Published in final edited form as:
Proc Am Control Conf. 2014 June ; 2014: 4198–4203. doi:10.1109/ACC.2014.6859462.

Hybrid Model Predictive Control for Sequential Decision Policies in Adaptive Behavioral Interventions

Yuwen Dong[1], **Sunil Deshpande**[1], **Daniel E. Rivera**[1], **Danielle S. Downs**[2], and **Jennifer S. Savage**[3]

Yuwen Dong: yuwen.dong@asu.edu; Sunil Deshpande: sdeshpa2@asu.edu; Daniel E. Rivera: daniel.rivera@asu.edu; Danielle S. Downs: dsd11@psu.edu; Jennifer S. Savage: jfs195@psu.edu

[1]Control Systems Engineering Laboratory (CSEL), School for Engineering of Matter, Transport, and Energy, Arizona State University, Tempe, AZ, USA

[2]Exercise Psychology Laboratory, Department of Kinesiology, Penn State University, University Park, PA, USA

[3]Center for Childhood Obesity Research and the Department of Nutritional Sciences, Penn State University, University Park, PA, USA

Abstract

Control engineering offers a systematic and efficient method to optimize the effectiveness of individually tailored treatment and prevention policies known as adaptive or “just-in-time” behavioral interventions. The nature of these interventions requires assigning dosages at categorical levels, which has been addressed in prior work using Mixed Logical Dynamical (MLD)-based hybrid model predictive control (HMPC) schemes. However, certain requirements of adaptive behavioral interventions that involve sequential decision making have not been comprehensively explored in the literature. This paper presents an extension of the traditional MLD framework for HMPC by representing the requirements of sequential decision policies as mixed-integer linear constraints. This is accomplished with user-specified dosage sequence tables, manipulation of one input at a time, and a switching time strategy for assigning dosages at time intervals less frequent than the measurement sampling interval. A model developed for a gestational weight gain (GWG) intervention is used to illustrate the generation of these sequential decision policies and their effectiveness for implementing adaptive behavioral interventions involving multiple components.

I. Introduction

Behavioral interventions seek to reduce unhealthy behaviors and ameliorate healthy ones through prevention and treatment [1]. These programs can combine various treatment arms which may be pharmacological, behavioral, or community-based in nature [1]. Traditional fixed interventions provide the same dosages of prevention or treatment components to all program participants without considering any individual dynamics. Recent efforts in medicine and behavioral health have suggested that tailoring treatment to the specific needs

of an individual may enable delivery of interventions with greater degrees of efficacy and adherence, and less waste of resource [2]; this is the motivating principle for *adaptive* interventions [3], which are also referred to as “just-in-time” interventions [4]. Adaptive interventions in which the dosage is adapted according to a participant's response over time constitute a form of feedback control system in behavioral health [1], [3]. The use of dynamical systems and control engineering methods to optimize adaptive behavioral interventions has been explored in [5]–[8].

Due to clinical and resource considerations, adaptive interventions which feature multiple intervention components (e.g., inputs), often require formulating and evaluating ‘decision rules’ that dictate the proper dosage sequence, that is, the order in which each component should be augmented, reduced or kept unchanged. This augmentation and/or reduction sequence specified in the sequential decision policies restricts how the dosages of intervention components can change over time. For example in a proposed gestational weight gain (GWG) behavioral intervention, an active learning component for physical activity may not be offered until the healthy eating active learning component has reached full dose [7]. Another example of sequential decisions can be found in an intervention to promote physical activity in older adults, where components of self-monitoring, initiation training, and maintenance must be introduced in an established pre-ordained sequence [9]. In addition, clinical considerations specify that at each decision point, the dosage of only one intervention component can be altered (if it is necessary). Finally, it is often clinically required that intervention decisions be made at frequencies other than the regular sampling interval. For example, the participant may visit the clinic every other Monday (the assessment cycle) while data is collected daily through self-monitoring.

In the behavioral medicine literature, these requirements or rules are generally implemented using *ad-hoc* heuristic methods. In prior work [7], hybrid model predictive control (HMPC) was used to implement the decision policies in the intervention to assign optimized categorical intervention dosages. The paper specifically dealt with application of HMPC to the GWG intervention, which can be modeled in terms of a network of production-inventory systems [10]. In particular, constraints as later shown in (14) and (15) were used to handle logic specification of sequential decision policies under the assumption that the dosage change can take place only one step at a time (i.e., move size constraints are ±1) [7]. However, for an arbitrary move size (as is often the case in adaptive interventions) the constraints postulated in prior work are inadequate to make sure that only one component incurs a dosage change at each assessment cycle.

