Published in final edited form as:
Stat Med. 2006 March 15; 25(5): 825–839.

Likelihood based tests for spatial randomness

Changhong Song[1,*] and **Martin Kulldorff**[2,†]
1*Department of Statistics, University of Connecticut, Storrs, CT, 06269, U.S.A.*

2*Department of Ambulatory Care and Prevention, Harvard Medical School and Harvard Pilgrim Health Care, 133 Brookline Avenue, 6th Floor, Boston, MA 02215, U.S.A.‡*

SUMMARY

Many different methods have been proposed to test the spatial randomness of a point pattern adjusting for an inhomogeneous background population. These tests can be classified into cluster detection tests, concerned with the detection and inference of local clusters, and global clustering tests, which collect evidence for clustering throughout the study region. This paper is mainly concerned about global clustering tests.

Some tests for spatial randomness are based on likelihoods, including the spatial and space-time scan statistics with variable window size and Gangnon and Clayton's weighted average likelihood ratio tests. Both of these tests perform well compared to other tests for cluster detection and global clustering respectively. In this study, we develop other likelihood based tests for global clustering and we explore the use of different weight functions with these tests. The power of these tests is evaluated using simulated data set and compared with existing methods.

Keywords

Likelihood ratio; Weight; Power; Disease clustering; Spatial statistics; Benchmark data

1 INTRODUCTION

Tests for spatial randomness are widely used in many areas of applications including agriculture [1], botany [2], ecology [3], geography [4], sociology [5] and public health [6,7]. In most situations, the interest is to detect whether the spatial distribution of the data is random or not after adjusting for some known spatial inhomogeneity. For example, when evaluating the spatial distribution of cancer incidence, it is necessary to adjust for the uneven spatial population density since there will obviously be more cancer cases in a one square mile area in the city than on the country side, simply because there are many more people living there.

Many different tests for spatial randomness have been proposed. When we are using these test statistics, it is important for us to know if they have good statistical power. The power of a test statistic depends on the type of clustering model generated under alternate hypothesis. There are two common kinds of clustering models used: localized clusters and global clustering [12,8]. For localized clusters, the relative risk of the disease at some location and its surrounding area is larger than the risk of other areas. For global clustering, there may be a lot of locations with disease hazard rate larger than other areas. Reasons for a localized cluster may be due to a geographical feature, such as a toxic waste site or a power plant. Reasons for global clustering

*Correspondence to: Changhong Song, Department of Statistics, University of Connecticut, 215 glenbrook road, U-4120, Storrs, CT, 06269, USA. Email: changhon@stat.uconn.edu.
†Email: martin kulldorff@hms.harvard.edu

can be that the disease is infectious or there are a lot of health hazards scattered in the study region, such as gas stations and hazardous waste sites.

Tests for spatial randomness can also be generalized into two categories: cluster detection tests and global clustering tests. The former is concerned with local clusters (or hot spot clusters), and is applied when we are interested in detecting the location of clusters and testing their statistical significance. The latter collects evidence of clustering throughout the entire region without evaluating the statistical significance of individual clusters. Examples of cluster detection test include Turnbull's CEPP [9] and the spatial scan statistic [10]. Examples of global clustering tests are Whittemore's Test [11], Besag-Newell's *R* [12], Cuzick-Edwards' [13] *k*-NN, Tango's Maximized Excess Events Test (MEET) [14], Swartz Entropy test [15] and Gangnon and Clayton's weighted average likelihood ratio tests [16]. This paper is mainly concerned with global clustering tests.

Tests based on likelihoods have been developed for both cluster detection and global clustering evaluation, including the spatial and space-time scan statistics with variable window size and Gangnon and Clayton's weighted average likelihood ratio tests. The spatial scan statistic uses Poisson or Bernoulli likelihood ratio and takes the test statistic as the maximum likelihood ratio over a large set of circular areas. Based on the Poisson likelihood ratio, Gangnon and Clayton proposed a weighted average likelihood ratio test (WALR)[16], as well as two likelihood based cluster detection tests, one of which is a penalized scan statistic that maximizes a weight adjusted likelihood rather than the likelihood itself [17].

For this paper, we develop a large set of likelihood based tests for global clustering, using different structural forms and different weight functions. The power is estimated for all test statistics and compared with the power of the spatial scan statistic, Gangnon and Clayton's *WALR* tests and Tango's MEET. In general, weighted likelihood tests have better power for local hot-spot clusters while weighted log likelihood tests have better power for global clustering models.

2 Likelihood Based Test Statistics

Spatial public health data is usually only available at the level of some aggregated geographical units, such as census tracts, zip-codes or counties. We will without loss of generality use 'county' to denote the geographical unit throughout this paper. The location of each county is represented by a single lat/long location, which could be its geographical or population weighted centroid.

2.1 Notation

First we label all the counties in the data with $i = 1, \ldots, I$, where I denotes the total number of counties. For county i, we denote c_i as the number of cases observed in that county, n_i as its population size, and a_i as its geographical area size. The distance between the centroids of counties i and j is denoted as $d_{i,j}$. When we order all counties by their distance from county i, we use the notation (j), $j = 0, \ldots, I - 1$ to denote the jth closest county to county i. Here $(j = 0)$ is county i itself since the distance is 0, $(j = 1)$ is county i's nearest neighbor, and so on. We denote $d_{i,(j)}$ as the distance between county i and its jth nearest neighbor, $c_{i,(j)}$ as the number of cases within the area of county i and its j nearest neighbor counties, and $n_{i,(j)}$ as the population size in county i and its j nearest neighbor counties. Next we denote $C = \sum_i c_i$ as the total number of cases, $N = \sum_i n_i$ as the total population size and $A = \sum_i ai$ as the total area of all counties. Last, we define J_i as the maximum cluster size for clusters centered on county i, defined in terms of the maximum number of neighboring counties that can be included. In this paper we use $J_i(p) = max\{(j) : n_{i,(j)} \le p{*}N\}$, where $0 < p < 1$ is a fixed constant defining a maximum circle size as a percent of the total population.

