SIZE IN MENTAL DEFICIENCY.*

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THE present study has been undertaken with two objects in view: first, to ascertain how size varies among fully grown defectives at different mental levels, and secondly, to provide a satisfactory background for further studies of particular parts, and especially of the nervous system.

In the past, various workers, searching for a neural basis of amentia, have measured, say, the thickness of the cerebral cortex or the brain volume, and, finding a diminution, have at once assumed that this must be causally related to the deficiency in intelligence. But clinical experience tells us that defectives are nearly always undersized in their entire somatic development, so that the diminution of any one part may be merely a local expression of a perfectly general shrinkage. This, of course, will not affect the data as such, but it may modify our deductions from those data. Clearly, if we are to be certain that the falling off in size of one particular part is not merely part of a general change we must first of all know what is happening elsewhere. To put it briefly, it is best, before studying particular parts, to study the whole. The present work is a general survey of size in patients who exhibit different degrees of intelligence ranging from complete idiocy up to the border line which separates the feeble-minded from the subnormal.

THE MATERIAL.

The material consists of 269 male defectives, their ages being in all cases above sixteen years. Senile aments and those with paralysis or definite malformations of the limbs and trunk were excluded. Hydrocephalics were not included, since in this condition head measurements must be regarded just as much an index of water volume as of brain size. On the other hand, microcephalics, whose small brains are but seldom the seat of ventricular dilatation, have been included. Whether or not the microcephalic belongs to a definite clinical type is a question with which we are not here concerned.

PART I.

LINEAR HEAD MEASUREMENTS IN MENTAL DEFICIENCY.

Two hundred and sixty-nine defectives of varying mental age were measured for head length, head breadth and ear to ear measurements.

Head length was measured by calipers from the most prominent point of the glabella to the occipital point. The measurement was made with Flowers type calipers and was read to the nearest quarter centimetre. Head length was measured with calipers to the nearest quarter centimetre at the

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widest part of the head above the plane of the ear-holes. 'Ear to ear' was measured with a tape-measure from the centre of each external auditory meatus, vertically over the top of the head. It was read to the nearest eighth-inch.

Head height proved in practice to be difficult to measure with reasonable accuracy. It was considered equally accurate to use a combination of head breadth and 'ear to ear,' assuming the coronal section of the head be half an ellipse. This gives the equation:—

$$\text{Head height} = \frac{2}{\pi} (\text{ear to ear}) - \frac{\text{head breadth}}{2}$$

Using this equation, head height was calculated to the nearest quarter centimetre.

Tables I-IV (see appendix) give the distributions of these measurements at different mental ages.

We may now proceed to extract the information in the following way. So as to be able to compare one measurement with another, say head length with head breadth, we first convert the data to bring all the general means to 100. This gives us the data in a form where the relative changes due to intelligence are left unaltered, but at the same time are made comparable between different measurements. We then find the average for each measurement at each mental age.

Now these means, say mean head length at mental age 4, are exact for that particular group of patients of mental age 4 which we have measured. But this group is only a sample of what we would have obtained had we measured all possible people of mental age 4. Another sample would give a slightly different result. It is not possible to say what the true mean really is, but it is possible to say within what limits it must lie. In fact, in work of this type, it is often much more important to know the limits within which it must lie than to know merely what our particular sample has as mean. Thus in head length, mental age 1 group, we can be sure that the true mean lies between 95.3 and 98.8, and this is much more informative than merely to know that our mean happens to be 97.051.

We have therefore expressed the data in such a way as to present the limits within which the true mean must lie.

The 'ribbon' representation in graph I shows the change in size at different mental ages for the four measurements. It shows a strip within which we can be sure that the true line of means will run. It is much preferable to the usual method of plotting a more or less irregular line, for the latter gives no clue as to what deviations from the line are statistically permissible. It will be seen in the graph, for instance, that the right-hand ends are trumpet-shaped. This is due to the fact that as the number of cases becomes fewer at this end, the mean is not specified with so much precision, and this is allowed for exactly by the opening out of the limiting lines.

GRAPH I.

GRAPH I, continued.