This paper improves the earlier HMPC formulation by addressing some of its existing shortcomings, and by consequence making it more generalizable. The HMPC formulation relies on the use of Mixed Logical Dynamical (MLD) framework for the control of hybrid systems developed in [11], and implements the improved three-degree-of-freedom (3 DoF) tuning formulation of Nandola and Rivera [5]. This work extends the traditional MLD framework by formulating the constraint equations associated with a user-specified dosage sequence table, restricting the manipulation to only one input at a time, and implementing a switching time strategy through the definition of mixed-integer linear inequalities, thus creating the functionality needed by the control algorithm to make sequential decisions

within a receding horizon framework. The GWG problem is used to exemplify the design of the improved HMPC controller through:

1. generation of a sequence table involving three intervention components,
2. manipulating one input at a time among the three, and
3. decisions on intervention components on a bi-weekly basis, despite daily sampling of intervention outcomes.

This paper is organized as follows. Section II presents the statement of the problem with a brief overview of GWG interventions and the proposed sequential decision policies in GWG problem as an illustration. Section III describes the 3 DoF HMPC scheme used as a decision framework in [5]. Section IV focuses on the design of the logic specified in the dosage sequence in Section II as an improved HMPC which includes generation of sequence tables, selection of one input change, and switching time strategy. Section V discusses the simulation results for a case study based on hypothetical parameters for GWG interventions implemented with the improved HMPC developed in Section IV. Section VI gives the summary and conclusions of this work.

II. Statement of the Problem

A. An Illustrative Problem: GWG Interventions

High pre-pregnancy body mass index (BMI) and excessive GWG represent major public health issues. They are associated with many negative outcomes, such as gestational diabetes, macrosomia, and birth defects [12], [13]. Interventions aiming to promote GWG within the 2009 Institute of Medicine (IOM) recommendations in Table I appear to reduce the risk of adverse outcomes in normal weight pregnant women; however, these interventions have been less successful in overweight and obese pregnant women who often exceed GWG IOM guidelines, which leads to the necessity to develop scalable, effective and efficient weight management interventions. Because adaptive interventions are tailored to the specific needs of each individual, an advantage to this design is that the potency of the intervention may be adjusted on an individual basis, which in turn, influences intervention effectiveness. Thus, adaptive interventions have the potential for improved outcomes by enhancing potency, increasing compliance, conserving resources (e.g., cost savings), and reducing any negative effects associated with treatment, in contrast to fixed interventions that may or may not work for individuals depending on their needs [3].

The conceptual linkages between the problem of adaptive interventions and control engineering can be understood by treating adaptive interventions as feedback control systems, with the outcome variable acting as the controlled variables, the intervention components representing the manipulated variables, and the decision policies serving the role of feedback control laws. The dynamics of the intervention without decision policies can be treated as “open-loop” to design the controller which can assign optimized dosages based on the participant's response. If measurements of external variables (e.g., stress, anxiety) that influence participant response are available, these can be incorporated in the controller algorithm in a feedforward manner [5]. Detailed development of an open-loop

model for HMPC serving as a decision policy for a GWG adaptive intervention is provided in [7], [10].

B. Sequential Decision Policies in GWG Interventions

In an adaptive intervention, there are usually many intervention components (u_1, …, u_n), the dosages of which are manipulated variables. As an illustration, we assume there are three (n = 3) components (u_1, u_2, u_3) which are augmented or reduced during the GWG intervention. These are Healthy Eating (HE) active learning (as $u_1(k) \in \{0, 1, 2, 3\}$), Physical Activity (PA) active learning (as $u_2(k) \in \{0, 1, 2, 3, 4\}$), and goal setting (as $u_3(k) \in \{0, 1\}$). In the baseline program, all these three components enter at the same time with a base dosage as $u_j(k) = 1$, $j \in \{1, 2, 3\}$.

The multiple intervention components in this problem require us to be faced with the decision regarding which component should be first augmented or reduced at each assessment cycle (considered in this illustration as on a bi-weekly basis) when the dosage can be updated per the individual's measured outcomes and performance. In this paper, we assume u_2 will be augmented from the baseline only when u_1 reaches its maximum dosage, while u_1 will not be reduced from full dosage until u_2 returns back to the base dosage (augmentation and reduction sequence above baseline). When it is necessary to decrease the dosage from the baseline, u_2 is reduced first, followed by u_1 and u_3; the augmentation sequence from zero dosage to baseline will be in the opposite order with u_3 increased to base dosage first, followed by u_1 and u_2 (augmentation and reduction sequence below baseline). At each assessment cycle, there will be only one intervention component augmented or reduced, indicating the necessity to involve the functionality of allowing the selection of one input change (if $\Delta u_i(k) \neq 0$, then $\Delta u_j(k) = 0$ for $j \neq i$; $i, j \in \{1, 2, \ldots, n\}$, where n is the number of intervention components, k is the sampling time). The logic in the sequential decision policies restricts how the future dosage can be specified, based on the current dosage the participant receives. The decision on how to properly assign the dosages will be dictated by the HMPC controller with the constraints associated with these logical conditions embedded into the dynamical model.