2.2 Structural form

Likelihood ratio based tests in this paper can be generalized into twelve different structural forms. The first one is a scan statistic defined as

$$LR_{mm} = \max_i \max_{j \le \mathcal{J}_i(p)} LR_{i,(j)} \tag{2.2.1}$$

where $LR_{i,(j)}$ shorthand for the likelihood ratio defined in the next section for the circle centered around county i and containing the j closest neighbors. The second form is

$$LR_{wm} = \sum_i w_i \max_{j \le \mathcal{J}_i(p)} LR_{i,(j)}. \tag{2.2.2}$$

The third function is defined as

$$LR_{ww} = \sum_i \sum_{j \le \mathcal{J}_i(p)} w_{i,(j)} LR_{i,(j)} \tag{2.2.3}$$

The next three forms are the same with the addition of an indicator function so that we only count the likelihood ratio if there are more cases than expected under the null hypothesis of no clustering. These are denoted by LR_{mm}^h, LR_{wm}^h and LR_{ww}^h respectively.

The remaining six structural forms are obtained by replacing the likelihood ration (LR) with the log likelihood ratio (LLR), denoting those tests as LLR_{mm}, LLR_{wm}, LLR_{ww}, LLR_{mm}^h, LLR_{wm}^h and LLR_{ww}^h respectively. We are using the subscripts *mm*, *wm* and *ww* to denote different combinations of maximizations and weighted sums.

2.3 Likelihoods

For each county i and its j closest neighbors, the Poisson based likelihood ratio for that potential cluster is

$$LR_{i,(j)} = \left(\frac{c_{i,(j)}/n_{i,(j)}}{C/N}\right)^{c_{i,(j)}} \left(\frac{(C-c_{i,(j)})/(N-n_{i,(j)})}{C/N}\right)^{C-c_{i,(j)}}$$

When only using the circles with more cases than expected, we have

$$LR_{i,(j)}^h = LR_{i,(j)} I(c_{i,(j)}/n_{i,(j)} > C/N)$$

The log likelihood ratios are $LLR_{i,(j)} = log(LR_{i,(j)})$ and $LLR_{i,(j)}^h = LLR_{i,(j)} I(c_{i,(j)}/n_{i,(j)} > C/N)$ respectively.

2.4 Weight functions

Nine different weight functions are used in this paper. The first is always identical to one, three depend only on the centroid of county i while five depend on both the centroid of county i and the number of neighbors j.

The first weight function is just the raw summation of likelihood ratios or log likelihood ratios, denoted as

$$w_{i,(j)}(ONE) = 1 \,\forall\, i, j$$

While we do not expect this weight to perform very well, we include it as a baseline comparison.

Gangnon and Clayton [16] proposed the following county based (CB) weight function for one of their likelihood based test statistics

$$w_{i,(j)}(CB) = \frac{1}{1 * \mathcal{J}_i}.$$

Like the one above, the next two weight functions also depend only on the centroid of county *i*. We call one the area based weight (AB) and the other the population based weight (PB). They are defined as

$$w_i(AB) = \frac{a_i}{A} \tag{2.2.4}$$

and

$$w_i(PB) = \frac{n_i}{N} \tag{2.2.5}$$

respectively.

The next weight function is called the distance based exponential weight (DE). Tango proposed this weight function for his Maximized Excess Events Test (MEET) [14] and it is defined as

$$w_{i,(j)}(DE) = e^{-4\left(\frac{d_{i,(j)}}{\lambda}\right)^2}, \tag{2.2.6}$$

where λ is a parameter reflecting the spatial scale of clustering. Usually, λ is set to range from a small value to half of the largest distance between two counties.

As an alternative to the distance based exponential weight, we have proposed a population adjusted exponential weight function (PE) for Tango's MEET [18]. This can also be used for likelihood based tests, and is denoted as

$$w_{i,(j)}(PE) = e^{-4\left(\frac{d_{i,(j)}}{\lambda_i}\right)^2}, \tag{2.2.7}$$

where $\lambda_i = d_{i,(t_i)}, t_i = max\{j : n_{i,(j)} \leq k\}$, and *k* is a parameter set by the user and can be viewed as a population measure for the scale of the clustering. *k* can be set to range from a small number such as 1% or 0.1% to 50% of the overall population. In this study, we set *k* to range from 5% to 50% of the overall population, because some counties have a population size over 1% of the overall population.

Based on nearest neighbor property, we can define another weight function as

$$w_{i,(j)}(NN) = \left(\frac{1}{(j)+1}\right)^s, \tag{2.2.8}$$

where *s* > 0 is a spatial scal parameter set by the user. In our calculation, we set the parameter *s* to range from 0.1 to 8. We call this weight function the nearest neighbor based weight (NN).

Gangnon and Clayton [16] proposed a distance adjusted area weight (DA) for their other likelihood based test statistic. This is defined as

$$w_{i,(j)}(DA) = \frac{a_i}{A}\frac{d_{i,(j+1)} - d_{i,(j)}}{d_{i,(\mathcal{J}_i(p)+1)}} \quad (2.2.9)$$

The above weight function can be modified to be based on the county population size rather than geographical area size. We name it distance adjusted population weight (DP) and define it as

$$w_{i,(j)}(DP) = \frac{n_i}{N}\frac{d_{i,(j+1)} - d_{i,(j)}}{d_{i,(\mathcal{J}_i(p)+1)}} \quad (2.2.10)$$

2.5 Maximization over weight function parameters

Some of the weight functions defined above depends on a spatial scale parameter λ, k or s, and each choice of that parameter leads to a different test statistic. A combined maximized test can be constructed that evaluates a large set of parameter values, adjusting the test for the multiple testing inherent in the many parameter values considered. Proposed by Tango for a different type of test for spatial randomness, the minimum p-value over the parameters is first calculated, and this minimum p-value is used as the test statistics, whos p-value is calculated by Monte Carlo simulations. Details of this maximization have been provided by Tango [14]. The maximized versions of the test statistics are denoted as $MLR_{ww}(XX)$ and $MLLR_{ww}(XX)$ respectively.

