLES (1912) in Missouri, have shown that meager feeding will reduce the quantity of milk that a cow will produce in a given lactation. WOLL (1902) has reported Danish records to prove that indoor housing decreases milk yield. DOLGICH has shown that working the cows in excess decreases the milk flow. If it is admitted, as it seems it must be that these known variations influence the potential capacities of cows for milk production, it is equally likely that the whole environmental complex in which the cow lives also influences the yield. The studies of PEARL and MINER (1919) indicate these environmental influences amount on an average to as much as one-third of the observed coefficients of variation of the milk of a cow. These environmental influences undoubtedly vary from lactation to lactation and from cow to cow. Such conditions make for regression, for it is only the fortunate coincidence that favors the full expression of the cow's milking capacities. If these considerations are granted as the true state of affairs, inheritance studies of milk production certainly should disregard this regression, as the inclusion of it (as is done by the use of these regression equations) would simply begot the true mechanism if not hide it altogether. Under these circumstances the student of inheritance should use the equation of the mean curve and not the regression equation to count the records of the individual cows, if possible choosing the best record of the series of records as in all probability this represents the fullest expression of the cow's genetic constitution.

THE CORRELATION OF THE MILK PRODUCTION OF A GIVEN AGE WITH THE TOTAL PRODUCTION OF TWO LACTATIONS

This question and the further extension of it to cover the case of the cow's whole lactation history are of especial interest to the practical dairyman. He is interested primarily in the total yield he may expect from a cow rather than the yield of individual lactation.

Two sets of correlations have been determined to answer this problem; one set on the data as above given where the correlation is between the lactation record of the given year and the lactation records of the given year plus the record of some other year; the second a special set of correlations for the correlation of the summed production for the first five lactations with their component parts.

The first set of correlations may fortunately be calculated without

the formation of the correlation tables from the constants already obtained for tables 11, 20 and 25. The mean and standard deviation of the sum of two components are known when $x = y + z$:

$$\overline{x} = \overline{y} + \overline{z}$$

$$\sigma_x = \sqrt{\sigma_y^2 + \sigma_z^2 + 2r_{yz}\sigma_y\sigma_z}$$

The product moments are (HARRIS 1917):

$$\Sigma(xy) = \Sigma(y^2) + \Sigma(yz)$$

$$\Sigma(xz) = \Sigma(z^2) + \Sigma(yz)$$

The calculation is then quite simple although tedious. The correlations found by this method are given in table 46.

The correlations are all very large when compared with correlations on most physiological material. They range from 0.7324 ± 0.0286 to $+0.9438 \pm 0.0116$. Such high values make it appear that the milk production of one lactation is the determining factor for the total milk production of two lactations when the given lactation is one of the components.

This dependence is well worth testing further. For this purpose a series of data including records of five lactations was chosen. Each of the cows whose records were selected began milking in the second year. The five lactations were consecutive and approximately a year apart depending on how long the previous lactations were continued. The lactation records for the five successive lactations were correlated with the total milk production for the five lactations. The correlation coefficients and other constants for these are given in table 47. These constants are calculated from tables 48 to 53.

The high correlation between the record of one lactation and the total production (first five lactation periods) is manifest. The correlations

TABLE 47

Age when lactation commenced	Mean milk production	Standard deviation for the milk production	Coefficients of variation for the milk production
2 years to 3 years	4159.1 $\pm$ 57.8	803.2 $\pm$ 40.8	19.31 $\pm$ 1.00
3 years to 4 years	4840.9 $\pm$ 86.1	1197.9 $\pm$ 60.9	24.74 $\pm$ 1.35
4 years to 5 years	5380.7 $\pm$ 78.8	1096.2 $\pm$ 55.7	20.37 $\pm$ 1.06
5 years to 6 years	5568.2 $\pm$ 87.1	1211.2 $\pm$ 61.6	21.75 $\pm$ 1.17
6 years to 7 years	5681.8 $\pm$ 91.4	1270.7 $\pm$ 64.6	22.36 $\pm$ 1.17
Total of five lactations	25613.6 $\pm$ 335.9	4672.0 $\pm$ 237.5	18.24 $\pm$.94

TABLE 46

Correlation between the lactation record of a given year and the lactation record of the given year plus the record of some other years.