III. HMPC as a Decision Framework

From an engineering perspective, to optimize an adaptive intervention, the controller assigns dosages of each intervention component to the participant as dictated by model dynamics, problem constraints, and disturbances. For such kind of problems, one has to be aware of the fact that they are hybrid in nature because the dosages of intervention components correspond to categorical values, and decisions are made in a discrete manner [5]. In this paper, the algorithm which takes into account a model of hybrid system is described as an MLD system [11] as follows,

$$x(k+1)=Ax(k)+B_1u(k)+B_2\delta(k)+B_3z(k)+B_dd(k) \quad (1)$$

$$y(k)=Cx(k)+d'(k)+\nu(k) \quad (2)$$

$$E_2\delta(k)+E_3z(k) \leq E_5+E_4y(k)+E_1u(k) - E_dd(k) \quad (3)$$

where *x* and *u* represent states and inputs of the system. *y* is the output and *d*, *d′* and ν represent measured disturbances, unmeasured disturbances and measurement noise signals, respectively of appropriate dimensions. δ and *z* are discrete and continuous auxiliary variables that are introduced in order to convert logical/discrete decisions into their equivalent linear inequality constraints. The effect of all unmeasured disturbances is lumped as *d′* in the measurement equation.

The 3 DoF approach used in this paper to tune the controller allows the user to adjust the speed of set point tracking, measured and unmeasured disturbance rejection independently [5] by varying parameters a_r, a_d and f_a respectively. These parameters can be adjusted between values 0 and 1. Details of the 3 DoF controller formulation can be found in [5]. This work relies on a quadratic cost function,

$$\min_{\{[u(k+i)]_{i=0}^{m-1},[\delta(k+i)]_{i=0}^{p-1},[z(k+i)]_{i=0}^{p-1}\}} \sum_{i=1}^{p} \|y(k+i) - y_r\|_{Q_y}^2 \quad (4)$$

$$\text{s.t } y_{\min} \leq y(k+i) \leq y_{\max}, i \in \mathcal{T}_p \quad (5)$$

$$u_{\min} \leq u(k+i) \leq u_{\max}, i \in \mathcal{T}_m \quad (6)$$

$$\Delta u_{\min} \leq \Delta u(k+i) \leq \Delta u_{\max}, i \in \mathcal{T}_m \quad (7)$$

and (1), (2), (3) hold true,

where $\mathcal{T}_p = \{1, \ldots, p\}$, *p* is the prediction horizon, $\mathcal{T}_m = \{0, \ldots, m-1\}$, *m* is the control horizon, y_r is the reference, Q_y the penalty weight on the tracking error.

The system characterized by categorical inputs in the previous section can be represented logically as:

$$\delta_i(k)=1 \iff z_i(k)=i-1; i \in \{1,2,3,4\} \quad (8)$$

$$u_1(k)=\sum_{i=1}^{4} z_i(k) \sum_{i=1}^{4} \delta_i(k)=1 \quad (9)$$

$$\delta_j(k)=1 \iff z_j(k)=j-5; j \in \{5,6,7,8,9\} \quad (10)$$

$$u_2(k)=\sum_{j=5}^{9} z_j(k)\sum_{j=5}^{9}\delta_j(k)=1 \quad (11)$$

$$\delta_\ell(k)=1 \iff z_\ell(k)=\ell-10;\ell\in\{10,11\} \quad (12)$$

$$u_3(k)=\sum_{\ell=10}^{11} z_\ell(k)\sum_{\ell=10}^{11}\delta_\ell(k)=1 \quad (13)$$

where (8) and (9) describe the dosage change for u_1, with $u_1(k) = 1$ as base dosage, $u_1(k) = 2$ and $u_1(k) = 3$ as two augmentations from the baseline, and $u_1(k) = 0$ as the reduction from the baseline; (10) and (11) describe the dosage augmentation and reduction for u_2 in a similar manner; (12) and (13) do so for u_3. (8) - (13) can be converted into linear inequalities as E matrices in (3). Further, sequential decision using HMPC in [7] was implemented with the following additional constraints only when $|\Delta u(k)| \leq 1$,

$$\Delta u_1(k)^2+\Delta u_2(k)^2+\cdots+\Delta u_n(k)^2 \leq 1 \quad (14)$$

$$|u_1(k)|+|u_2(k)|+\cdots+|u_n(k)| \leq 1. \quad (15)$$

The ensuing section will develop a generalized procedure to formulate HMPC for sequential decision policies.