2.6 Maximum number of neighbors

The last parameter J_i in the formulas in section 2.2 is the number of neighbors to maximize over or to do the summation over. This parameter can be set in different ways, such that the circle includes a fixed number of counties, a fixed geographical area size or a fixed population size. In this study we chose the latter, so that In this paper we use $J_i(p) = max\{(j) : n_{i,(j)} \leq p * N\}$, where $0 < p < 1$ is a fixed constant defining a maximum circle size as a percent of the total population. We will use two different values of p, so that either a maximum of 10 or 50 percent of the population is included. We denote these tests as $LR_{xx}(XX, 10)$ and $LR_{xx}(XX, 50)$ respectively for the likelihood ratio based test statistics and equivalently for the log likelihood ratio and maximized versions.

2.7 Existing test statistics

Most of the test statistics evaluated in this paper are new, but a few of them have been proposed before. The spatial scan statistic [10] is defined by LR_{mm}. Note that since this test takes the maximum likelihood ratio over both the j neighbors and centroids i, it does not matter whether one uses the likelihood ratio or the log likelihood ratio. While the values of the test statistic are different, the p-values are always identical. Two of the other likelihood based test statistics were proposed by Gangnon and Clayton [16], $LR_{ww}(CB)$ and $LR_{ww}(DA)$ respectively. These two tests, as well as the weights proposed by Tango [14] for his excess events test, served as the inspiration for many of the new tests statistics evaluated in this paper.

3 Tango's Maximized Excess Events Test (*MEET*)

Compared to other tests for global spatial clustering, Tango's MEET has in other studies been shown to have very good power [8,19]. Hence, we will for comparison purposes include this test as well, even though it is not based on a likelihood formulation.

For some weight function $w_{i,j}$, Tango's Excess Events Test (EET) is a weighted sum of excess events defined as

$$EET = \sum_i \sum_j w_{i,j}(c_i - n_i \frac{C}{N})(c_j - n_j \frac{C}{N}). \tag{3.1}$$

Tango proposed two weight functions, $w_{i,j}(DE) = e^{-4(\frac{d_{i,j}}{\lambda})^2}$ [14] and $w_{i,j} = e^{-\frac{d_{i,j}}{\lambda}}$ [20], where λ is a measure of the spatial scale of clustering. These two weight functions have similar power [18], so we only include results for the former weight function in this paper.

Tango's *EET* depends on a scale parameter λ. To be able to detect clustering irrespectively of its geographical scale, Tango [14] proposed the maximized excess events test (*MEET*). With the DE weight it is defined as

$$MEET(DE) = \min_{0 \le \lambda \le U} P(EET(DE, \lambda) > eet(DE, \lambda) \mid H_0, \lambda), \tag{3.2}$$

where $eet(DE, \lambda)$ is the observed values of $EET(DE, \lambda)$ conditioning on λ and U is an upper limit on λ. Basically, the maximized test is using the minimum of the profile p-values as the test statistics adjusting for the multiple testing resulting from the many parameter values considered.

4 Power evaluation

4.1 Benchmark data

A simulated benchmark data set is used to evaluate the statistical power of the different test statistics. This data set is based on the 1990 female population in the 245 counties and county equivalents in the northeastern United States, consisting of the states of Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New York, New Jersey, Pennsylvania, Delaware, Maryland and the District of Columbia. The total population was 29,535,210. The smallest size population is 2,319 in Forest County, Pennsylvania, and the largest size population is 1,231,176 in Kings County (Brooklyn), New York. The benchmark data has been described in detail elsewhere [8], and here we just give a brief description. The benchmark data sets can be downloaded at 'http://satscan.org/datasets/'.

First, 99,999 random data sets were generated under the null hypothesis by randomly and independently allocating 600 cases to counties with probabilities proportional to their population size. The null data is used to estimate the critical values, which is the cut-off point for the significance at the 0.05 level. For each of 26 alternative clustering model, 10,000 random data sets were generated to estimate the power. Two types of clustering models were used, localized hot spot clusters and global chain clustering.

For the global chain clustering process, the counties are tied together sequentially on a chain that passes through each county exactly once, after which it reconnects with the first county on the chain, forming a Hamiltonian cycle, as shown on a map in an earlier publication [8]. Spatial clustering is generated by first randomly locating 300 cases on the map according to the null hypothesis. Each of these original cases then generates one additional 'twin' case. The first kind of global clustering model is constructed by assigning the twins to the same county, so that they are a distance zero apart. In the second type of global clustering the twin may be assigned to a neighboring county along the chain, with the county selected so that there is a fixed population adjusted distance between the twins, with 0.5%, 1%, 2%, 4% or 8% of the total population between them. For the third type of global clustering models, the distance was instead selected at random according to the exponential distribution with mean 0.5%, 1%, 2%, 4% and 8% respectively. The chain does not imply that the disease itself spreads around the chain, just that twin cases are located in either of the two directions, as defined by the chain.

Hot spot clusters were generated by setting the relative risk in some counties to be larger than 1. Three different sets of hot spot clusters are constructed in a rural, urban and mixed area respectively. In each area, there are five different sized clusters with 1, 2, 4, 8 and 16 counties respectively for each of these three sets. The center of the rural cluster is Grand Isle County in Vermont. The center of the mixed cluster is Allegheny County (Pittsburgh) in Pennsylvania. The center of the urban cluster is New York County (Manhattan) in New York. Detailed information about the hot spot clusters are provided in Table 1.

