ENERGY DEPENDENCE OF THE HOUSFIELD NUMBER

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SUMMARY

A standard definition is proposed for the Hounsfield number. Any CT number can be converted to the Hounsfield scale after performing a simple calibration using air and water. The energy dependence of the Hounsfield number \( H \) is given by the expression

\[
H = \frac{(H_C + H_p Q)}{(1 + Q)} ,
\]

where \( H_C \) and \( H_p \) are the Compton and photoelectric coefficients of the material being measured, expressed in Hounsfield units, and \( Q \) is the "quality factor" of the scanner. \( Q \) can be measured by performing a scan of a single calibrating material, such as a KI solution.

By applying this analysis to dual-energy scans, the Compton and photoelectric coefficients of an unknown substance may easily be obtained. This can lead to a limited degree of chemical identification.

INTRODUCTION

We have developed an analytic technique which is useful for understanding the meaning of the Hounsfield number, and for interpreting dual-energy scans. It is based on the long-recognized observation that the Compton and photoelectric components of x-ray attenuation, individually have the same energy dependence over a wide range of materials and energies. Thus, if the Hounsfield number is broken up into two parts, Compton and photoelectric, with each part normalized to water, they will not be energy dependent.

When used for analyzing dual-energy scans, our technique has certain advantages over that of Rutherford et al., which is based on effective energy. First and most important, the present method is analytical rather than direct. Second, the calibration procedure is easier. Third, it avoids the uncertainty connected with the use of a single effective energy.

The effective energy of a CT scanner has been defined as that monochromatic energy at which a given material will exhibit the same attenuation coefficient as is measured on the scanner. The difficulty is that this energy is not the same for all materials, because substances with a stronger photoelectric absorption will give greater weight to the lower part of the spectrum. For example, Brooks and Di Chiro have calculated that the effective energy of the EMI Mark I scanner is 73.6 kV for water and 71.6 kV for bone. Of course this difference is not great, and a single effective energy can provide an approximate fit for a range of materials. Our analysis sidesteps this question, and instead utilizes an easily-measurable, well-defined "quality factor" to describe the x-ray spectrum. From the quality factor, one may derive the attenuation coefficient of water, the effective energy for water, and the "contrast scale". We believe that the quality factor is a preferable way to characterize the effective spectrum of a given scanner.

DEFINITION OF HOUSFIELD NUMBER

Throughout this paper we use Hounsfield notation. That is, the attenuation coefficient \( \mu \) is normalized to water and Hounsfield values \( H \) are obtained by subtracting unity and multiplying by 1000. On this scale the attenuation of water is zero, and each unit represents 0.1% of the attenuation of water. The equation defining the Hounsfield scale is

\[
\frac{\mu}{H_w} = 1 + H/1000 ,
\]

where \( H_w \) is the attenuation coefficient of water.

The advantages of the Hounsfield scale are that small differences in tissue attenuation are more readily apparent and the numbers are more easily stored in the computer and printed. Although a scale factor of 500 was originally used by EMI, the trend now is toward 1000, since the conversion process is simpler and finer differences can be seen. In the following treatment, \( H \) will be used for the ideal Hounsfield number as defined by Eq. 1, and \( H_m \) for the uncorrected CT number as measured on a given machine. The next question is how to relate the measured \( H_m \) to the ideal Hounsfield scale. Some scanners use the Hounsfield scale, so that nominally speaking, no conversion is required. This can be verified by checking the air and water readings, which should be -1000 and 0 respectively. If this is not the case, we can write

\[
H = 1000 \left( \frac{H_m - H_a}{H_w - H_a} \right) ,
\]

where \( H_a \) is the measured number for air and \( H_w \) is the measured number for air. Eq. 2 is based on a linear relation between the CT number and attenuation. It is valid as long as the attenuation of the unknown does not itself cause beam-hardening effects or other nonlinearities.

