

# Principles of physics in surgery: the laws of flow dynamics physics for surgeons – Part 1

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## Abstract

In the field of medicine and surgery many principles of physics find numerous applications. In this article we have summarized some prominent applications of the laws of fluid mechanics and hydrodynamics in surgery. Poiseuille's law sets the limits of isovolaemic haemodilution, enumerates limiting factors during fluid resuscitation and is a guiding principle in surgery for vascular stenoses. The equation of continuity finds use in non-invasive measurement of blood flow. Bernoulli's theorem explains the formation of post-stenotic dilatation. Reynolds number explains the origin of murmurs, haemolysis and airflow disturbances. Various forms of oxygen therapy are a direct application of the gas laws. Doppler effect is used in ultrasonography to find the direction and velocity of blood flow. In this first part of a series of articles we describe some applications of the laws of hydrodynamics governing the flow of blood and other body fluids.

**Keywords** Hydrodynamics · Poiseuille's · Equation of continuity · Bernoulli's · Reynolds number · Doppler effect

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## Introduction

“Science is organized knowledge”

Herbert Spencer

“Observation is a passive science, experimentation is an active science.”

The basics of all the mechanisms of operation of living and non-living things in nature are guided by laws of pure sciences i.e. Physics, Chemistry and Mathematics. This article is an endeavour to understand the various common pathophysiologic mechanisms and events in surgery with application of basic laws of physics.

## Aim

The present review article is based on the application of laws of fluid mechanics to the flow and stasis of various body fluids and gaseous and fluid diffusion.

## Materials and methods

The application of various laws was considered after discussion with various experts in the field of biomedical engineering and biophysics and the most important principles which were relevant to clinical surgery were considered.

A systematic Medline search (1960–2006) was carried out using keywords such as Poiseuille's law, the Equation of continuity, Bernoulli's principle, Reynolds number, Dalton's law, Van't Hoff's law, Boyle's law, Doppler effect, etc. Additional references were obtained from the bibliographies of the selected articles.

## Applications of individual laws

### Poiseuille's law

Poiseuille's law [1] relates to the rate ( $Q$ ) of blood flow through a blood vessel with the difference in blood pressure at the two ends ( $\Delta P = P_1 - P_2$ ), the radius ( $r$ ) and the length ( $L$ ) of the artery, and the viscosity ( $\eta$ ) of the blood (Fig. 1). This law can be expressed in the following algebraic equation:

$$\text{Volume flow rate} = \frac{\pi (\text{Pressure difference}) (\text{radius})^4}{8 (\text{viscosity}) (\text{length})}$$

$$\text{Or } Q = \pi r^4 \Delta P / 8L\eta$$

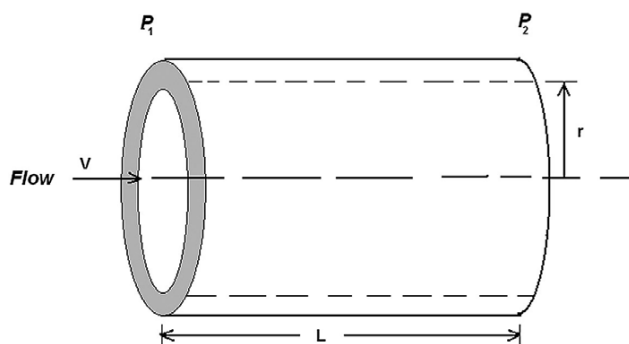
An increase in the diameter of a vessel from 1 mm to 2 mm will increase the flow rate by 16 times (for  $r = 2$ ;  $r^4 = 16$ ) if other factors shown above remain constant. The following examples explain its clinical applications:

- 1) According to Poiseuille's law, the rate of laminar flow of a fluid like blood is inversely proportional to its viscosity. Blood viscosity is decreased at reduced haematocrits increasing the flow through the vessels and hence the oxygen delivery to tissues. However, with dilution of blood the density of RBCs also falls. By careful titration of these two opposing effects it has been found that the "optimum haemodilution" to deliver the most oxygen to tissues is 30% [2]. This is the principle of isovolaemic haemodilution (IHT). It is achieved by replacing a certain calculated volume of the patient's blood depending on the current haematocrit, by a crystalloid solution. IHT has been successfully utilized in the treatment of Branch Retinal Vein Occlusion [3] and in surgery on liver and other organs associated with significant blood loss. Anaesthetist collects patients blood preoperatively, in blood collecting bags, and maintains the blood pressure by infusing crystalloids to achieve IHT. Once the bleeding from surgery is controlled, patients own blood collected before is transfused.
- 2) Rapid fluid administration is often the key to successful resuscitation of a bleeding patient. Some

of the important factors affecting the rate of fluid resuscitation include the diameter of the intravenous tubing, the size and length of the venous cannulae, the fluid viscosity and the site of administration. According to Poiseuille's law, flow is proportional to the fourth power of the radius of the catheter and the larger its diameter, the faster one can infuse a solution through it. Central venous placement reduces 'L', the length of the vessels traversed before reaching the right atrium and assures rapid flow. Importantly, the diameter of the intravenous tubing used may be the rate-determining factor in fluid delivery. A 16 gauge cannula (internal diameter 1.30 mm) can in fact achieve a flow rate 3.5 times greater than that of an 18 gauge (internal diameter 0.95 mm). Blood-infusion tubing allows twice the flow compared to standard intravenous tubing and is an easily available substitute when rapid fluid resuscitation is needed. Conversely if too high an injection rate like a jet, is used through a small needle, gas comes out of solution and may form bubbles (due to dissolution of gas from blood at low pressure at the level of jet, which results from high velocity; see Bernoulli's theorem).

In order to keep 'L' small choose a short infusion tube. A better venipuncture site would be closest to the right atrium on a wider vein like the cephalic, basilic, jugular or subclavian veins rather than on the long saphenous vein at the ankle, which is relatively narrow and is far from the heart. It is best to use crystalloids first rather than colloids to minimize viscosity ( $\eta$ ). Higher flow rate is possible by applying a pressure cuff around the infusion bottle or raise its height from the patient to maximize ' $\Delta P$ '.

- 3) In surgical correction for arterial narrowing where there are two or more sites of block or narrowing, it is best to first open up the proximal large diameter arteries. Since flow varies with the fourth power of radius, even a fractional increase in radius in a large artery can result in a greater increase in blood flow than a much greater fractional diameter increase in a smaller distal vessel.



**Fig. 1** Demonstrating Poiseuille's law

### Equation of continuity

The equation of continuity [4] (for an 'ideal fluid' that is incompressible and lacks viscosity), states that flow velocity is inversely proportional to the area of the conduit cross-section. As the fluid moves from a wider pipe (area =  $A_1$ , velocity =  $V_1$ ) into a narrower pipe (area =  $A_2$ , velocity =  $V_2$ ) or a constriction, a corresponding volume must move a greater distance forward in the narrower pipe and thus has a greater speed (It is based on the basic law of 'conservation of mass' i.e. the rate at which mass enters a system is equal

to the rate at which mass leaves the system) (Fig. 2).

$$R_v = A_1 V_1 = A_2 V_2 \text{ [} R_v \text{ is the volume flow rate]}$$

This principle can be used to non-invasively estimate the area of stenotic mitral valves and has a correlation coefficient of 0.91 compared with invasive angiographic measurements [5].

### Bernoulli's principle

Bernoulli's principle [6] formulated by Daniel Bernoulli (1700–1782) states that as the speed of a moving fluid (liquid or gas) increases, the pressure within the fluid decreases and vice versa. This is an application of the law of 'conservation of mechanical energy' (which in turn is made up of potential energy and kinetic energy) to the flow of an ideal fluid. It is expressed by the following equation:

$$\frac{1}{2} \rho v^2 + \rho gh + p = \text{Constant}$$

Where:  $\rho$  = density of fluid,  $v$  = velocity of the fluid,  $g$  = gravitational acceleration,  $h$  = elevation and  $p$  = pressure gradient across the conduit.