4.2 Power for parameter dependent tests

Three weight functions *DE*, *PE* and *NN* depend on a parameter. Tables 2–5 show the estimated power of these tests for different parameter values. For each test and clustering model, the highest power among the parameter values is noted in bold face. The average power of each test statistic is reported in the last column of the table. For comparison, the power of the maximized test with the same weight function is reported in the last row for each test. The maximized versions perform well in all cases, with the average power falling at most slightly below the best performing parameter specific test and in many cases considerably better than the worst performing test in the same calss.

Table 2 shows estimated power of the three weighted likelihood ratio tests for the global chain clustering models. When the distance between cases increase, which means the spatial scale of clustering is larger, $LR_{ww}(DE, 50, \lambda)$ has higher power for larger values of λ and vice versa. For $LR_{ww}(NN, 50, s)$, the same is true for smaller values of *s*. This is not surprising since a small *s* gives more weight to the counties far away. For $LR_{ww}(PE, 50, k)$ a large *k* is always better, except for the zero distance clustering model.

Table 3 shows the power of the three weighted log likelihood ratio tests for global chain clustering models. For $LLR_{ww}(DE, 50, \lambda)$ and $LLR_{ww}(PE, 50, k)$, a small parameter value is better for small scale clustering while a large parameter is better for large scale clustering. For $LLR_{ww}(NN, 50, s)$, the trend is reversed.

Table 4 illustrates the power of weighted likelihood ratio tests for hot spot clusters. When the clustering models have more counties and a larger population, $LR_{ww}(DE, 50, \lambda)$ and $LR_{ww}(NN, 50, s)$ show similar trends as in Table 1, with a larger *lambda* and a smaller *s* providing higher power for large clusters. $LR_{ww}(PE, 50, k)$ also shows a similar pattern, which indicates that a large parameter value is good for large clusters and a small parameter is good for small clusters.

Table 5 illustrates the power of weighted log likelihood tests for hot spot clusters. These tests have with some excpetions lower power than the equivalent likelihood ratio based tests in Table 4. The $LLR_{ww}(DE, 50, \lambda)$ tests are good for detecting urban clusters if the right parameter value is choosen. $LLR_{ww}(PE, 50, k)$ has good power for rural clusters when *k* is small, for mixed clusters when *k* is medium sized and for the very largest urban cluster when *k* is large. For $LLR_{ww}(NN, 50, s)$, a small *s* is good for clusters with more counties and a large *s* is good for clusters with less counties.

4.3 Power for parameter independent tests

Table 6 shows estimated power of the parameter independent tests for the global chain clustering. To make comparison easier, the highest power for each clustering model is highlighted. The best performing likelihood based tests were $MLLR_{ww}(DE, 50)$ and $MLLR_{ww}(NN, 50)$, which have similar power as Tango's MEET(DE), which has previously been shown to have very good power for these alternative models compared to seevral other

global clustering tests [19]. Several of the other likelihood based tests also perform well for global chain clustering.

Table 7 shows the estimated power for the hot-spot cluster alternatives. Overall, the best performing test was $LR_{wm}{}^{h}(PB, 50)$, followed by $LR_{wm}{}^{h}(PB, 10)$, $LR_{wm}{}^{h}(ONE, 50)$, $LR_{mm}{}^{h}(50)$ and $MLR_{ww}(DE, 50)$. The first and third of these have higher power for large clusters while the latter two have higher power for small clusters. The log likelihood based tests have in general lower power than the equivalent likelihood based test statistics, with the exception of the scan statistics LR_{mm} and LLR_{mm} for which the two are identical by definition. Tests based on a maximum circle size of 10 percent have slightly higher power for the smaller and medium size hot-spot clusters but much lower power for the two largest urban clusters, compared to the equivalent tests with a 50 percent maximum.

5 Discussion

In this study, we have developed and evaluated several likelihood based tests for spatial randomness. Several of the new tests perform very well, similar to the best performing tests from earlier power comparisons [8,16,19]. By looking at different components of the test statistics, we were in many cases able to determine what component parameters work well for different types of alternative hypotheses.

In two previous studies [8,19] we systematically evaluated eight test statistics using the same benchmark data: Besag-Newell's *R* [12], Bonetti-Pagano's *M* [21], Cuzick-Edwards' *k*-NN [13], Moran's *I* [22], Swartz' Entropy Test [15], Spatial scan statistic, Tango's *MEET* and Whittemore's Test [11]. The power varies greatly for these different test statistics. Tango's *MEET* has the best power for global clustering and the spatial scan statistic has the best power for hot-spot clusters. In this paper we have shown that a couple of likelihood based test statistics have as good power as Tango's *MEET* for global clustering, while several likelihood based tests have about the same power as the spatial scan statistic for hot-spot alternatives.

While several tests have very good power for the various hot-spot cluster alternatives, there is a major advantage of the spatial scan statistic is that we can pinpoint the location causing the rejection of the null hypothesis [10]. This means that the p-value of the test can be assigned to the detected cluster and not only to the study region as a whole. Since the tests based on weighted summations use evidence from all parts of the map to decide whether to reject the null hypothesis or not, no such cluster specific p-values can be assigned from those tests. Since the scan statistics also have among the best power of the tests evaluated, we recommend its use when the interest is in the detection of hot-spot clusters and the evaluation of their statistical significance.

For global clustering, Tango's $MEET(DE)$, $MLLR_{ww}(DE, 50)$ and $MLLR_{ww}(PE, 50)$ all have very good power, and we recommend using either of these if the main interest is to evaluate whether there exist clustering in general throughout the study region. Note that $MLLR_{ww}(DE, 50)$ and $MLLR_{ww}(PE, 50)$ both depend on the maximum population bound of 50%, and the strength of these two tests varies with different choice of the population bound. When the maximum is smaller such as 10%, the power may decrease when the scale of the global clustering is large, but increase if if the scale of clustering is small.