IV. An Improved HMPC to Address the Sequential Decision Policies

A. Generation of Sequence Table

Table II summarizes the proposed dosage sequence according to the description mentioned in Section II.B, which elucidates how the dosages will change during the intervention. For instance, if the participant is receiving the intervention dosage with sequence 2, then in the next assessment cycle, she can 1) get her dosage augmented to sequence 3, 4, or 5 based on the move size $\Delta u_2(k)$, 2) get her dosage reduced to sequence 1 or 0 according to the move size $\Delta u_1(k)$, or 3) remained unchanged. The HMPC will be able to determine the optimized discrete dosages which follow the logic of dosage sequence and meet the move size $\Delta u(k)$.

With the help of information in Table II, the following logical conditions are generated and embedded into the dynamical model so that only the dosage combinations in Table II are selected,

$$\Omega=(\delta_2\wedge\delta_6\wedge\delta_{11})\oplus(\delta_3\wedge\delta_6\wedge\delta_{11})\oplus(\delta_4\wedge\delta_6\wedge\delta_{11})\oplus(\delta_4\wedge\delta_7\wedge\delta_{11})\oplus(\delta_4\wedge\delta_8\wedge\delta_{11})\oplus(\delta_4\wedge\delta_9\wedge\delta_{11})\oplus(\delta_2\wedge\delta_5\wedge\delta_{11})\oplus(\delta_1\wedge\delta_5\wedge\delta_{11})\oplus(\delta_1\wedge\delta_5\wedge\delta_{10}) \quad (16)$$

where $(\delta_2 \wedge \delta_6 \wedge \delta_{11})$ is the dosage in baseline program (dosage sequence 0), in which $\delta_2 = 1$ means $u_1 = 1$ is selected, $\delta_6 = 1$ means $u_2 = 1$, and $\delta_{11} = 1$ means $u_3 = 1$; $(\delta_3\wedge\delta_6\wedge\delta_{11})$ represents the dosage sequence 1; and the like. The 9 combinations in (16) are the 9 dosage

sequences in Table II. Ω in (16) can be expressed in the linear inequalities in (3). This limits the possibilities of the dosage combinations to 9 instead of a possible $C_4^1 \times C_5^1 \times C_2^1 = 40$ combinations. For problems with larger dimensions than those shown in this example, the generation of Table II and its corresponding logical conditions in (16) can be efficiently automated.

B. Selection of a Single Input in a Multi-Input Scenario

The adaptive interventions usually require that only one component exhibits a dosage change at each assessment cycle. This is necessary because it can prevent the participant from being uncomfortable due to any dramatic intervention adaptation and hence unable to follow up with the pace of the intervention. This basically implies that the controller can only choose one input to incur change at each sampling time, and its change also has to follow the logic in the sequential decision policies above. In order to achieve this on the basis of the sequence table, additional binary variables φ and its associated logical specifications are introduced. They are converted into linear inequalities, and are implemented by either appending them to (3) or by overwriting (7). The number of the additional binary variables corresponds to the number of the manipulated inputs.

In the GWG interventions illustrated above, three binary variables (φ_1, φ_2 and φ_3) are augmented into the vector of binary variables δ in (3). The selection of one input change can be logically expressed as follows,

$$\phi_1(k)=1 \iff \begin{cases} |\Delta u_1(k)|>0, \\ \Delta u_2(k)=\Delta u_3(k)=0 \end{cases} \quad (17)$$

$$\phi_2(k)=1 \iff \begin{cases} |\Delta u_2(k)|>0, \\ \Delta u_1(k)=\Delta u_3(k)=0 \end{cases} \quad (18)$$

$$\phi_3(k)=1 \iff \begin{cases} |\Delta u_3(k)|>0, \\ \Delta u_1(k)=\Delta u_2(k)=0 \end{cases} \quad (19)$$

$$\phi(k) \odot \Delta u(k)_{\min} \leq \Delta u(k) \leq \phi(k) \odot \Delta u(k)_{\max} \quad (20)$$

$$\phi_1(k)+\phi_2(k)+\phi_3(k) \leq 1 \quad (21)$$

$$\text{where } \phi(k)=[\phi_1(k)\phi_2(k)\phi_3(k)]^T \quad (22)$$

$$\Delta u(k)=[\Delta u_1(k)\Delta u_2(k)\Delta u_3(k)]^T \quad (23)$$

$$\Delta u(k)_{\max}=[\Delta u_1(k)_{\max}\Delta u_2(k)_{\max}\Delta u_3(k)_{\max}]^T \quad (24)$$

$$\Delta u(k)_{\min}=[\Delta u_1(k)_{\min}\Delta u_2(k)_{\min}\Delta u_3(k)_{\min}]^T \quad (25)$$

and ⊙ is the Hadamard product, k is the sampling time. The logical specifications in (17) - (20) can be expressed as linear inequalities related with $u(k)$ over the m control horizon, and $\varphi(k)$ over the p prediction horizon; (21) is augmented after the linear inequalities of binary variables $\delta(k)$ in (3) over the p prediction horizon. Please note that the move size constraints in (24) and (25) are defined as time-varying vectors for the general case.