From the graph we may note, firstly, that mental age certainly has some effect on head size since we cannot draw a horizontal line to run through the ribbon. It is clear that the left-hand end must turn down. As intelligence falls off (to the left), we see that head size, as shown generally in all the measurements, falls off also, but the falling off in size is most marked at the lower (idiot) end. It is important to note that the general form of the curves is similar.

In addition, an index of brain volume was calculated by multiplying together head length, head breadth and head height and dividing by 1,000 for convenience. There is no point, in this investigation, in applying one of the formulæ, such as Lee's, for finding the most probable brain volume in cubic centimetres. Since we wish merely to compare group with group, an index will serve the same purpose equally well.

The actual distribution of the brain volume index is given in the appendix (table V). Treating the data as before, we find the graph to be similar to those above but to have a much steeper slope. This, however, is to be expected since here we are dealing with volume (three-dimensional), while previously we were dealing with linear (one-dimensional) quantities. In this case we would expect the volume measurement to have a slope about three times as steep as the others.

To make the brain volume measurements comparable with the others, we must either cube the latter, or take the cube root of the former. Since linear quantities are the easier to handle, we will take the cube root of the brain volume as the linear representative of the data (appendix, table VI).

Treating the new data as before, we find the following (graph II).

GRAPH II.

This, it will be seen, has much the same slope and general form as those of the other head measurements. All five curves seem to be similar, and if plotted on top of one another we obtain the following (graph III).

GRAPH III.

The curves are all obviously similar, except for the small random sampling irregularities which are to be expected.

CONCLUSIONS.

With regard to head measurements generally, we may note that the following points are established by the data:—

- (1) Head size falls off generally with decreasing intelligence.
- (2) The falling off in size is quite general and affects equally head length, head breadth, head height, and ear to ear measurements. Brain volume, when made comparable, also alters in just the same way.
- (3) The falling off is most marked in the 'idiot' region.

Further conclusions must be postponed till after Part II has been considered.

APPENDIX TO PART I.—TABLES OF DATA AND STATISTICAL METHODS USED.

THE MEASUREMENTS.—*The general effect of errors in measurements.*

In work of this type, there are three types of errors which may be present. They are (1) biased, (2) random, (3) correlated. Their effects on the data will be quite different.

(1) Biased errors, i.e. reading all measurements too high or too low, will have no particular effect, since we want simply to compare group with group. Such comparison is unaffected by a systematic change.

(2) Random errors refer to the small discrepancies between repeated measurements which are always found in any series of measurements. Associated with them are the errors introduced by grouping, i.e. by measuring 'to the nearest unit.' Random errors introduce a certain extra 'blurriness' to the data. It must be remembered, however, that in biological data there is always a certain blurriness of the data in any case. The 'scatter' is measured by the variance. Random errors have the effect of increasing the variance. If we are comparing group with group so as to compare the means, then the effect of introducing random errors is precisely the same as if we had dealt with a smaller number in our sample—that is to say, it is equivalent to wasting some of our cases. In the present series, the random and grouping errors waste about one per cent. of the total information—that is to say, our series of 269 cases measured with our technique and grouping is equivalent to a series of 266 cases measured with infinite accuracy. It does not appear to be generally realized that the effect of increasing accuracy can be greatly overestimated in some types of biological work.

In all the measurements, the results have been grouped to a unit equal to about a third or a quarter of the standard deviation. This means that, in grouping units, the variance is about 9 to 16 (squared) units. The variance introduced by grouping is 0.088. The latter is less than one per cent. of 9, so that the wastage of information is less than one per cent., as stated above.

The discussion of errors above refers to errors of measurement of size. In the case of the measurement of intelligence there is also bound to be some error. Most cases were measured with Burt's Revision of the Binet-Simon test, some with Porteus' maze, and some with both. In the latter case the two estimates were averaged. The Porteus' maze seems to give results a little higher than Burt's Revision, but this error cannot affect the systematic differences between the mental-age groups, since the maze tests were scattered more or less at random over all mental ages.