Character*	Correlation	Character	Correlation
2 and (2 + 3)	+0.8443 ± .0143	3 and (2 + 3)	+0.9190 ± .0077
2 and (2 + 4)	.8133 ± .0173	4 and (2 + 4)	.9300 ± .0069
2 and (2 + 5)	.8030 ± .0202	5 and (2 + 5)	.9341 ± .0073
2 and (2 + 6)	.7961 ± .0224	6 and (2 + 6)	.9433 ± .0038
2 and (2 + 7)	.8404 ± .0208	7 and (2 + 7)	.9296 ± .0096
2 and (2 + 8 and 9)	.7808 ± .0252	8 and 9 and (2 + 8 and 9)	.9289 ± .0088
2 and (2 + 10 and over)	.8026 ± .0379	10 and over and (2 + 10 and over)	.9438 ± .0116
3 and (3 + 4)	.8954 ± .0109	4 and (3 + 4)	.9048 ± .0099
3 and (3 + 5)	.8489 ± .0157	5 and (3 + 5)	.9071 ± .0100
3 and (3 + 6)	.8463 ± .0179	6 and (3 + 6)	.9005 ± .0118
3 and (3 + 7)	.8613 ± .0176	7 and (3 + 7)	.8924 ± .0139
3 and (3 + 8 and 9)	.8503 ± .0182	8 and 9 and (3 + 7)	.9009 ± .0123
3 and (3 + 10 and over)	.9337 ± .0198	10 and over and (3 + 10 and over)	.8686 ± .0379
4 and (4 + 5)	.8643 ± .0143	5 and (4 + 5)	.8976 ± .0109
4 and (4 + 6)	.8559 ± .0165	6 and (4 + 6)	.7324 ± .0286
4 and (4 + 7)	.8816 ± .0141	7 and (4 + 7)	.9233 ± .0094
4 and (4 + 8 and 9)	.7992 ± .0220	8 and 9 and (4 + 8 and 9)	.8788 ± .0138
4 and (4 + 10 and over)	.7845 ± .0371	10 and over and (4 + 10 and over)	.7739 ± .0386
5 and (5 + 6)	.9239 ± .0089	6 and (5 + 6)	.9363 ± .0076
5 and (5 + 7)	.8715 ± .0143	7 and (5 + 7)	.8979 ± .0115
5 and (5 + 8 and 9)	.8661 ± .0149	8 and 9 and (5 + 8 and 9)	.8886 ± .0117
5 and (5 + 10 and over)	.8740 ± .0215	10 and over and (5 + 10 and over)	.8793 ± .0207
6 and (6 + 7)	.9077 ± .0106	7 and (6 + 7)	.9007 ± .0104
6 and (6 + 8 and 9)	.8534 ± .0151	8 and 9 and (6 + 8 and 9)	.8669 ± .0138
6 and (6 + 10 and over)	.8690 ± .0206	10 and over and (6 + 10 and over)	.8377 ± .0251
7 and (7 + 8 and 9)	.8681 ± .0135	8 and 9 and (7 + 8 and 9)	.9053 ± .0099
7 and (7 + 10 and over)	.8006 ± .0281	10 and over and (7 + 10 and over)	.8139 ± .0265
8 and 9 and (8 and 9 + 10 and over)	.8703 ± .0147	10 and over and (8 and 9 + 10 and over)	.8682 ± .0149

* Under character is recorded first the age at which the first lactation record was made. The and signifies this lactation record was correlated with the sum of the two lactation records made at the ages indicated in the parenthesis. The age at which the first lactation was made is recorded as the term in parenthesis. The plus sign signifies that this record was added to the record made at the age indicated as the second term of the parenthesis.

TABLE 47 (continued)

Age when lactation commenced	Correlation of individual lactation records and five-lactations total production	Correlation ratios of individual lactation records and five-lactations total production	Regression equations. Eight-months lactation and five-lactation total production
2 years to 3 years	$+0.7416 \pm .0323$	$+0.7517 \pm .0312$	$Y_T = 7671.9 + 4.3139y_2$
3 years to 4 years	$.8418 \pm .0209$	$.8856 \pm .0155$	$Y_T = 9719.5 + 3.2833y_3$
4 years to 5 years	$.8613 \pm .0186$	$.8796 \pm .0162$	$Y_T = 5861.4 + 3.6710y_4$
5 years to 6 years	$.8250 \pm .0230$	$.8561 \pm .0192$	$Y_T = 7893.5 + 3.1824y_5$
6 years to 7 years	$.8205 \pm .0234$	$.8468 \pm .0203$	$Y_T = 8471.9 + 3.0170y_6$
Total of five lactations			

are so high as to lead to the conclusion that the record of one lactation in a cow's life serves to determine the records for the total production in the cow's life.