As we know, the strength of the test statistics depend not only on the clustering models, but also on the underlying population. This power evaluation used the female population in 245 counties in Northeastern United States as the underlying population. It is possible that the relative strength of the various test statistics evaluated may be different if a different underlying population is used. The same is true to an even large extent when it comes to the choice of alternative hypotheses, and there are probably some clustering models for which the results

will be very different. Similar studies may be done for more typical geographical regions with different clustering models. The type of power evaluations done in this paper are, in spite of these limitations, very important though. For any practical application, a test must be chosen, and the power estimates presented in this paper provides some help with that choice. It should be noted though, that as long as one of the better tests is chosen, it may not matter much exactly which one.

References

1. Moran PAP. Random associations on a lattice. Preceedings of the Cambridge Philosophical Society 1947;43:321–328.
2. Nagashima H, Terashima I, Katoh S. Effects of plant-density on frequency-distributions of plant height in cheopodium-album stands. Annals of Botany 1995;75:173–180.
3. Meirmans PG, Vlot EC, Den Nijs JCM, Menken SBJ. Spatial ecological and genetic structure of a mixed population of sexual diploid and apomictic triploid dandelions. Journal of Evolutionary Biology 2003;16:343–352. [PubMed: 14635873]
4. Fotheringham AS, Charlton M, Brunsdon C. The geography of parameter space: an investigation of spatial non-stationarity. International Journal of Geographical Information Systems 1996;10:605–627.
5. Cortese CF, Falk RF, Cohen JK. Further considerations on the methodological analysis of segregation indices. American Sociological Review 1976;41:630–637.
6. Nkhoma ET, Hsu CE , Hunt VI, Harris AM. **Detecting spatiotemporal clusters of accidental poisoning mortality among Texas counties, U.S., 1980 2001***International Journal of Health Geographics* 2004; **3**: 25.
7. Klassen AC, Kulldorff M, Curriero F. Research Geographical clustering of prostate cancer grade and stage at diagnosis, before and after adjustment for risk factors. International Journal of Health Geographics 2005;4:1. [PubMed: 15649329]
8. Kulldorff M, Tango T, Park P. Power comparisons for disease clustering tests. Computational Statistics and Data Analysis 2003;42:665–684.
9. Turnbull BW, Iwano EJ, Burnett WS, Howe HL, Clark LC. Monitoring for clusters of disease: application to leukemia incidence in upstate New York. American Journal of Epidemiology 1990;132 (1):S136–S143. [PubMed: 2356825]
10. Kulldorff M. A spatial scan statistic. Communications in Statistics: Theory and Methods 1997;26:1481–1496.
11. Whittemore AS, Friend N, Brown BW, Holly EA. A test to detect clusters of disease. Biometrika 1987;74:631–635.
12. Besag J, Newell J. The detection of clusters in rare diseases. Journal of the Royal Statistical Society 1991;A154:143–155.
13. Cuzick J, Edwards R. Spatial clustering for inhomogeneous populations. Journal of the Royal Statistical Society 1990;m2:73–104.
14. Tango T. A test for spatial disease clustering adjusted for multiple testing. Statistics in Medicine 2000;19:191–204. [PubMed: 10641024]
15. Swartz JB. An entropy-based algorithm for detecting clusters of cases and controls and its comparison with a method using nearest neighbors. Health and Place 1998;4:67–77. [PubMed: 10671012]
16. Gangnon RE, Clayton MK. A weighted average likelihood ratio test for spatial clustering of disease. Statistics in Medicine 2001;20:2977–2987. [PubMed: 11568953]
17. Gangnon RE, Clayton MK. Likelihood-based tests for localized spatial clustering of disease. Environmetrics 2004;15:797–810.
18. Song C, Kulldorff M. **Tango's maximized excess events test with different weights** Under review.
19. Song C, Kulldorff M. Power evaluation of disease clustering tests. International Journal of Health Geographics 2003;2:9. [PubMed: 14687424]
20. Tango T. A class of tests for detecting 'general' and 'focused' clustering of rare diseases. Statistics in Medicine 1995;14:2323–2334. [PubMed: 8711272]

21. Bonetti M, Pagano M. The interpoint distance distribution as a descriptor of point patterns, with an application to spatial disease clustering. Statistics in Medicine 2005;24:753–773. [PubMed: 15523703]

22. Moran PAP. Notes on continuous stochastic phenomena. Biometrika 1950;37:17–23. [PubMed: 15420245]

Table 1

Description of the hot-spot alternative cluster models, with the counties included, population size, the expected number of cases under null and alternate hypotheses respecctcively and the relative risk (RR).

	Counties	Counties included	Population	$E[c \mid H_0]$	$E[c \mid H_A]$	RR
Rural clusters	1	Grand Isle,VT	2,675	0.05	10	192.89
	2	above + Franklin, VT	22,911	0.46	12	27.03
	4	above + Clinton, NY, Chittenden, VT	132,343	2.69	18	7.05
	8	above + Lamoille, VT, Washington, VT, Essex, NY, Addison, VT	204,829	4.16	22	5.35
	16	above + Orleans, VT, Franklin, NY, Caledonia, VT, Orange, VT, Essex, VT, Rutland, VT, Warren, NY, Windsor, VT	360,275	7.32	28	3.90
Mixed clusters	1	Allegheny, PA	710,196	14.43	39	2.85
	2	above + Washington, PA	817,050	16.41	42	2.70
	4	above + Beaver, PA, Westmoreland, PA	1,108,440	22.52	51	2.40
	8	above + Butler, PA, Armstrong, PA, Lawrence, PA, Fayette, PA	1,352,284	27.47	58	2.24
	16	above + Greene, PA, Indiana, PA, Clarion, PA, Mercer, PA, Somerset, PA, Venango, PA, Cambria, PA, Jefferson, PA	1,684,327	34.22	67	2.10
Urban clusters	1	New York, NY	786,178	15.97	42	2.73
	2	above + Hudson, NJ	1,072,181	21.78	50	2.43
	4	above + Bronx, NY, Kings, NY	2,953,077	59.99	100	1.81
	8	above + Queens, NY, Bergen, NJ, Essex, NJ, Richmond, NY	5,018,909	101.96	150	1.63
	16	above + Union, NJ, Nassau, NY, Passaic, NJ, Rockland, NY, Westchester, NY, Morris, NJ, Middlesex, NJ, Monmouth, NJ	7,627,173	154.94	209	1.53

NIH-PA Author Manuscript
NIH-PA Author Manuscript

Table 2

Estimated power of the parameter based weighted likelihood ratio for the global chain clustering.