C. Switching Time Strategy

This section describes the algorithm for making the control decisions at a known integer multiple T_{sw} of the system sample time T_s. For the GWG problem, the decisions are made on a bi-weekly basis. This requirement is different from specifications of multirate control as all variables in this paper are sampled at the same rate. The algorithm used in this paper is referred to as a switching time strategy and is achieved by enforcing control move size $\Delta u(k)$ to be zero over the control horizon except when decisions have to be made. By taking the receding horizon nature of HMPC into account, this can be written as time-dependent linear equality constraint: $A_{T_{SW}}(k)u = 0$, where $A_{T_{SW}}(k)$ has a block-diagonal structure generated at each sampling instant k using the following steps:

1. Given the control horizon m and switching time $T_{sw}(\leq m)$, calculate the number of blocks $\texttt{numblocks} = \left\lfloor \frac{m-1}{T_{SW}} \right\rfloor + 1$. If the $\texttt{numblocks} \geq 3$, define number of block excluding the first and last block: $\texttt{midblocks} = \texttt{numblocks} - 2$. The term 'switching sample' will be used to denote sample when the control is allowed to change its value i.e. $\texttt{iff rem}\left(\frac{k}{T_{SW}}\right) = 0$, where $\texttt{rem}$ is the remainder.
2. The matrix $A_{T_{SW}}$ is populated by 0, 1 or − 1 to implement the move size restriction, and its size is determined by length of control horizon and size of $\texttt{numblocks}$: $A_{T_{SW}}(k) \in \mathbb{R}^{(T_{SW}+\text{midblocks}\times(T_{SW}-1)+\text{rem}(\frac{m-1}{T_{SW}})+1)\times m}$.
3. The rows in $A_{T_{SW}}(k)$ corresponding to switching samples will be set to zero; otherwise the rows will be populated to implement $\Delta u(k + i) = 0$.
4. Finally, the first sample $u(k)$ is assigned the previously calculated optimal value i.e. $u(k) = u^*$ when k does not correspond to the switching sample (as per the receding horizon framework).

While mathematically the switching time strategy is similar to move blocking strategies used in MPC [14], the difference (besides the process of generating the matrix $A_{T_{SW}}(k)$) lies in the fact that in move blocking strategies, the idea is to decrease the dimension of the decision variable to reduce the computational burden, whereas here the requirement is to

apply controls only at specified samples while the size of the decision variable remains the same. This is enforced as a constraint in the optimization problem shown in (4).

In summary, this section introduced three specific concepts: the generation of a sequence table, the selection of a single input in a multi-input scenario, and a switching time strategy. The ensuing section shows a simulation study applying these concepts for a simulated hypothetical GWG intervention.

V. Simulation Results and Discussion

In this section, a case study for GWG interventions is presented. The simulation is based on a hypothetical 25-year-old female with pre-gravid body mass 75 kg, 160 cm in height, which classifies her as overweight (BMI=29.30). The open-loop model for GWG interventions is described in [7], [10], and is depicted in Fig. 1. This is a conceptual framework for an adaptive GWG intervention which continues to be modified, but for the purpose of this paper, the proposed sequential decision policies will be used to illustrate how the improved HMPC assigns the optimized dosages and follows the proposed sequential decision policies described in Section II.B. For the sake of simplicity, we only focus on the effects that intervention components and self-regulation play on the perceived behavioral control (PBC) inflow in the Theory of Planned Behavior (TPB) models. The 2009 IOM guidelines in Table I is used as reference trajectory for HMPC controller. We claim that with no intervention, this woman will have a ramp increase in her energy intake (EI) from day 14 to day 91 and her EI will keep constant throughout the remaining of the pregnancy. The participant is sedentary at the time of conception, and potentially engaged in less PA from the second to third trimester as she gains weight in the absence of intervention. The intervention can help improve her physical activity level (PAL) in the second trimester, and however she will still decrease her PA during her latter stage of pregnancy. These are two PA disturbances for intervention and no intervention case, which will lower her energy expenditure (EE) in the energy balance model.