The bearing of this conclusion on the mechanism behind the formation of milk is clearly of much significance. The mammary glands may be considered to function because of the interaction of the two variables environment and heredity. Environment may be considered to influence this functioning in two stages; the period during the growth of the cow up to the time she calves and the time after calving when milk is actually secreted. Since this study deals with the records of the same individual cow for successive lactations the inherited complex for her potential milk-producing capacity may be said to be given and constant throughout this work.

The highest possible milk production, or what might be considered the full expression of the hereditary complex, for a given cow may be considered as brought about by ideal environmental conditions for that cow. Taking this definition of the ideal environmental conditions it follows that environmental changes from this condition however slight they may be, are detrimental to the full expression of the mammary gland's possibilities. In a herd like the one with which these data deal no extremely detrimental conditions for any individual cow will exist. The reason for this is clear when it is remembered that the herd has always been kept in the same location under the direction of the same manager thoroughly versed in the feeding and handling of cattle.

Some measure of the influence of environment on the correlation coefficients in the third division into which the mammary functions were divided may be obtained by comparing such coefficients for one lactation period and those covering several lactations. Thus if we correlate

TABLE 48
Correlation surface for amount of eight-months milk production for the first five lactations in the cow's life and the amount of eight-months milk production at two years old for pure-bred Jersey cows.

		Total production for first five lactations (eight-months periods)																						Total
2 years old		15000-16000	16000-17000	17000-18000	18000-19000	19000-20000	20000-21000	21000-22000	22000-23000	23000-24000	24000-25000	25000-26000	26000-27000	27000-28000	28000-29000	29000-30000	30000-31000	31000-32000	32000-33000	33000-34000	34000-35000	35000-36000	36000-37000	
		1	1	2	1	1	1	3	2	2	4	1	1	1	1	1	1	1	1	1	1	1	1	1
2000-2500																								1
2500-3000																								3
3000-3500																								18
3500-4000																								15
4000-4500																								20
4500-5000																								21
5000-5500																								5
5500-6000																								3
6000-6500																								2
Total		1	1	3	2	5	4	4	6	10	5	6	4	8	5	9	3	5	2	2	1	1	1	88

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TABLE 49
*Correlation surface for amount of eight-months milk production for the first five lactations in the cow's life
 and the amount of eight-months milk production at three years old for pure-bred Jersey cows.*

Total production for first five lactations (eight-months periods)		Total	
2500-3000	1	1	2
3000-3500	1	1	10
3500-4000	1	1	12
4000-4500	2	2	14
4500-5000	2	2	15
5000-5500	1	1	12
5500-6000	1	1	5
6000-6500	1	1	9
6500-7000	1	1	4
7000-7500	1	1	3
7500-8000	1	1	2
Total	1	1	88

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TABLE 50

Correlation surface for amount of eight-months milk production for the first five lactations in the cow's life and the amount of eight-months milk production at four years old for pure-bred Jersey cows.

Total production for first five lactations (eight-month periods)		Total	
4 years old			
3000-3500	1	1	3
3500-4000		1	4
4000-4500		1	15
4500-5000	2	4	10
5000-5500		3	21
5500-6000		1	11
6000-6500		2	6
6500-7000		3	11
7000-7500		1	5
7500-8000		1	1
8000-8500		1	1
Total	1	1	88

TABLE 51
Correlation surface for amount of eight-months milk production for the first five lactations in the cow's life
and the amount of eight-months milk production at five years old for pure-bred Jersey cows.

Total production for first five lactations (eight-months period)		Total	
3000-3500	I	I	4
3500-4000	I	I	7
4000-4500	I	I	5
4500-5000	I	I	13
5000-5500	I	I	15
5500-6000	I	I	11
6000-6500	I	I	13
6500-7000	I	I	7
7000-7500	I	I	9
7500-8000	I	I	2
8000-8500	I	I	2
Total	I	I	88

TABLE 52
Correlation surface for amount of eight-months milk production for the first five lactations in the cow's life
and the amount of eight-months milk production at six years old for pure-bred Jersey cows.