In order to calibrate the numerical scale of a scanner, all that is required is to measure the CT number of air and water. Other readings can then be converted to the Hounsfield scale using Eq. 2. There are two cautions, however: First, the amount of air used for calibration should not be so large as to alter the beam-hardening or cause overloading errors. On the other hand, it must be sufficient to obtain a good average reading. A hole 1.5-2 cm in diameter is a good compromise.

Second, the calibration should be performed under similar conditions to the measurement of the unknown material. E.g., if materials inside the skull are being measured, the air and water calibration should be performed inside a phantom skull. This is necessary because the presence of bone, or other deviations from water, introduces beam-hardening effects which are not fully corrected in most scanners. The assumption of linearity is still valid in such a case, but the calibration constants will be different.

Investigators who are interested in quantitative analysis should realize that machine numbers generally
contain some experimental error, and that a proper calibration and correction, using Eq. 2, should be performed.

**ENERGY DEPENDENCE**

To examine the energy dependence of \( H \) we must invoke two further assumptions. First we assume that the attenuation coefficient \( \mu \) is measured with the same spectrum \( S(E) \), regardless of the nature of the material being measured. This assumption is obviously true for a water-box scanner, provided only small amounts of material other than water are present. The effective spectrum is the exit spectrum of the x-ray beam multiplied by the spectral response of the detector. (It is an important fact of x-ray theory that the additional attenuation produced by a small amount of material in a beam is determined by the exit spectrum, regardless of where the material is located.)

If large amounts of other materials are present, such as bone or air, or if a non-water-box scanner is used, beam-hardening effects will alter the exit spectra. Therefore the measuring spectrum will be different for different points. Even for a given point, different rays have different spectra, so that the effective spectrum is some kind of average over the rays through the point. In this case, our analysis will be valid to the extent that the average spectrum is the same for all materials.

Corrections for beam-hardening do not hurt the assumption. These corrections are generally of two types. One type is a compensating "wedge" that tends to equalize all ray-paths directly. Another type is linearization of the raw data, which helps compensate for the differences in spectra. In either case the effective spectra for different rays are partly equalized, and so the assumption of a constant spectrum is strengthened.

Now consider the monochromatic attenuation coefficient \( \mu(E) \). Its energy dependence may be simplified if we write

\[
\mu(E) = \mu_c(E) + \mu_p(E),
\]

where \( \mu_c(E) \) is the Klein-Nishina function for Compton scattering, multiplied by the electron density, and \( \mu_p(E) \) represents photoelectric absorption, and also includes coherent scattering and binding energy corrections.\(^7\)\(^8\) Now the energy dependence of \( \mu_c(E) \) is the same for all elements, by definition. Our second assumption is that this is also true for \( \mu_p(E) \). This is illustrated in Fig. 1, where we see that the photoelectric cross-section follows an \( E^{-2} \) dependence for a number of common elements. The fact that photoelectric attenuation follows a common energy dependence was recognized at least as early as 1937, and has been applied to computed tomography by Alvarez and Macovski\(^2\) and Dubal and Wiggli.\(^4\) While the explicit energy function is not needed in our analysis, we note in passing that our value of -2.8 for the exponent differs somewhat from earlier determinations.

If we normalize \( \mu_c(E) \) and \( \mu_p(E) \) to water, the energy dependence drops out and we are left with ratios that depend only on the material. Following Mayneord,\(^9\) we can relate these normalized coefficients to the electron density \( \rho_e \) and the effective atomic number \( Z^* \) by

\[
\frac{\mu_c}{\mu_{cw}} = \frac{\rho_e}{\rho_{cw}},
\]

\[
\frac{\mu_p}{\mu_{pw}} = \left( \frac{\rho_e}{\rho_{pw}} \right) \left( \frac{Z^*}{Z_{cw}^*} \right)^{3.1},
\]

where the subscript \( w \) refers to water. We have dropped the argument \( E \) from the above ratios to emphasize that they are independent of energy.