The case study assumes the participant enters the intervention with baseline program at week 14 and starts engaging in self-regulatory behaviors (e.g., weighing herself to monitor GWG, using dietary records to monitor EI and pedometer to monitor PAL). The dosage of the intervention components is adapted every two weeks until week 36.

Table III summarizes the model parameters in the simulation studies, including the behavioral parameters, time constants τ_i, time delays θ_i, gains assumed for the participant, and the filter parameters respectively. The definition of these variables can be found in [7], [10]. All values are hypothetical but can be selected such that the simulated responses mimic those of an actual participant. The parameters for the HMPC are as follows: $p = 60$ and $m = 35$, $Q_y = 10$, $\alpha_r = 0.9$, $\alpha_d = 0.3$, $f_a = 0.9$, the sampling time for the participant to measure her GWG and monitor dietary record is T_s=1 day, the assessment cycle is T_{sw}=14 days. The move size constraints for the manipulated variables at the assessment cycle are $\Delta u(k)_{max} = [1\ 2\ 1]^T$ and $\Delta u(k)_{min} = [-1\ -2\ -1]^T$ to illustrate that the improved HMPC no longer limits the move size to ±1.

The simulation result in Fig. 2 depicts the predicted maternal weight gain, energy intake, energy expenditure, the dosages of the intervention components, respectively. The whole process can be divided into four stages. The first stage occurs before the intervention begins. We can see that the participant has a ramp energy intake increase as she is aware of her pregnancy and she will remain sedentary throughout the first trimester. The intervention starts at week 14 with the participant receiving the baseline intervention, indicating the second stage which involves the dosage changes below the baseline program ($u(k) \leq 1$). The participant's weight is within the IOM guidelines at the beginning of the interventions, and that explains why her dosages get reduced for the first and second assessment cycle (week 16 and 18), with $u_2(k)$ decreased to zero first, followed by $u_1(k)$. However, the reduced intervention is not enough for this participant due to her high EI leading to GWG above the IOM guidelines around week 20. Thus, her intervention is augmented to the baseline program for the week 20 and 22, respectively. The dosage changes in the second stage meets the reduction and augmentation sequence below the baseline of the proposed sequential decision policies described in Section II.B. In the third stage, HMPC controller realizes that this base dose is still not enough for this high EI and low PA participant, requiring intervention adapted above the baseline ($u(k) \geq 1$). As a result, $u_1(k)$ is the first to be augmented with $-1 \leq \Delta u_1(k) \leq 1$; $u_2(k)$ will not be augmented until two weeks after $u_1(k)$ reaches maximum dose ($u_1(k) = 3$). Because the move size constraints of u_2 are ±2, it is augmented by two doses from its base dosage at week 28. At day 214, the participant's weight is within the upper bound of the IOM guidelines; HMPC controller reduces the dosage of u_2 first, while still keeping u_1 at its maximum dosage. The dosage change during the third stage follows the proposed sequential decision policies above the baseline. The fourth stage starts with the termination of the intervention at week 36. The previous effect of intervention and self regulation keep the participant engaging in a healthy diet and active exercise life style. Despite of her reduced EE, the final EE is still above the initial one.

VI. Summary and Conclusions

In this paper, we present the application of an improved HMPC for adaptive behavioral interventions where the system is described using the MLD framework. This work focuses on the design of a HMPC controller that systematically specifies the magnitude of intervention dosages which conform to sequential decision rules. In order to describe the procedures for how to embed the logical conditions and specifications associated with the dosage sequence into the MLD model, GWG interventions are used as an illustration with corresponding sequential decision policies being proposed. The HMPC-based intervention is able to adjust the dosages of the intervention components “just-in-time,” relying on the dynamical model and measured outcomes and performance during the intervention to improve the participant's response, increase the effectiveness of the intervention and achieve reduced waste of resources.

This work demonstrates the potential for real-world applications of an adaptive intervention by employing a control-oriented approach. The improved HMPC was illustrated on an intervention for managing gestational weight gain in pregnancy; however it can be applied to any other adaptive behavioral intervention problem with multiple intervention components which also require a certain dosage sequence.

Acknowledgments

Support for this work has been provided by the Office of Behavioral and Social Sciences Research (OBSSR) of the National Institutes of Health, the National Institute on Drug Abuse (NIDA), and the National Heart, Lung, and Blood Institute (NHLBI) through grants R21 DA024266, K25 DA021173 and R01 HL119245.