The general effect of small errors in measuring mental age will be to diminish the differences between the measurements obtained at the different age groups. But since the variance in measuring mental age would be fairly constant in different investigations, while on the one hand it has the effect of lowering the differences found between various mental ages, yet these differences will still be comparable with one another, both between different investigations and between different investigators.

Owing to the fact that there are no suitable tests for very low-grade patients, the cases with mental age of less than three years (0, 1 and 2 years) have been grouped.

The measurements have all been done by one person.

It may be objected that in this series there are no normal controls. These have been omitted deliberately for two reasons. Firstly, the series does not need outside controls, since we are comparing the mental age groups with each other, and each group acts as control to the other. The normals, if used at all, would be used not so much as controls, but rather to extend the range of mental age. They have not been included here owing to the practical impossibility of obtaining comparable material. To use nurses, for instance, would be obviously absurd, since they are deliberately selected for size. It was considered safer to leave the range restricted than to attempt to extend it by including material of doubtful validity. To include a group which is not comparable would be to vitiate the whole work.

There is only one other possible source of error. It is that this institution, in admitting patients, may tend to take certain types, such as small and feeble idiots, or large and strong feeble-minded patients. In such a case, our collection of idiots, for instance, would tend in size to be below the general average of idiots. From a knowledge of the administrative side, however, we are satisfied that no such selection of types exists with regard to this institution.

TABLE I.

The distribution of *head length* for various mental ages is:—

		Mental age in years									
		1	3	4	5	6	7	8	9	10	11
Head length in cm.	21.75	...	...	...	1						
	21.50										
	21.25	...	...	...	...	1					
	21.00	...	...	...	...	1	...	..	...	1	
	20.75	...	...	1	1	2					
	20.50	...	...	2	...	1	...	2			
	20.25	...	3	1	2	1	1	...	2	1	1
	20.00	1	1	1	1	1	2	2	...	...	2
	19.75	2	2	2	...	3	1	4	2		
	19.50	5	3	3	1	3	6	5	3	1	
	19.25	1	8	4	3	1	3	...	2	1	
	19.00	3	4	7	6	2	4	4	3	2	
	18.75	3	5	3	6	2	2	3	1	1	1
	18.50	4	8	...	7	1	2	4	4		
	18.25	4	3	4	2	4	1	2	1		
	18.00	4	1	2	1	2	2	...	1	1	
	17.75	1	2	2	2	1					
	17.50	1	2	1	2	...	1				
	17.25	5	2	4	...	2					
	17.00	2	...	1							
	16.75	...	...	...	1	1					
	16.50	...	...	...	1						
	16.25	1	1								
	16.00	...									
	15.75	1									
		38	45	38	37	29	25	26	19	8	4

TABLE II.

The distribution of *head breadth* for various mental ages is:—

[illegible]

TABLE IV.

The distribution of *head height* for various mental ages is:—

[illegible]

TABLE VI.

The distribution of *cube root-brain volume* for various mental ages is:—

		Mental age in years									
		1	3	4	5	6	7	8	9	10	11
Cube root of brain volume	1.80	...	...	0.2	0.4						
	1.78	...	...	0.8	1.6						
	1.76	...	...	1.0	...	2.0	...	...	...	1.0	
	1.74	...	...	1.0	...	1.7	0.5	1.0	...	1.4	
	1.72	1.0	...	0.5	1.0	1.3	1.9	...	...	1.6	0.5
	1.70	0.2	3.0	1.9	0.2	1.0	3.6	0.4	0.2	...	0.5
	1.68	2.8	2.0	5.6	2.8	2.0	1.0	2.6	2.8		
	1.66	1.8	6.0	1.2	2.8	3.6	3.8	5.4	3.8	...	0.6
	1.64	3.4	5.6	4.4	3.4	1.8	4.0	3.8	2.0	0.4	1.4
	1.62	1.6	4.0	3.2	1.6	3.2	4.5	4.0	4.0	2.4	
	1.60	3.8	4.3	4.8	9.8	3.4	2.3	5.1	1.2	0.2	1.0
	1.58	1.4	2.9	2.2	3.0	...	0.4	3.1	0.8		
	1.56	3.2	4.1	3.4	2.7	3.3	0.3	0.6	2.5	1.0	
	1.54	5.6	3.5	4.2	1.4	2.1	0.7	...	1.4	0.7	
	1.52	4.2	3.6	1.6	0.3	1.6	...	...	0.3	0.3	
	1.50	2.4	3.4	1.0	0.8	...	1.0				
	1.48	2.0	0.6	...	2.6	0.7					
	1.46	1.6	2.0	...	1.6	0.3					
	1.44	0.3	...	1.0	0.3	0.3	1.0				
	1.42	1.1	...	...	0.7	0.7					
	1.40	0.6									
	1.38										
	1.36										
	1.34	1.0									