6 years old	Total production for first five lactations (eight-months period)																						Total
	15000-16000	16000-17000	17000-18000	18000-19000	19000-20000	20000-21000	21000-22000	22000-23000	23000-24000	24000-25000	25000-26000	26000-27000	27000-28000	28000-29000	29000-30000	30000-31000	31000-32000	32000-33000	33000-34000	34000-35000	35000-36000	36000-37000	
3000-3500	1	2																					3
3500-4000		1	1						1														8
4000-4500					3	1	1																3
4500-5000				1	1	2	2	1	3	2			1										13
5000-5500						1	1	3		1	2	1	1	2		1							14
5500-6000								1	5	1	1	1	1		2								12
6000-6500											2	2	3	1	3	1	1						13
6500-7000									1	1	1			1	1	1	1	1				1	7
7000-7500														1	2	1	1			1			7
7500-8000													1	1	1		2	1		1			6
8000-8500																			1				1
8500-9000																							0
9000-10000																			1				1
Total	1	1	3	2	5	4	4	6	10	5	6	4	8	5	9	3	5	2	2	1	1	1	88

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TABLE 53

Lactations correlated	Correlation coefficients	Average correlation coefficients	Correlation coefficients for given lactation and total of five lactations minus the given lactation	
			1st and 2 + 3 + 4 + 5	2nd and 1 + 3 + 4 + 5
1 and 2 1 and 3	+0.5991 ± .0461 .6007 ± .0459	1st lactation and 2nd, 3rd, 4th, 5th	0.5676 ± .0486	0.6475 ± .0418
1 and 4 1 and 5	.5671 ± .0488 .5033 ± .0537	2nd lactation and 1st, 3rd, 4th, 5th	.6284 ± .0430	.7352 ± .0331
2 and 3 2 and 4	.7785 ± .0284 .5851 ± .0473	3rd lactation and 1st, 2nd, 4th, 5th	.6445 ± .0416	.7768 ± .0285
2 and 5 3 and 4	.5509 ± .0501 .6213 ± .0442	4th lactation and 1st, 2nd, 3rd, 5th	.6098 ± .0451	.7075 ± .0359
3 and 5 4 and 5	.5775 ± .0480 .6657 ± .0400	5th lactation and 1st, 2nd, 3rd, 4th	.5744 ± .0480	.6924 ± .0375

the milk production of two-year-olds with the total production of the next four lactations and compare these correlations with those for the two-year-olds correlated separately with each of the other lactations we will have a measure of the action of environment in affecting milk production of the cow as against the relatively steady environment over a longer period. The data of table 47 furnish the constants for the necessary calculations of the long-time lactation periods. The calculations for the short time involve the calculation of twelve correlations. These data are shown in table 53.

The correlation coefficients of the given lactation with the total of the first five minus the given lactation are in general about 0.1 higher than the mean correlation coefficient as found by averaging the correlations of the separate lactation with the given one. The consistency of the difference rather than the amount leads to the conclusion that this difference while slight is biologically significant. Such being the case the environment may be roughly said to effect a lowering of the correlation coefficient which on an average amounts to from one-seventh to one-eighth the correlation coefficient of the average environment under which these cows live.

It is impossible to get any accurate means of measuring the relative influence of the environmental conditions of the younger years. From the history of the herd, the standard conditions for all the young stock and the similarity of the feeding, etc., it seems doubtful if such environmental differences as do exist in the calf period are of as great consequence to the subsequent milk production as are such differences in the actual lactation cycle.

These results are in conformity with those of GALTON (1895) for widely different material on the differential action of "nurture and nature" in favoring the much greater influence of nature over nurture in the somatic expression of a character. The intensity with which a given cow maintains a given plane of milk production throughout life indicates a fair degree of precision in the mechanical operation of the inherited factors which appear in large measure to govern the amount of milk produced.