This principle explains the phenomenon of 'post-stenotic dilatation'. This condition arises from the fact that the sudden widening at the end of the stenosis results in a conversion of high kinetic energy into high potential energy, dissipated in the form of increased lateral pressure and turbulence. The increased lateral pressure can deform the vessel wall, increasing the diameter beyond its normal size. This further increase in diameter exacerbates the dissipation of energy, because the change in diameter from stenosis to downstream vessel becomes even greater. The result is a potentially progressive dilatation distal to a stenosis that can severely weaken the vessel wall.

### Reynolds number

Reynolds Number (Re) [7] proposed by Osborne Reynolds in 1883 predicts the conditions under which laminar flow

of a fluid is disrupted and turbulence occurs. It is calculated as:

$$Re = (v.d.\rho) / \eta$$

Where: mean velocity of flow =  $v$ , diameter of vessel =  $d$ , density of fluid =  $\rho$  and viscosity of fluid =  $\eta$  are known.

Murmurs occur in the heart when laminar flow is replaced by turbulence and this occurs at a Reynolds number above 2000. Turbulence can also damage red blood cells causing haemolysis as in large arterio-venous fistulas. One should be cognizant of these facts while choosing the right diameter of a tube delivering blood at a high speed e.g. in an Extracorporeal membrane oxygenator (ECMO). It has been found that a 10 French arterial cannula is best suited to deliver the blood flow rates needed [8].

The conditions of turbulence as predicted by Re are also important with interventions to increase the airflow. In case of hyperbaric airway pressures, turbulence due to greater velocity also raises airway resistance. This can offset the benefit of raising the flow rate and airway pressure, resulting in less than expected ventilation.

### Dalton's law and Henry's law

Dalton's law [9] propounded by English scientist John Dalton (1766–1844), states that the total pressure of a gaseous mixture is equal to the sum of the partial pressures of its components. For example: a mixture containing three gases a, b and c, the total pressure would be given by

$$P = p_a + p_b + p_c$$

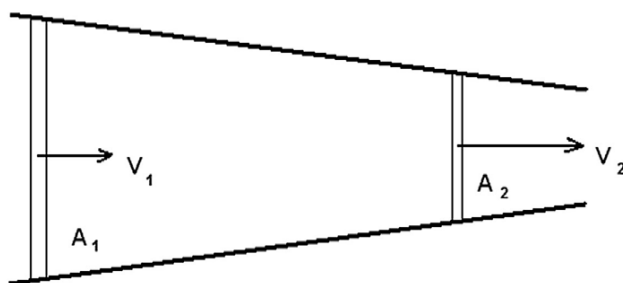
Where:  $p_a$ ,  $p_b$ ,  $p_c$  are the respective partial pressures of gases a, b and c.

The Alveolar air equation used to calculate the  $P_{a-O_2}$  at any barometric pressure is a modification of Dalton's law of partial pressure as it also takes into account the partial pressure exerted by water vapours as the air we breathe in is saturated during its passage through the respiratory tract (the partial pressure exerted by 'vapours' depends only on the temperature and not on the size of the container).

Alveolar air equation:

$$P_{a-O_2} = F_{i-O_2} (P_{\text{Barometric}} - P_{H_2O}) - (1.25 \times Pa_{CO_2}).$$

Henry's law [10] formulated by English chemist William Henry (1774–1863), states that gas enters into physical solution in direct proportion to the partial pressure exerted by that gas. Combining the application of these two laws allows the anaesthesiologist to raise the partial pressure of oxygen in blood to very high levels by giving 100% oxygen instead of the usual 21% from room air. Dalton's law allows the increase of partial pressure in the inspired gas mixture and Henry's law allows the dissolution of a greater amount of oxygen in blood due to its presence at a higher partial pressure.