		Fixed distance					Exponential distance					
	0.00	**0.5%**	**1%**	**2%**	**4%**	**8%**	**0.5%**	**1%**	**2%**	**4%**	**8%**	**average**
$LR_{ww}(DE, 50, \lambda)$												
λ = 66,000	0.83	0.49	**0.38**	**0.28**	**0.18**	**0.11**	0.54	0.43	**0.33**	**0.24**	**0.16**	**0.36**
λ = 32, 000	0.89	**0.51**	0.37	0.26	0.15	0.10	0.57	**0.45**	**0.33**	0.23	0.14	**0.36**
λ = 23, 000	0.92	**0.51**	0.36	0.24	0.14	0.09	**0.58**	**0.45**	0.32	0.21	0.13	**0.36**
λ = 15, 000	0.93	0.48	0.32	0.19	0.11	0.07	**0.58**	0.42	0.29	0.18	0.12	0.34
λ = 10, 000	0.94	0.47	0.29	0.17	0.09	0.07	0.56	0.40	0.26	0.17	0.10	0.32
λ = 7, 000	0.94	0.40	0.23	0.13	0.07	0.06	0.50	0.35	0.22	0.14	0.09	0.28
λ = 4, 000	0.92	0.33	0.16	0.09	0.06	0.05	0.43	0.28	0.17	0.10	0.07	0.24
λ = 500	**0.95**	0.27	0.10	0.05	0.04	0.05	0.37	0.22	0.13	0.08	0.06	0.21
$MLR_{ww}(DE, 50)$	0.91	0.46	0.32	0.21	0.13	0.09	0.53	0.40	0.29	0.19	0.13	0.33
$LR_{ww}(PE, 50, k)$												
k = 50% *population*	0.90	**0.52**	**0.38**	**0.25**	**0.16**	**0.10**	**0.58**	**0.45**	**0.33**	**0.23**	**0.15**	**0.37**
k = 40% *population*	0.90	0.51	0.37	**0.25**	0.15	0.09	0.57	0.44	0.32	0.22	0.14	0.36
k = 30% *population*	0.91	0.51	0.36	0.23	0.14	0.08	0.57	0.44	0.31	0.21	0.13	0.35
k = 20% *population*	0.93	0.51	0.34	0.21	0.12	0.07	**0.58**	0.41	0.29	0.19	0.12	0.34
k = 15% *population*	0.94	0.49	0.32	0.19	0.10	0.07	0.57	0.39	0.28	0.18	0.11	0.33
k = 10% *population*	0.95	0.46	0.28	0.15	0.08	0.06	0.54	0.39	0.25	0.15	0.10	0.31
k = 5% *population*	**0.97**	0.40	0.21	0.10	0.06	0.06	0.51	0.34	0.20	0.12	0.08	0.28
$MLR_{ww}(PE, 50)$	0.93	0.47	0.33	0.22	0.13	0.09	0.54	0.41	0.29	0.20	0.13	0.34
$LR_{ww}(NN, 50, s)$												
s = 0.1	0.81	0.48	**0.38**	**0.28**	**0.19**	**0.12**	0.52	0.43	**0.33**	**0.24**	**0.16**	0.36
s = 0.25	0.83	0.49	**0.38**	**0.28**	**0.19**	**0.12**	0.54	0.43	**0.33**	**0.24**	**0.16**	0.36
s = 0.5	0.88	**0.50**	**0.38**	0.27	0.17	0.11	0.56	**0.44**	**0.33**	**0.24**	0.15	**0.37**
s = 1	0.93	**0.50**	0.35	0.23	0.14	0.09	0.57	**0.44**	0.31	0.21	0.13	0.36
s = 1.2	0.95	**0.50**	0.34	0.21	0.13	0.08	**0.58**	0.43	0.30	0.20	0.13	0.35
s = 1.4	0.96	0.49	0.32	0.20	0.11	0.08	0.57	0.42	0.29	0.19	0.12	0.34
s = 1.6	0.96	0.47	0.29	0.17	0.10	0.07	0.57	0.40	0.26	0.17	0.11	0.33
s = 1.8	**0.97**	0.44	0.26	0.15	0.09	0.06	0.55	0.38	0.24	0.15	0.09	0.31
s = 2	**0.97**	0.42	0.24	0.13	0.08	0.06	0.53	0.36	0.22	0.13	0.08	0.29
s = 2.4	0.96	0.38	0.19	0.10	0.06	0.05	0.48	0.32	0.19	0.11	0.07	0.27
s = 2.8	0.96	0.34	0.16	0.08	0.05	0.05	0.45	0.29	0.17	0.10	0.06	0.25
s = 4	0.95	0.29	0.12	0.06	0.05	0.05	0.40	0.25	0.14	0.08	0.06	0.22
s = 8	0.95	0.27	0.11	0.05	0.04	0.05	0.38	0.22	0.13	0.08	0.06	0.21
$MLR_{ww}(NN, 50)$	0.91	0.45	0.32	0.22	0.14	0.11	0.51	0.38	0.28	0.20	0.13	0.33

Table 3

Estimated power of the parameter based weighted log likelihood ratio tests for global chain clustering.