For a single element, \( Z^* \) is the actual atomic number \( Z \). For a mixture containing \( n_1 \) atoms/cm\(^3\) of element 1, \( n_2 \) atoms/cm\(^3\) of element 2, etc., \( Z^* \) is

\[
Z^* = 3.1 \sqrt{(n_1 Z_1^* + n_2 Z_2^* + \ldots)/\rho_e}
\]

In the following analysis it will be convenient to use Hounsfield notation for the Compton and photoelectric coefficients. Thus we define \( H_c \) and \( H_p \) so that

\[
\frac{\mu_c}{\mu_{cw}} = 1 + \frac{H_c}{1000}
\]

\[
\frac{\mu_p}{\mu_{pw}} = 1 + \frac{H_p}{1000}
\]

In other words, \( H_c \) and \( H_p \) describe the Compton and photoelectric coefficients exactly the way \( H \) describes total attenuation, except they are not energy dependent.

\( H_c \) and \( H_p \) may easily be calculated from tabulated data on Compton and photoelectric cross-sections, remembering that "photoelectric" also includes coherent scattering and binding energy corrections. One merely divides the attenuation coefficient for the desired material by that of water, using any convenient energy, and then converts to Hounsfield notation using Eq. 6.

By integrating Eq. 3 over the detected energy spectrum \( S(E) \) and converting to Hounsfield notation, we obtain

\[
H = \left( H_c + \frac{H_p}{Q} \right)/(1 + Q).
\]

Eq. 7 shows in a very simple manner how the Compton and photoelectric components add together to give the total measured attenuation. \( Q \) is the single parameter of the scanner which determines what the Hounsfield number of a given material will be. It is analogous to effective energy, in that it describes the x-ray spectrum used for the measurement, but it does not depend on the material being measured. \( Q \) is defined mathematically in the Appendix. Physically, it is the ratio of the photoelectric and Compton components of the attenuation of water, as measured with the spectrum \( S(E) \). Since it characterizes the beam spectrum, or "quality", \( Q \) might be called the CT quality factor (not to be confused with the quality factor used in radiation therapy). For the EMI Mark I scanner, \( Q = 5-15\% \), depending on tube voltage and amount of bone present.

**MEASUREMENT OF QUALITY FACTOR**

The simplest way to determine \( Q \) is by scanning a known sample - that is, one for which the quantities \( H_c \) and \( H_p \) may be calculated from tabulated data. It
is preferable to choose a sample with significant photoelectric absorption, so that the $Q$-dependence of Eq. 7 will be strong. After calculating the true Hounsfield number $H$ for the sample, one may obtain $Q$ by solving Eq. 7 as follows:

$$Q = \frac{(H - H_{c})}{(H_{p} - H)}. \quad (8)$$

APPLICATION TO DUAL-ENERGY SCANNING

We now indicate how Eq. 7 can be used to simplify the analysis of dual-energy data. This discussion is applicable both to scans made at two x-ray voltages, e.g., 100 kVp and 140 kVp, and to scans made with a split-detector. First, at each energy, air and water readings must be obtained to calibrate the Hounsfield scale (Eq. 2). Second, the quality factor must be measured at each energy (Eq. 8).

Now consider an unknown substance for which we want to determine the Compton and photoelectric coefficients, $H_{c}$ and $H_{p}$. If scans are made at two energies, we may express the resulting Hounsfield numbers as

$$H_{1} = \frac{(H_{c} + H_{p}Q_{1})}{(1 + Q_{1})},$$

$$H_{2} = \frac{(H_{c} + H_{p}Q_{2})}{(1 + Q_{2})}, \quad (9)$$

where subscript 1 refers to the high energy and 2 refers to the low energy. These equations may be solved for $H_{c}$ and $H_{p}$ to yield

$$H_{c} = H_{1} - (H_{2} - H_{1})Q_{1}/(Q_{2} - Q_{1}),$$

$$H_{p} = H_{2} - (H_{2} - H_{1})(Q_{2} - Q_{1})/(Q_{2} - Q_{1}). \quad (10)$$

Eq. 10 may be applied to every image point, so that complete Compton and photoelectric images are obtained, or a single calculation may be performed for an area of interest, using average CT numbers for the entire area.