The comparison of the accuracy with which this control mechanism works with that of similar secretions in other species has a good deal of direct bearing on the subject. The only data available are for the egg of the domestic fowl. This egg, like milk, is composed of a number of substances going to make up the whole. It differs from milk in being recorded in number of eggs produced and not in pounds. The eggs are

further produced in clutches, each clutch extending over a short time in the year. These clutches are recurrent and might be considered to parallel the successive lactation cycles. Egg production differs from milk production in a further vital particular. The stimulation of the secretion in an egg depends on the passage down the oviduct of an egg yolk or other foreign body. The amount of egg albumen, etc., secreted depends on the number of egg yolks released from the ovary and not on any innate property of albumen secretion as does the secretion of the parts to form milk. Remembering these differences it is still of a good deal of interest to compare the correlation which may exist between successive periods of egg production and the total number produced in a given period and the correlations for successive periods of milk production and the production for the first five lactations as derived in table 47.

Correlations obtained by HARRIS and BLAKESLEE (1918) for the successive monthly egg productions of White Leghorn pullets with their first annual production, range from $+0.372 \pm .030$ to $0.695 \pm .018$ with an average of 0.556. The correlations of the successive yearly productions and the total of the first five lactations shown in table 47 range from $+0.7416 \pm .0323$ to $0.8613 \pm .0186$. The difference between these correlations is striking. In the proper calculation of this difference it must be remembered that the correlations for egg production is for the twelve-month period where the production for the individual-month correlation contributes only one-twelfth to the total annual egg production while that for the milk production contributes about one-fifth to the total production of the five lactations. The correlation would therefore be expected slightly larger for the milk production. This increased size of the milk correlations over the egg correlations is very small relative to the whole difference, as the confirmatory results of table 38 (page 159) show. The correlations for the milk production of the successive lactations, therefore, represent in concrete terms that a greater reliance may be placed in the milk records of one lactation as measuring the cow's capacity than can be placed in the monthly egg records as measuring the hen's capacity for annual production. Such being the case, if we generalize this conclusion in its ultimate terms, the causative mechanism behind milk production works with greater fineness and precision than does the mechanism for egg production. * Since this mechanism seems in its broadest sense to be of hereditary origin in the two cases, it follows that in the cow this hereditary complex is less influenced in its action by environment than is the action of the material stuffs of the fowl for egg production.

PRACTICAL ASPECT OF THE CORRELATIONS FOR MILK PRODUCTION OF ONE
LACTATION RECORD WITH ANOTHER LACTATION RECORD

The constants deduced in table 47 have a good deal of practical value to the dairyman and to the student of farm management. To illustrate, suppose a herd which has 1000 two-year-olds which have just completed their first lactation, is chosen at random. The manager of this herd will wish to know what animals to save for future lactations and the student of farm management desires to know what may be expected for the total production of the animals to enable him to determine the plane to which the herd should be culled for the greatest profit to the owner. The equations derived from table 4 on page 120 allow the determination of the distribution of the cows as to their eight-months yield of milk if the one thousand two-year-old cows are drawn at random from a population similar to that of the Jerseys of these data. The equation for this curve may be repeated for sake of convenience of those who desire to check the results.

$$y = 71.7407 \left(1 - \frac{x^2}{16,291,354} \right)^{10.648}$$

From the actual eight-months milk production of table 47 the equation to determine the sum of the expected lactation yields of the first five lactations is given by

$$Y_T = 7671.9 + 4.3139y_2$$

where Y_T is the total expected yield of the five lactations and y_2 is the actual yield of the two-year-old cow.

Perhaps one of the easiest ways to table the necessary information for its readiest uses is to have the summation of both the number of cows and their expected total production. This summation to be from both ends of the range of milk produced. The data for such a comparison are given in table 54.

If all the cows in this herd are kept to their sixth lactation the average production for each cow for the five lactations would be 25,070 pounds of milk. Noting the summed number of cows in column four if the 217 poorest producers are culled from the herd the cows left in the herd will produce on an average 26,408 pounds or on an average 5,389 pounds more milk than the culled cows. Again if the dairyman decides to cull out 543 of the thousand animals the production of the 457 remaining will be 28,192 pounds or 5,747 pounds more than the culled cows on the average for the summed productions for the five lactations.