		Fixed distance					Exponential distance					
	0.00	**0.5%**	**1%**	**2%**	**4%**	**8%**	**0.5%**	**1%**	**2%**	**4%**	**8%**	**average**
$LLR_{ww}(DE, 50, \lambda)$												
λ = 66, 000	0.55	0.40	0.34	0.27	**0.20**	**0.14**	0.42	0.37	0.30	0.23	0.17	0.31
λ = 32, 000	0.75	0.47	0.38	0.28	**0.20**	0.12	0.52	0.43	0.34	0.25	**0.18**	0.36
λ = 23, 000	0.89	0.52	**0.40**	**0.29**	0.19	0.12	0.60	0.49	0.37	**0.26**	**0.18**	0.39
λ = 15, 000	0.98	**0.56**	**0.40**	0.27	0.17	0.10	0.68	0.53	**0.38**	0.25	0.17	0.41
λ = 10, 000	**1.00**	**0.56**	0.37	0.23	0.15	0.09	0.72	0.54	0.37	0.24	0.16	0.40
λ = 7, 000	**1.00**	0.54	0.33	0.20	0.13	0.09	0.74	0.53	0.35	0.22	0.14	0.39
λ = 4, 000	**1.00**	0.51	0.26	0.15	0.10	0.07	0.78	0.54	0.32	0.19	0.12	0.37
λ = 500	1.00	0.42	0.15	0.07	0.06	0.06	**0.81**	0.50	0.26	0.14	0.08	0.32
$MLLR_{ww}(DE, 50)$	**1.00**	0.55	0.36	0.25	0.17	0.12	0.79	**0.55**	0.37	0.25	0.16	**0.42**
$LLR_{ww}(PE, 50, k)$												
k = 50% *population*	0.76	0.47	**0.39**	**0.29**	**0.19**	**0.12**	0.53	0.44	**0.35**	**0.25**	**0.17**	0.36
k = 40% *population*	0.80	0.48	0.38	0.28	0.18	0.11	0.54	0.44	**0.35**	0.24	**0.17**	0.36
k = 30% *population*	0.85	0.50	0.38	0.27	0.17	0.10	0.56	**0.46**	**0.35**	0.24	0.16	**0.37**
k = 20% *population*	0.92	**0.51**	0.37	0.24	0.14	0.08	0.59	**0.46**	**0.35**	0.23	0.14	**0.37**
k = 15% *population*	0.95	0.50	0.35	0.22	0.12	0.07	0.60	0.45	0.34	0.21	0.13	0.36
k = 10% *population*	0.98	0.48	0.31	0.18	0.10	0.07	**0.61**	0.45	0.30	0.19	0.12	0.34
k = 5% *population*	**1.00**	0.38	0.20	0.10	0.07	0.06	0.57	0.37	0.22	0.14	0.10	0.29
$MLLR_{ww}(PE, 50)$	0.99	0.49	0.36	0.24	0.16	0.10	0.60	0.45	0.33	0.23	0.15	**0.37**
$LLR_{ww}(NN, 50, s)$												
s = 0.1	0.47	0.36	0.31	0.26	**0.20**	**0.14**	0.37	0.33	0.28	0.22	0.17	0.28
s = 0.25	0.51	0.37	0.33	0.26	**0.20**	**0.14**	0.39	0.34	0.29	0.23	0.17	0.30
s = 0.5	0.63	0.41	0.35	0.27	**0.20**	**0.14**	0.44	0.38	0.31	0.24	0.17	0.32
s = 1	0.96	0.51	0.39	**0.29**	**0.20**	0.12	0.62	0.49	0.37	0.26	**0.18**	0.40
s = 1.2	**1.00**	0.56	**0.41**	0.28	0.18	0.12	0.70	0.54	0.40	**0.27**	**0.18**	0.42
s = 1.4	**1.00**	0.61	**0.41**	0.26	0.17	0.11	0.74	0.60	0.42	**0.27**	0.17	0.43
s = 1.6	**1.00**	**0.63**	0.39	0.24	0.15	0.10	0.83	0.63	**0.43**	0.26	0.16	**0.44**
s = 1.8	**1.00**	**0.63**	0.36	0.20	0.12	0.08	0.86	**0.64**	0.42	0.24	0.15	0.43
s = 2	**1.00**	0.61	0.32	0.16	0.10	0.08	**0.87**	**0.64**	0.40	0.22	0.13	0.41
s = 2.4	**1.00**	0.56	0.25	0.12	0.08	0.07	0.86	0.60	0.35	0.19	0.11	0.38
s = 2.8	**1.00**	0.51	0.21	0.10	0.07	0.06	0.85	0.56	0.31	0.17	0.10	0.36
s = 4	**1.00**	0.45	0.17	0.08	0.06	0.06	0.83	0.53	0.28	0.15	0.09	0.33
s = 8	**1.00**	0.42	0.15	0.07	0.06	0.06	0.82	0.50	0.26	0.14	0.09	0.32
$MLLR_{ww}(NN, 50)$	**1.00**	0.57	0.35	0.23	0.17	0.12	0.82	0.58	0.38	0.24	0.16	0.42

Table 4

Estimated power of the parameter based weighted likelihood ratio tests for hot spot clusters.