References

1. Rivera DE, Pew MD, Collins LM. Using engineering control principles to inform the design of adaptive interventions: A conceptual introduction. Drug and Alcohol Dependence. 2007; 88S:S31–S40. [PubMed: 17169503]
2. Murphy SA, Collins LM, Rush AJ. Customizing treatment to the patient: Adaptive treatment strategies. Drug and Alcohol Dependence. 2007; 88S:S1–S3. [PubMed: 17350181]
3. Collins LM, Murphy SA, Bierman KL. A conceptual framework for adaptive preventive interventions. Prevention Science. 2004; 5(3):185–196. [PubMed: 15470938]
4. Kumar S, Nilsen W, Pavel M, Srivastava M. Mobile health: Revolutionizing healthcare through transdisciplinary research. Computer. 2013; 46(1):28–35.
5. Nandola NN, Rivera DE. An improved formulation of hybrid model predictive control with application to production-inventory systems. IEEE Trans Control Systems Tech. 2013; 21(1):121–135.
6. Deshpande, S.; Nandola, N.; Rivera, DE.; Younger, J. Proceedings of the 2011 American Control Conference. San Francisco, CA: 2011. A control engineering approach for designing an optimized treatment plan for fibromyalgia; p. 4798-4803.
7. Dong, Y.; Rivera, DE.; Downs, DS.; Savage, JS.; Thomas, DM.; Collins, LM. Proceedings of the 2013 American Control Conference. Washington, D.C, USA: 2013. Hybrid model predictive control for optimizing gestaional weight gain behavioral interventions; p. 1970-1975.June 17 - June 19
8. Davison, DE.; Vanderwater, R.; Zhou, K. Proceedings of the 2012 American Control Conference. Montréal, Quebec, Canada: 2012. A control-theory reward-based approach to behavior modification in the presence of social-norm pressure and conformity pressure; p. 4046-4052.June 27 - June 29
9. Hekler EB, Buman MP, Poothakandiyl N, Rivera DE, Dzierzewski JM, Aiken-Morgan A, McCrae CS, Robers BL, Marsiski M, Giacobbi PR. Exploring behavioral markers of long-term physical activity maintenance: A case study of system identification modeling within a behavioral intervention. Health Education and Behavior. 2013; 40(IS):51S–62S. [PubMed: 24084400]
10. Dong, Y.; Rivera, DE.; Thomas, DM.; Navarro-Barrientos, JE.; Downs, DS.; Savage, JS.; Collins, LM. Proceedings of the 2012 American Control Conference. Montréal, Quebec, Canada: 2012. A dynamical systems model for improving gestational weight gain behavioral interventions; p. 4059-4064.June 27 - June 29
11. Bemporad A, Morari M. Control of systems integrating logic, dynamics and constraints. Automatica. 1999; 35(3):407–427.
12. Beyerlein A, Schiessl B, Lack N, von Kries R. Optimal GWG ranges for the avoidance of adverse birth weight outcomes: a novel approach. Am J Clin Nutr. 2009; 90:1552–1558. [PubMed: 19812177]
13. Institute of Medicine. Weight gain during pregnancy: reexamining the guidelines. The National Academies Press; Washington, DC: 2009.
14. Cagienard R, Grieder P, Kerrigan EC, Morari M. Move blocking strategies in receding horizon control. Proceedings of the 43rd IEEE Conference on Decision and Control (CDC). Dec.2004 2:2023–2028.

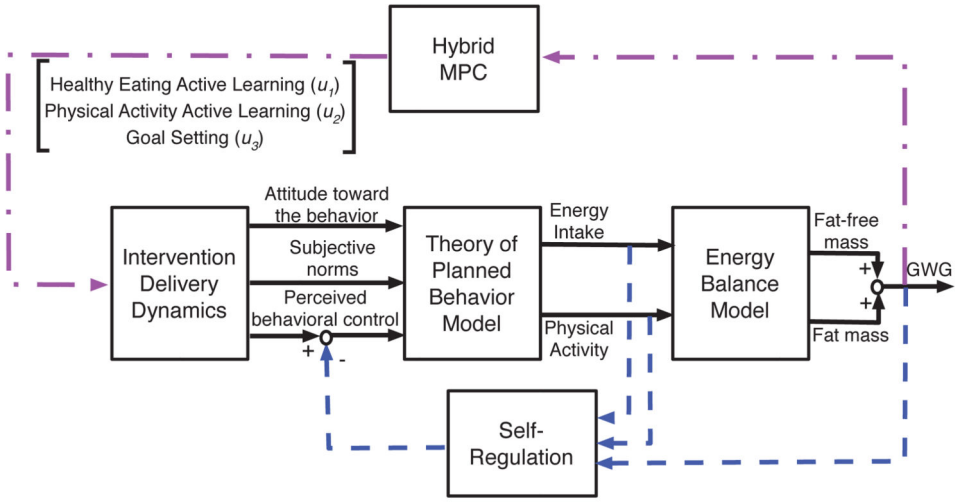

Fig. 1.