TABLE 54
Actual milk production of one thousand two-year-old cows and the expected five-lactation yield for any division of that herd or per cow for any division.†

Actual milk production	Cows producing given amount	Expected five-lactation yield	Summed number of cows lowest to highest production	Expected summed five-lactation yield for cows in herd lowest to highest production	Summed number of cows highest to lowest production	Expected summed five-lactation yield for cows in herd highest to lowest	Expected production per cow lowest to highest	Expected production per cow highest to lowest
1375 to 1625	1	14143	1	14,143	1000	25,060,737	14,143	25,070*
1625 to 1875	2	15221	3	41,355	999	25,050,000	14,822	25,076
1875 to 2125	5	16300	8	128,721	997	25,028,382	15,794	25,068
2125 to 2375	12	17378	20	335,173	992	24,941,016	16,734	25,146
2375 to 2625	23	18457	43	753,397	980	24,734,564	17,649	25,240
2625 to 2875	38	19535	81	1,408,071	957	24,316,340	18,538	25,401
2875 to 3125	57	20614	138	2,681,904	919	23,571,666	19,400	25,644
3125 to 3375	78	21692	217	4,381,681	862	22,387,833	20,229	25,979
3375 to 3625	98	22770	314	6,604,300	783	20,688,056	21,019	26,408
3625 to 3875	112	23849	426	9,265,303	686	18,465,437	21,761	26,926
3875 to 4125	118	24927	543	12,195,510	574	15,803,438	22,445	27,524
4125 to 4375	114	26006	657	15,161,471	457	12,874,227	23,093	28,192
4375 to 4625	102	27084	759	17,922,976	343	9,908,266	23,604	28,920
4625 to 4875	84	28163	843	20,281,662	241	7,146,761	24,056	29,698
4875 to 5125	63	29241	906	22,121,166	157	4,788,135	24,416	30,517
5125 to 5375	43	30320	949	23,422,180	94	2,948,571	24,683	31,371
5375 to 5625	26	31398	975	24,247,950	51	1,647,557	24,864	32,254
5625 to 5875	14	32477	989	24,711,391	25	821,787	24,974	33,163
5875 to 6125	7	33555	996	24,936,881	11	358,346	25,032	34,096
6125 to 6375	3	34634	999	25,029,352	4	132,856	25,057	35,054
6375 to 6625	1	35712	1000	25,069,737	1	40,385	25,070*	36,058

* It will be noted that this figure does not correspond exactly with that of table 49. The reason for this is that table 54 is based on the whole number of two-year-olds with a mean production of 4032.89 (see table 4) while table 49 is based on 88 of these two-year-olds with a mean production of 4159.1. The corresponding total five-lactation expected records are 25,069.4 and 25,613.6.

† All calculations in this table were carried out to three decimal places, otherwise the results were found to be only approximately correct. The data however are recorded to the nearest whole number.

As the records dealt with in this paper are all for the first eight months of lactation it follows that most cows will extend beyond this limit and produce more milk. In using this table to determine what cows may be kept profitably, this fact should be kept in mind. If the dairyman has determined the complete cost of producing his milk a knowledge of the price he receives for it will allow him to determine at once from column three what cows he should keep in the herd.

THE RELATIVE CONTRIBUTION OF ONE YEAR'S MILK PRODUCTION TO THE TOTAL PRODUCTION OVER THE FIRST FIVE LACTATIONS

A little thought on the subject will convince one that even though there is a substantial correlation between the lactation records of the individual lactation and the total production over several lactations it by no means follows that the individual cows necessarily contribute the same relative amounts of milk to the individual lactations and to the total of the five-lactation groups. The problem may be quantitatively analyzed by the determination of the correlation between the total of the five lactations of the individual cows and the deviation of the pounds of milk produced by them in the single lactations from their probable value. The calculations of this correlation may be most easily made by the method of HARRIS (1909, 1918). The formula for the correlations is given by the equation

$$r_{xz} = \frac{r_{xy} - v_x/v_y}{\sqrt{1 - r_{xy}^2 + (r_{xy} - v_x/v_y)^2}}$$

where the y 's represent the individual lactation records and the x 's the totals for the five lactations. The five constants for these are tabled in table 55.

TABLE 55

Correlation between the total milk production of the five-lactation records and the deviation of the individual records from their probable values.