In the table above, the distribution gives what would have been obtained had the root been obtained from the original (ungrouped) data. This gives fractional frequencies, but avoids the systematic error which would be introduced by taking the root from the data after grouping.

We now proceed to reduce the figures to a common basis. This may be done in several ways. For our purpose, probably the best way is to multiply the measurements by some factor so that the general means all come to 100. Thus, the general mean of all head length measurements is 18.873 cm. Consequently if we multiply all the measurements by 100/18.873 the general mean will come to 100. This method makes a more effective use of the data than, say, trying to adjust them to some rather hypothetical normal.

We find the means and variances in each column directly from the tables of distributions and correct them to the standard form by multiplying them

by $100/18.878$ and $(100/18.878)^2$ respectively. Thus for head length, mental age 1 year, we find the standardized mean to be 97.05 and the corresponding variance to be 28.95.

To compare these means, it is probably best to keep all the mental age groups quite separate. One could fit a polynomial and compare these to find the trend, but to apply any test of significance we would have to assume that the residual variance was constant. It appears from the data that this is definitely not so.

If we were going to use some further analysis of the data, then we would have to use a polynomial fitted by least squares, otherwise any tests of significance applied would be inaccurate. In our case, however, we merely need the form of the regression line, and this can be better obtained by a free hand curve. If we were to fit a polynomial, we would have the further disadvantage that a parabola would be too arbitrary, while a cubic or curve of higher degree would show too much tendency to waggle.

The method adopted here is to keep each mental age group quite separate and to compare group with group and measurement with measurement simply by direct comparison, using the standard deviation as a test of significance. This method is not so sensitive as a method using compounded data, but the latter may in our case be too sensitive and might give erroneous conclusions.

To find the limits of the mean we calculate the standard deviation of the mean. Then, using 'Student's' t distribution, we find the distance of the 5% point from the mean. Thus, for mental age 1, $n'=38$, $n=37$, 5% point=2.02. Multiplying the standard deviation of the mean by this, we then add and subtract this from the mean and thus find the (5%) limits within which the true mean is almost certain to lie.

We thus obtain the following limits for the means:—

TABLE VII.

		Head length	Head breadth	Ear to ear
Mental age in years	1	95.3—98.8	96.0—99.3	95.0—98.8
	3	98.1—100.8	97.4—99.8	97.7—100.7
	4	97.9—101.3	99.4—102.5	99.3—102.6
	5	97.7—101.2	98.7—102.1	97.5—101.3
	6	98.7—103.5	98.9—102.9	98.5—102.7
	7	99.6—102.6	99.3—102.3	99.9—102.8
	8	100.6—103.4	99.5—102.2	100.2—103.0
	9	99.6—102.9	99.0—102.4	98.9—102.7
	10	98.4—106.6	97.2—106.7	97.7—108.8
	11	99.0—110.3	98.7—103.9	95.5—108.0

		Head height	Brain volume	$\sqrt[3]{\text{Brain volume}}$
Mental age in years	1	94.4—98.9	87.0—96.4	95.3—98.7
	3	97.6—101.4	93.9—101.0	97.8—100.3
	4	98.9—102.9	97.4—106.6	99.1—102.1
	5	96.6—100.9	93.9—103.9	97.8—101.2
	6	98.2—102.7	97.1—109.1	98.9—102.8
	7	99.8—103.6	97.4—106.6	100.0—103.4
	8	100.1—103.9	101.3—107.7	100.5—102.5
	9	98.5—103.6	98.2—106.6	99.4—102.2
	10	97.3—110.2	94.1—122.1	98.1—106.9
	11	93.6—110.5	93.6—122.0	98.1—107.2

PART II.