General diagram of the dynamical systems model for GWG adaptive interventions. Blue dash line represents the effect of self-regulation, and the pink dash-dot line stands for the output of the HMPC decision policies.

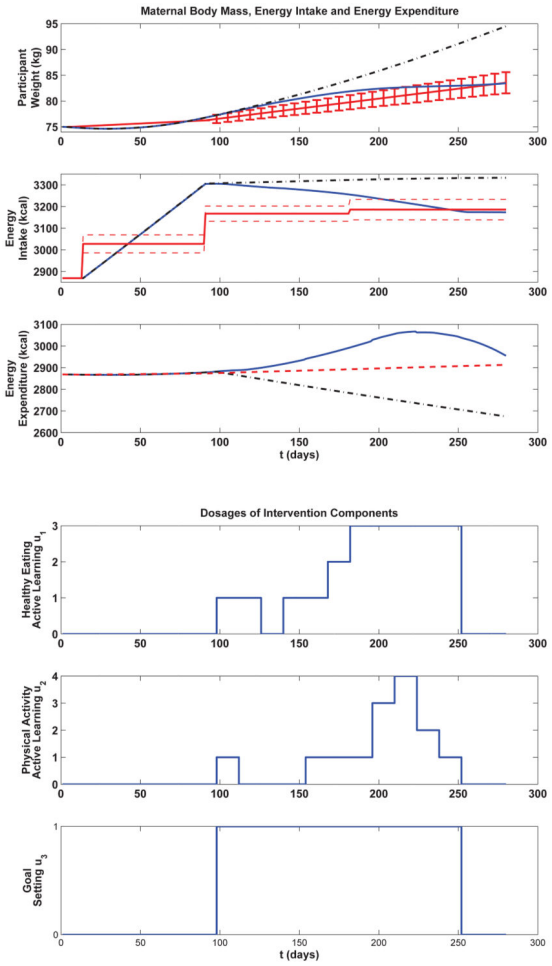

Fig. 2.

Simulation responses for the maternal body mass, energy intake and energy expenditure, and the intervention components dosages. Red lines represent the 2009 IOM guidelines applied on a daily basis; the blue solid line represent the case with intervention and self-regulation while the black dashed line represents the case with no intervention.

Table I

Target GWG recommended by the 2009 Institute of Medicine guidelines [13].

Classification	Pre-gravid BMI (kg/m^2)	Target GWG (kg) Trimester	
		1	2 - 3
Underweight	<20	0.5 - 2.0	11.4 - 15.8
Normal	20 - 25	0.5 - 2.0	9.1 - 13.0
Overweight	25 - 30	0.5 - 2.0	6.0 - 8.6
Obese	>30	0.5 - 2.0	4.4 - 7.0

Table II

Dosage sequence table for the hypothetical gestational weight gain (GWG) behavioral intervention example.

Dosage Sequence	u_1	u_2	u_3	Description
-3	0	0	0	Reduction of u_3
-2	0	0	1	Reduction of u_1
-1	1	0	1	Reduction of u_2
0	1	1	1	Baseline
1	2	1	1	First augmentation of u_1
2	3	1	1	Second augmentation of u_1
3	3	2	1	First augmentation of u_2
4	3	3	1	Second augmentation of u_2
5	3	4	1	Third augmentation of u_2

Table III

Model parameters for the simulation studies. Time constants (τ_i) are in units of days [7], [10].

Parameter	EI-TPB	PA-TPB	Parameter	EI-TPB	PA-TPB
b_1	3	1	e_1	6	4
n_1	2	7	m_1	3	8
p_1	1	4	c_1	2	2
τ_1	1	30	γ_{11}	1	0.7
τ_2	1	30	γ_{22}	1	0.5
τ_3	1	10	γ_{33}	1	0.7
τ_4	1	20	β_{41}	1	0.34
τ_5	1	30	β_{42}	1	0.27
$\theta_1 \ldots \theta_3$	0	0	β_{43}	1	0.13
$\theta_4 \ldots \theta_6$	0	0	β_{53}	1	0.08
θ_7, θ_8	0	0	β_{54}	1	0.42
$k_{11} \ldots k_{1n}$	0	0	$\theta_{11} \ldots \theta_{1n}$	0	0
$k_{21} \ldots k_{2n}$	0	0	$\theta_{21} \ldots \theta_{2n}$	0	0
$k_{31} \ldots k_{3n}$	0.0008	0.0045	$\theta_{31} \ldots \theta_{3n}$	0	0
λ_r	80	100	λ_d	90	155