Age when correlation records were made	Correlation	Correlation $\div$ probable error
2 years and total	-0.2897 $\pm$.0659	4.39
3 years and total	+ .1905 $\pm$.0693	2.75
4 years and total	- .0667 $\pm$.0716	.93
5 years and total	- .0240 $\pm$.0719	.33
6 years and total	+ .0087 $\pm$.0719	.12

These correlations of table 55 change their sign three times, two of them being plus and three minus; their magnitudes range between $+0.1905 \pm .0693$ and $-0.2897 \pm .0659$. As adjudged by their probable errors only two of the correlations can be considered significant. Taken at their face value these coefficients would seem to indicate that the highest producers at two years contribute relatively less to the total production over several lactations than was their expected share and that these same high producers contribute relatively more in their second lactation to this total than was to be expected of them. In the other lactations each individual of a given grade contributed approximately its proportionate expected share to the total production. These relations of the deviations from their probable yields of the first and second lactations are hard to account for in terms of the known physiology of the mammary gland. The parallel already indicated of the logarithmic nature of growth and milk production with age seems to point to a possible causal relation between growth of the mammary gland and the quantity of its secretion. Should this prove true the problem of this differential relative contribution of the high producers at two years old and at three becomes a problem of the relative duration of growth of the mammary gland in the high, medium and low producers and not a cyclic action of different sets of factors for milk production itself. This means that the factors which govern milk production have their regulating power maintained in the same relative strength throughout the life of the cow with the exclusion of any group of factors acting within this life time for any short period.

SUMMARY

The purpose of this paper is to present quantitative data, along with the biometrical analysis of the same, on the normal variations and correlations of milk production during the cow's life. The necessity of such data and analyses is well recognized now, in physiological and genetic investigations of a quantitative subject. The economic value of milk gives this investigation a significant relation to improvement in yield and in the selection of cows to remain within the herd. The materials used for this study are peculiarly suited to the ends in view. They are a strictly homogeneous group of milk records of normal pure-bred Jersey cows maintained under the conditions of a large farm, managed by a well-trained man thoroughly versed in his business. While the conclusions derived from this study are stated in general form, it is not presumed that the conclusions have more generality than the data on which

they are based. It is altogether probable however that the conclusions drawn do apply to most of the other dairy breeds of cattle.

The conclusions of most moment may be summarily stated as in the following paragraphs. Unless otherwise stated, the conclusions apply to the pure-bred Jersey herd the data for which are given in this paper.

The frequency polygons for milk production are only slightly unsymmetrical. The range of variation is large, one cow producing less than 1500 pounds in her eight-months lactation period, another cow producing more than 10,000 pounds for her eight-months lactation period. The height of the modal ordinate for the frequency polygons is greatest in the earlier and later years of the cow's life. Only one mode is evident in any of the histograms. From this it appears likely that these curves are unimodal curves. Too much stress should not be laid on this however since certain of the distributions appear quite flat-topped.

The analytical constants show that the eight-months-milk-production curves are of the PEARSON types I, II and IV. The skewness of these curves is all in the positive direction and rather small in amount. This small size of the skewness is of considerable interest to those investigators who deal with advanced-registry data since it would indicate that these data can with but small error be fitted by normal curves to determine certain of their needed constants.

The correlation coefficient between age at the commencement of test and milk production in comparison with the correlation ratio for the same variables shows that the regression of milk production on age is far from linear. This regression curve when analyzed turns out to be a logarithmic function. Attention is called to the fact that this same kind of logarithmic function also describes the relation of growth to age. This paralleling of the two functions may indicate that a causal relation exists between them such that increased milk production chiefly depends on increase in size of the mammary gland caused by growth.

The standard deviations of milk production progressing from the heifer to the aged cow are described by a cubic parabola. This regular change of the standard deviation of milk production groups shows that the curve formed by them falls into the heteroscedastic, nomic class as defined by PEARSON.

Correlation tables for milk production of the different age groups are presented.

The means derived from these tables remain, within the limits of random sampling, the same for any given age at test irrespective of the age at which the same cows were subsequently tested.

The standard deviations for the various age groups also remain at the same value within the limits of sampling irrespective of the age at which a subsequent test may be made.