BODY WEIGHT, ARM LENGTH AND FOOT LENGTH IN MENTAL DEFECTIVES AT DIFFERENT MENTAL AGES.

Continuing the investigation of size in mental deficiency, we have also measured body weight, arm length and foot length, so as to obtain an idea of what the rest of the body, apart from the head, is doing.

THE METHODS.—In examining the weight record of each patient, it was obvious that each person's weight is rather variable, there being steady upward and downward trends for no particular reason. 'Body weight' was therefore obtained by taking a fair average value of the monthly weight record of each patient. The weights were afterwards grouped to the nearest 7 lbs.

The right arm and right foot were measured in 256 of the total series, the original number being reduced owing to transfers or the unsuitability of the subjects.

The arm was measured from the olecranon to the tip of the styloid process of the ulna. These limits were chosen as they can be identified with certainty and terminate in comparatively sharp points.

The foot was measured from the heel to the tip of the big toe. The instrument used resembled that employed by the shoemaker for measuring feet, but the scale was graduated specially to read to the nearest quarter centimetre. This grouping, however, proved unnecessarily fine and the measurements were subsequently regrouped.

The general discussion of the question of errors was given in Part I and need not be repeated. Exactly the same principles are applicable here.

The full data and statistical methods used are given in the appendix as they are not of general interest,

Analyzing the data as before (Part I), and graphing the results, we find the mean lengths at various mental ages to be, for arm and foot:—

GRAPH 1.

GRAPH 2.

To obtain the graphs, we have taken the average at each mental age, and have plotted, not the actual averages found in our sample, but the limits within which the true mean must lie. In this way one is able to assess the significance of changes much more easily than if one were given merely the line of the actual averages. The 'ribbon' means that the true line must run somewhere inside it.

The 'body weight' results are similar but are much steeper in slope. This is to be expected since weight is a 'cubic' unit, while 'length' is linear. One would expect 'weight' to have about three times the slope of 'length.' To make the results comparable to the arm and foot results, we may take the cube root of body weight and use this as our estimate of size.

Dealing, not with body weight, but with its cube root, we find this quantity to vary with intelligence as shown in the following graph:—

GRAPH 3.

It will be seen that the general form of the three curves is similar. This is confirmed by plotting all three curves on top of each other.

GRAPH 4.

The three curves are entirely similar except for small non-systematic, random sampling irregularities.

From the graph above, we may draw the following preliminary conclusions:—

- (1) All three body measurements change with alterations of mental age. Measurements of patients of lower mental age tend to be smaller than those with higher mental age.
- (2) All three measurements seem to change equally, i.e. the shrinkage appears to be perfectly general and not limited to any particular part of the body.
- (3) The falling off in size is most marked in the 'idiot' region, but there is clearly (graph 4) some falling off in the higher grades.

These conclusions are very similar to those obtained from an examination of head measurements (Part I). We will therefore compare the four head with the three body measurements.

On examining the data (see appendix B), it is found that the data of Parts I and II fall into two distinct groups:—

- (A) Head length, head breadth, 'ear to ear' and cube root brain volume.
- (B) Cube root body weight, arm length and foot length.

In group A, the different measurements all seem to vary with mental age in the same way, and the three measurements in group B are also similar to

one another, but group A shows differences from group B. The latter has generally a steeper slope than group A. This may be shown by combining the four measurements in group A together and the three in group B (see appendix C).

Group A and group B are graphed below:—

GRAPH 5.

GROUP A.

GROUP B.

A shows what happens when all the head measurements are combined together. B shows the combined body measurements.

It will be noticed that this grouping, which has been found simply by an examination of the numerical data, also corresponds to a natural anatomical grouping, since group A consists of all the measurements related to the skull or brain, while group B contains the measurements obtained from the rest of the body.

An examination of the above graph shows that the 'body' group of measurements (B) actually changes more for a given change of mental age than does the 'brain' group of measurements (A). This difference in slope is statistically significant (appendix B).