If desired, one may go further and calculate electron density and effective atomic number, using Eqs. 6 and 4. In fact $H_{c}$ is the electron density, expressed in Hounsfield notation, and $H_{p}$ is a simple function of effective atomic number. While previous dual-energy articles have emphasized the importance of effective atomic number, the present author feels there are some advantages to staying with the Hounsfield notation: $H_{c}$ and $H_{p}$. Either set of parameters contains the same information, and the choice rests with the reader.

Another possibility is to perform chemical identification. Each element is characterized by a particular ratio of photoelectric and Compton coefficients. If we assume that an elevated Hounsfield number arises because only one chemical element has been added to soft tissue (or water), it is possible to determine the element and its concentration. For example, this method may be used to distinguish between iodine and calcium, and to measure the amount present. Going further, by assuming that a volume element contains two known elements, one may determine both concentrations.

There are two additional points to be made about chemical identification. First, there is no point in taking measurements at more than two energies, as has been proposed recently, since there are only two physical parameters to be determined. Second, the possibility of chemical identification is based on the assumption of "tight packing," i.e., that the densities involved are consistent with those in a normal solution that fills the entire volume element. Theoretically there is a mathematical solution for any chemical element added to water that will produce the measured values $H_{c}$ and $H_{p}$ (two equations - two unknowns). However, this generally will involve densities which either underfill or overfill the volume element. There is only one mathematical "solution" which corresponds to a physical space-filling "solution".

NOISE ANALYSIS

Equations (10) are simple linear equations, so that a noise analysis is easy to perform. If $Q_{1}$ and $Q_{2}$ are the uncertainties (standard deviations) in $H_{1}$ and $H_{2}$, then the uncertainties in $H_{c}$ and $H_{p}$ are given by

$$Q_{c} = \sqrt{\frac{2}{Q_{1} + Q_{2}} (Q_{1} + 2Q_{2}) \frac{Q_{1} + Q_{2}}{Q_{2} - Q_{1}}},$$

$$Q_{p} = \sqrt{\frac{2}{Q_{1} + Q_{2}} (Q_{1} + 2Q_{2}) \frac{Q_{2} - Q_{1}}{Q_{2} - Q_{1}}} \quad (11)$$

In order to obtain a numerical illustration of these uncertainties, we take $Q_{1} = 12$% and $Q_{2} = 12$, as found for an EMI Mark I scanner, and assume that $Q_{1} = Q_{2} = 1 H$, a reasonable estimate for data averaged over an area of about 200 sq. mm. This results in $Q_{c} = 4 H$ and $Q_{p} = 37 H$.

The reason for the increased noise in the Compton and photoelectric coefficients may be seen by studying Eq. 10. In the limit $Q_{1} \rightarrow 0$ (not far from practice), $H_{c} \rightarrow H_{1}$, in the limit $Q_{2} \rightarrow \infty$ (quite far from practice), $H_{p} \rightarrow H_{2}$. Thus, in these limits there would be no noise increase. In practice, however, the quantity $H_{1} - H_{2}$ must be applied to each term as a correction, particularly to $H_{p}$. This correction term is fairly noisy, being the difference of two similar quantities and in the case of $H_{p}$, dominates the noise.

In other words, while the computation of Compton and photoelectric coefficients is useful for purposes of chemical identification, it will not help to reveal a lesion which is barely distinguishable. For that purpose, a simple display of the two original scans $H_{1}$ and $H_{2}$, and possibly $H_{1} - H_{2}$, is best.

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