As would be expected from their derivation and as actually checks out to be the case the coefficients of variation do not change within the age group for the cows which are retained in the herd for longer or shorter periods. These facts show that the cows which were kept in the herd as two-year-olds were all equally likely to be kept in the herd until they became aged cows as far as the milk production of these two-year-olds was concerned. Generalizing this statement cows which were kept in the herd at any given age were all equally likely to be kept in the herd at any other age as far as the milk record of any of those cows at the given age was concerned. This point removes any possible objection to the accuracy of the correlations for the milk productions at the various ages at least so far as concerns the cows remaining in the herd on the basis of their milk production being high.

The correlation coefficients for the milk production of one lactation with that of another lactation at another age range from $+0.2144 \pm .0919$ to $+0.7306 \pm .0284$. The signs of these correlation coefficients are all plus. The numerical values are large enough to be significant. Each set of correlation coefficients for the milk production at a given age with the milk productions at other ages in the cow's life deviate only slightly from a line which is approximately linear, taking a nearly constant value for each age. The mean value for these correlation coefficients of milk production of one lactation at any age with the milk production of another lactation at another age is $+0.5352$.

This mean value of the coefficient of correlation seems to be generally applicable to milk production for on somewhat similar data GAVIN finds results which are comparable to these.

These correlation coefficients are high for this kind of data as may be seen from the fact that correlation coefficients of similar nature for egg production range lower and have a lower mean $+0.4458$ than do these correlation coefficients for milk production.

The magnitude of these correlation coefficients in comparison to those on egg production put in terms of the physiology of the two sets of glands, mammary and ovary, means that the mammary gland works with more mechanical precision than does the ovary.

The linearity of all the regressions is established by the calculation of the correlation ratios and their comparison with the corresponding correlation coefficients.

From the means, standard deviations and correlation coefficients are formed the straight-line equations for the prediction of the amount of milk production of any given lactation at any age from these known variables at another age. These equations are given in table 43. Examination of these equations seems to show that most stress is laid on the milk production at two years old by the coefficients, indicating that the milk production at this age represents more accurately than any other the future possibilities of the cow. Similar analysis reveals the fact that correction of the milk records to about the age of maximum productivity of the cow gives the nearest approach to the correct milk production for that cow.

The correlation coefficients of milk production at a given age with the total milk production of two lactations are presented in table 46. These correlations prove to be very high, ranging from $+0.7324 \pm 0.0286$ to $+0.9438 \pm 0.0116$. Such high values make it appear that the milk production of one lactation is the determining factor for total milk production of two lactations when the given lactation is one of the components.

This conclusion is confirmed by a study of a smaller series of data where the first five lactation records are known, the first commencing at two years old and the other four commencing approximately a year apart. These correlation coefficients for milk production of a lactation at a given age with the total milk production over the five lactations range from $+0.7416 \pm 0.0323$ to $+0.8613 \pm 0.0186$. Each regression is linear as is shown by a comparison of the correlation coefficients and correlation ratios.

A. means of measuring the effect of environmental changes on the coefficient of correlation of the milk production of a lactation at a given age with the milk production of a lactation at another age is given. This measure depends on the difference in the correlation coefficients of the milk production of a given age correlated with the milk productions of several successive lactations and the correlation of the milk production of the given lactation and the total milk production of the other lactations. In the first case the environmental changes of the different lactations as they affect milk production make themselves felt in the correlation coefficients; in the second case the use of the total milk production gives a mean of these environmental effects. The outcome of this study shows the environmental changes reduce the correlation one-ninth to one-eighth of what it would be under the conditions of an average environment. This result makes it clear that for these Jerseys heredity

plays a greater part in their milk production from lactation to lactation than environment.

The correlation coefficients for the milk production of one lactation with that over several lactations, furnishing as it does the basis for the regression formula by which milk production over long periods may be predicted from shorter ones, is of the greatest practical importance in culling the herd of the mediocre animals. The use of these coefficients is illustrated in table 54.

The relative contribution of the different lactations to the total milk production over a long period shows that in the first lactation the high-producing cows do not produce quite as much relatively as they would be expected to produce. In the second lactation these same high-producing cows produce relatively more than they would be expected to produce. Thereafter the contribution of any class is at random. The significance of the relative contribution of the first two years is only slight. In view of these facts the conclusion may be drawn, that the factors which govern milk production have their regulatory power maintained in the same relative strength throughout the life of the cow to the exclusion of any group of factors acting within this life for any short period.

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