**Equation of Continuity**

**Fig. 2** Depicting equation of continuity

This principle is also the basis of hyperbaric oxygen therapy. However since too much oxygen is also harmful, it is necessary to control the time for which hyperbaric oxygen is administered depending on the pressure used. Hundred percent oxygen at two atmospheres absolute for 6 hours results in cough, decreased vital capacity, substernal chest pain, and areas of patchy atelectasis. At two atmospheres, exposure is limited to 2 hours. Exposure to 100% oxygen at three atmospheres absolute for 3 hours results in a grand mal seizure [11]. Therefore, exposure is limited to 90 minutes at three atmospheres absolute.

### Boyle's law

Boyle's law [12] discovered by Robert Boyle in 1662, states that at a constant temperature, the volume and the pressure of a gas are inversely proportional. Thus a larger amount of oxygen can be delivered to a patient by increasing the pressure of the inspired gas. This law also explains the dissolution of nitrogen in blood of a diver at high pressure under the sea, which bubbles out into the tissues on rapid reduction in pressure when a diver reaches above the sea level, resulting in decompression sickness or 'Bends'.

### Doppler effect

Doppler effect [13] was proposed by Austrian physicist Johann Christian Doppler in 1842. The Doppler effect is change in the frequency of echo signals whenever there is a relative motion between the sound source and the reflector. This principle is used in Doppler ultrasonography, which depends on measurement of the relative change in the reflected ultrasound frequency when compared to the transmitted frequency. In case of Doppler ultrasonography because of the Doppler effect, transmitted sound waves encountering a group of red cells moving toward the transducer are scattered back at a frequency higher than the incident frequency

producing a positive frequency shift. The opposite effect occurs when a given frequency transmitted into the tissues encounters red cells moving away (Figs. 3a and 3b).

The Doppler effect in tissues may be expressed as following equation (Fig. 4).

$$f_o - f_e = F_\Delta = (2 \cdot f_o \cdot V \cdot \cos \theta) / c$$

Where:  $f_o$  = transmitted frequency,  $f_e$  = reflected frequency,  $F_\Delta$  = change in frequency,  $V$  = relative velocity of the reflector and  $c$  = velocity of sound through the medium.

The frequency shift, also called the "Doppler shift" ( $F_\Delta$ ) of ultrasound will depend on the transmitted frequency ( $f_o$ ), the velocity ( $V$ ) of the moving blood and the angle of the transducer ( $\theta$ ) with the direction of the blood flow. A higher returning frequency due to blood flow towards the ultrasonic probe is called a "positive Doppler shift". The flow of blood away from transducer results in a "negative frequency shift".

The Doppler effect finds its application commonly in confirming the diagnosis and localizing the site of lesion in peripheral vascular disease and in deep or superficial vein incompetence [14]. It is also commonly used to monitor foetal well being during the antenatal period [15].

In addition to providing the information about the *rate* and *direction* of flow, it can also tell us the *nature* of flow. In most of the cardiovascular system, including the heart and great vessels, the flow is normally laminar. In contrast, turbulent or non-laminar flow is present when there is some obstruction or narrowing. This turbulence is detected as green areas superimposed on the red and/or blue areas on the Doppler ultrasound [16].

### Van't Hoff's law

The Van't Hoff's [17] law states that "for the same solvent and for the same solute at a constant external pressure the osmotic pressure ( $P$ ) is directly proportional to the molar concentration of the solute ( $C$ ) or is inversely proportional