Before we sum up, we may first state that throughout the paper, before discussing any finding, we have first made sure, by actual testing, that the finding in question is statistically significant. The reader may rest assured that we are not discussing any points which are really insignificant.

To sum up Parts I and II, we see that consideration of the above data establishes the following points:—

- (1) All the measurements made change with mental age, all decreasing with lower intelligence.
- (2) When the measurements are reduced to a comparable basis, the forms of the curves are roughly the same, all showing at first a slight fall, and then a more rapid fall as idiocy is reached.
- (3) The measurements fall into two natural groups which correspond with the division into 'head size' indices and 'body size' indices. These two groups show significant differences between them.
- (4) As intelligence falls, body size falls off more rapidly than head size.

GENERAL CONCLUSIONS.

(1) When we come to consider the general significance of these findings, a point of importance at once emerges. If there is present a general change of size with intelligence, then it follows that attempts to localize intelligence by the employment of methods which restrict the investigation to a comparison of measurements made on defectives with the same measurements made on normal subjects can have only a limited value. Further, the data they provide must be interpreted with the greatest caution. To measure some part—perhaps the cerebral cortex—and having found that it increases as intelligence increases, to conclude that the part concerned must therefore be connected causally with intelligence is clearly an assumption which cannot be upheld. To use a *reductio ad absurdum*, we have found that foot length falls off with decreasing mental age, yet no one would accept the conclusion that intelligence must be localized in the feet! Further, we have seen that 'body' measurements actually fall off more than 'brain' measurements, but this does not indicate that intelligence is more a property of the body than of the brain.

For the method commonly in vogue which we have referred to above, we must therefore substitute one which will yield more precise information. Although in this paper we are not primarily concerned with technique, yet it may be mentioned that the method of partial correlations provides a better approach to the statistical treatment of the subject.

(2) The data show, as clinical experiences teaches, that the deficiency manifests itself just as much in the body as in the brain, if not more. Mental deficiency, therefore must be regarded as essentially a general phenomenon, affecting the whole individual, rather than a localized disturbance affecting merely one small part.

(3) The fact that as we go down the intelligence scale the body shrinks faster than the brain, is compatible with the hypothesis that mental deficiency is essentially an arrest of development. We may recall that as the various stages of growth are retraced in the normal from childhood to infancy so we also find that the body shrinks faster than the brain.

APPENDIX TO PART II.—TABLES OF DATA AND STATISTICAL METHODS USED.

APPENDIX A.

TABLE 1.

The distribution of *body weight* for various mental ages is:—

		Mental age in years									
		1	3	4	5	6	7	8	9	10	11
Body weight in lbs.	175	...	...	...	...	...	...	...	2		
	168										
	161	...	...	1	1	...	...	...	1	...	1
	154	...	...	1	1	2	1	3	1		
	147	...	1	4	1	2	1	1	...	...	1
	140	...	1	2	5	5	...	3	3	2	
	133	1	3	3	2	4	4	5	2	2	
	126	4	9	4	6	3	5	3	3	1	1
	119	3	6	8	4	6	3	2	2	...	1
	112	6	9	8	4	4	4	2	3		
	105	3	5	3	7	1	6	4	2	1	
	98	4	6	2	3	1	1	2	...	1	
	91	1	3	1	2	1	...	...	...	1	
	84	7	2	1							
	77	3	...	...	...	...	...	1			
	70	3									
	63	2	...	...	1						
	56										
	49	1									

TABLE 2.

The distribution of *cube root-body weight* for various mental ages is:—

[illegible]

TABLE 5.