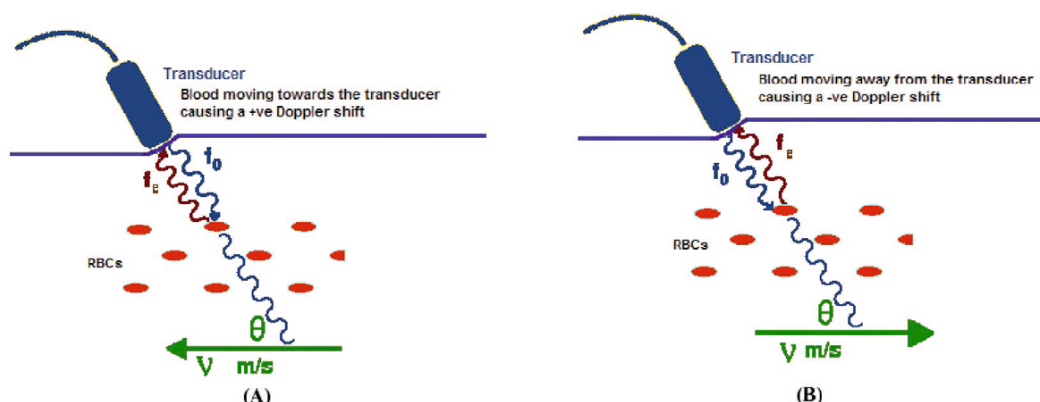
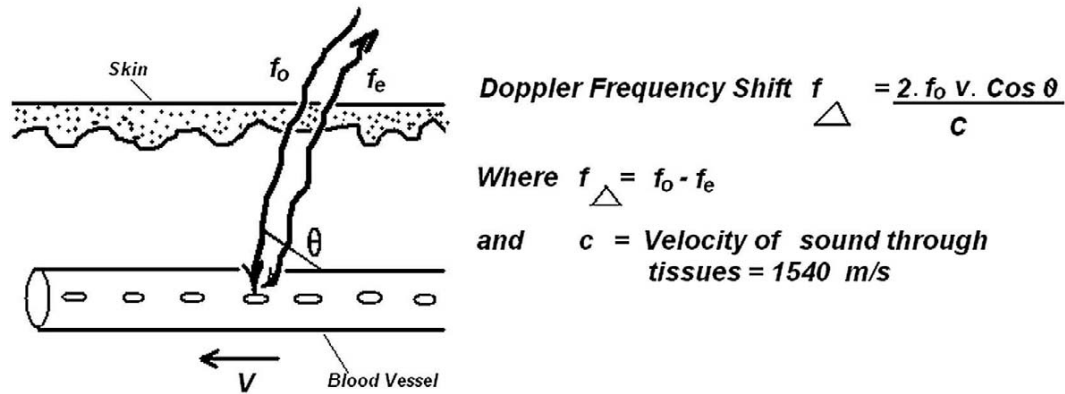


Fig. 3 Explaining the relationship of direction of blood flow to Doppler frequency shift

### Doppler Equation



**Fig. 4** Showing factors determining Doppler frequency shift

to the dilution (V).”

$$P \propto C$$

$$\propto 1/V$$

Based on the above law the effusion of fluid by osmotic barrier occurs in an abscess where by the action of lysosomal enzyme the cellular lipoproteins, phospholipids are broken down to smaller peptides, fatty acids, amino acids, nucleic acids, etc. This results in an increase in the number of solute particles in the fluid. Because osmotic property is a colligative property that is dependent on solute concentration per unit volume of solvent, liquefactive necrosis in pus formation results in an increase in volume of the solvent indirectly due to increase in the concentration of various solutes. Thus a rise in hydrostatic pressure due to drawing of fluid from surrounding tissue causes an increase in the size of the abscess until the turgor pressure of the abscess equalizes the arteriolar pressure causing cessation of blood flow to the abscess and the surrounding tissue. At this point the tissue surrounding the abscess undergoes necrosis. This is a major reason, in addition to the active bacterial insult, why a pyogenic membrane surrounding the abscess cavity is composed of necrotic ‘*avascular*’ tissue through which no antibiotics, immunoglobulins or polymorphs can penetrate effectively causing bacterial proliferation to continue unabated. Hence, the removal of the necrotic tissue should be till fresh bleeding occurs from the abscess wall. This will hasten the resolution of sepsis by allowing access of the polymorphs, immunoglobulins and antibiotics which not only will kill the bacteria but will also cause release of growth factors like PDGF (from platelets and polymorphs), FGF (from polymorphs), VEGF (macrophages), EGF (from macrophages). These growth factors help in various phases of wound healing. A surgeon draining an abscess ought to be cognizant of these physical facts. Hence, it is well advised to remove the pyogenic membrane by gentle curetting with a curette

scoop or a gauge wrapped on a finger till fresh bleeding is seen from the abscess margins.

**Conflict of interest** The authors do not have any disclosable interest

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