		Body weight	$\sqrt[3]{\text{Body weight}}$
Mental age in years	1	76.0—87.9	91.0—95.7
	3	92.4—99.8	97.6—100.2
	4	98.4—108.1	99.6—102.2
	5	95.0—106.1	98.3—102.2
	6	102.3—112.5	100.9—104.3
	7	96.8—106.6	99.2—102.4
	8	98.6—112.1	99.6—104.1
	9	103.5—121.0	101.4—106.7
	10	88.6—116.4	96.2—105.6
	11	91.5—143.2	97.7—113.7

		Arm length	Foot length
Mental age in years	1	92.1—97.1	90.8—96.1
	3	97.9—100.9	98.1—101.8
	4	97.0—101.1	96.8—101.2
	5	97.0—101.5	98.8—102.9
	6	99.9—104.9	98.3—104.5
	7	100.1—104.4	98.7—104.9
	8	100.5—106.1	99.6—104.3
	9	99.6—105.4	101.4—107.8
	10	95.1—107.8	94.7—107.2
	11	100.7—114.0	102.6—111.9

APPENDIX B.

The measurements are divisible into two natural groups.—We calculate, by the method of least squares, the slopes b of the various regression lines of the standardized measurements on mental age. We also calculate the variances of these estimates. If p is any particular measurement, we find the following:—

TABLE 6.

p	Measurement	b	V_b	n
1	Head length	+0.5770	0.014732	269
2	Head breadth	+0.4012	0.010908	„
3	Ear to ear	+0.5169	0.013770	„
4	$\sqrt[3]{\text{brain volume}}$	+0.5281	0.010822	„
5	$\sqrt[3]{\text{body weight}}$	+0.9845	0.017126	„
6	Arm length	+0.9698	0.021852	256
7	Foot length	+1.0273	0.026423	„

By inspection we see that the values of b fall into two groups: (1) around 0.5 and (2) around 1.0. We may test the significance of this separation in the following way:—

If the seven values of b are really derived from the same material, the differences will be due solely to random sampling, and $\sum_{p=1}^7 \frac{(b_p - \bar{b})^2}{V_p}$ will

be distributed as is χ^2 for $n = 6$. Actually we find $\chi^2 = 27.30$ and P is less than 0.01. Clearly the b 's are not homogeneous.

If the data really consist of two separate parts, $p = 1$ to 4, and $p = 5$ to 7, then if $b_1 = \text{mean } b \text{ from } 1 \text{ to } 4$, $\sum_{p=1}^4 \frac{(b_p - \bar{b}_1)^2}{V_p} + \sum_{p=5}^7 \frac{(b_p - \bar{b}_2)^2}{V_p}$ will be distributed as is χ^2 for $n = 5$. We find $\chi^2 = 1.48$ and $P = 0.92$. The discrepancy has completely vanished, showing that seven values of b must be divided into two groups, and showing that each sub-group is apparently homogeneous.

APPENDIX C.

Combining the Measurements.—We may combine the data simply by finding the general mean at each mental age. To find the variance of these combined means is not so simple. To take group A, the variance of the sum of four quantities is equal to the sum of the four variances plus twice the

product $r_{st} \sqrt{(V_s V_t)}$, the latter being applied to all the possible pairs ($\frac{n(n-1)}{2}$ in number) and summed. This latter part, involving the correlations between all the data in pairs, involves much computation. Since, however, the variances are all practically equal (apart from sampling variations) and as the correlations will all be about +0.5, and as we are not going to apply further tests, we may simplify the latter to the expression, for the variance of the mean of n quantities,

$$V(\text{grand mean}) = \frac{n + 1}{2n^2} (\text{sum of variances of means}).$$

On examining the data in group A, we find that the variances of head height are consistently above the others, and we find, by testing, that the variances of our final (combined) estimates are actually larger when head height is included than when it is left out. Head height is thus not sufficiently precise to 'pay its way,' and its inclusion, instead of adding to the final precision, actually lowers it. In group A, we have therefore omitted head height when combining them.

By following exactly the same lines as before, we find the limits of the means to be:—

TABLE 7.

		Group A	Group B
Mental age in years	1	95.8—98.5	91.7—95.8
	3	98.0—100.1	98.1—100.7
	4	99.3—101.8	98.2—101.3
	5	98.3—101.1	98.4—101.8
	6	99.2—102.5	100.2—104.1
	7	100.0—102.4	99.7—103.5
	8	100.5—102.5	100.4—104.3
	9	99.6—102.1	101.5—106.0
	10	99.4—105.7	97.1—105.1
	11	100.1—105.1	103.4—110.2