	Rural clusters					Mixed clusters					Urban clusters					
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	average
$LR_{ww}(DE, 50, \lambda)$																
$\lambda = 66,000$	0.99	0.97	0.94	0.95	**0.95**	0.89	0.90	0.92	**0.94**	0.95	0.79	0.81	0.82	0.91	0.95	0.91
$\lambda = 32,000$	0.99	0.98	**0.96**	**0.96**	**0.95**	0.91	0.92	0.93	**0.94**	**0.96**	0.83	0.85	0.86	0.93	0.96	0.93
$\lambda = 23,000$	0.99	**0.99**	**0.96**	**0.96**	**0.95**	0.92	0.92	**0.94**	**0.94**	0.95	0.86	0.87	0.88	0.94	**0.97**	**0.94**
$\lambda = 15,000$	0.99	**0.99**	**0.96**	**0.96**	0.93	0.93	**0.93**	0.93	**0.94**	0.93	0.89	0.89	0.91	0.95	**0.97**	**0.94**
$\lambda = 7,000$	**1.00**	**0.99**	**0.96**	0.92	0.81	0.93	0.92	0.90	0.87	0.80	0.94	0.94	**0.94**	**0.96**	0.94	0.92
$\lambda = 4,000$	**1.00**	**0.99**	0.93	0.80	0.56	0.94	0.90	0.80	0.69	0.54	**0.96**	**0.95**	**0.94**	0.94	0.84	0.85
$\lambda = 500$	**1.00**	**0.99**	0.90	0.73	0.52	**0.95**	0.91	0.78	0.66	0.51	0.94	0.84	0.57	0.32	0.19	0.72
$MLR_{ww}(DE, 50)$	**1.00**	**0.99**	0.95	0.94	0.93	0.93	0.91	0.91	0.92	0.93	0.93	0.93	0.91	0.94	0.94	**0.94**
$LR_{ww}(PE, 50, k)$																
k = 50% *population*	0.99	0.98	0.96	0.96	0.96	0.92	0.93	0.95	0.96	**0.97**	0.83	0.84	**0.85**	**0.92**	**0.95**	**0.93**
k = 40% *population*	0.99	0.98	0.96	0.96	0.96	0.92	0.93	0.95	0.96	**0.97**	0.83	**0.85**	0.84	0.91	0.93	**0.93**
k = 30% *population*	0.99	**0.99**	0.96	0.97	0.97	0.93	0.93	0.95	0.96	**0.97**	0.84	**0.85**	0.84	0.89	0.89	**0.93**
k = 20% *population*	0.99	**0.99**	0.97	0.97	0.97	0.94	0.94	0.95	**0.97**	**0.97**	0.85	**0.85**	0.81	0.83	0.75	0.92
k = 15% *population*	0.99	**0.99**	0.97	0.97	0.97	0.94	**0.95**	**0.96**	**0.97**	**0.97**	0.84	0.83	0.74	0.71	0.64	0.90
k = 10% *population*	0.99	**0.99**	**0.98**	**0.98**	**0.98**	**0.95**	**0.95**	**0.96**	**0.97**	**0.97**	0.86	0.83	0.68	0.61	0.54	0.88
k = 5% *population*	**1.00**	**0.99**	**0.98**	**0.98**	**0.98**	0.94	0.94	0.93	0.93	0.91	**0.89**	0.83	0.61	0.48	0.37	0.85
$MLR_{ww}(PE, 50)$	0.99	**0.99**	**0.98**	**0.98**	0.97	0.93	0.94	0.95	0.96	**0.97**	0.86	0.83	0.82	0.89	0.93	**0.93**
$LR_{ww}(NN, 50, s)$																
$s = 0.1$	0.99	0.97	0.94	0.94	0.94	0.89	0.90	0.92	0.93	0.95	0.78	0.80	0.82	0.90	**0.94**	0.91
$s = 0.25$	0.99	0.98	0.94	0.95	**0.95**	0.90	0.91	0.92	**0.94**	0.95	0.81	0.82	0.83	0.90	**0.94**	0.91
$s = 0.5$	0.99	0.98	0.95	0.95	**0.95**	0.91	0.92	0.93	**0.94**	**0.96**	0.85	0.85	0.84	**0.91**	**0.94**	0.93
$s = 1$	**1.00**	**0.99**	**0.96**	**0.96**	**0.95**	0.93	0.93	**0.94**	**0.94**	0.95	0.89	0.88	0.86	**0.91**	**0.94**	**0.94**
$s = 1.2$	**1.00**	**0.99**	**0.96**	**0.96**	**0.95**	0.94	**0.94**	**0.94**	**0.94**	0.95	0.90	0.89	**0.87**	**0.91**	0.93	**0.94**
$s = 1.4$	**1.00**	**0.99**	**0.96**	**0.96**	**0.95**	0.94	**0.94**	**0.94**	**0.94**	0.94	0.91	0.90	**0.87**	**0.91**	0.92	**0.94**
$s = 1.6$	**1.00**	**0.99**	**0.96**	**0.96**	0.94	0.94	**0.94**	0.93	**0.94**	0.94	0.92	0.90	**0.87**	0.90	0.91	**0.94**
$s = 1.8$	**1.00**	**0.99**	**0.96**	0.95	0.94	0.94	**0.94**	0.93	0.93	0.92	0.93	**0.91**	0.86	0.89	0.89	0.93
$s = 2$	**1.00**	**0.99**	**0.96**	0.95	0.93	**0.95**	**0.94**	0.92	0.92	0.91	0.93	**0.91**	0.85	0.87	0.86	0.93
$s = 2.4$	**1.00**	**0.99**	**0.96**	0.94	0.91	**0.95**	0.93	0.91	0.90	0.88	**0.94**	**0.91**	0.83	0.84	0.81	0.91
$s = 2.8$	**1.00**	**0.99**	0.95	0.93	0.88	0.94	0.93	0.89	0.87	0.84	**0.94**	**0.91**	0.81	0.80	0.75	0.90
$s = 4$	**1.00**	**0.99**	0.93	0.88	0.78	0.94	0.92	0.85	0.79	0.72	**0.94**	0.90	0.75	0.67	0.55	0.84
$s = 8$	**1.00**	**0.99**	0.90	0.75	0.55	**0.95**	0.91	0.79	0.67	0.52	**0.94**	0.86	0.60	0.37	0.21	0.73
$MLR_{ww}(NN, 50)$	**1.00**	**0.99**	0.95	0.94	0.93	0.93	0.92	0.91	0.92	0.93	0.92	0.88	0.82	0.87	0.91	0.92

Table 5

Estimated power of the parameter based weighted log likelihood ratio tests for hot spot clusters.

	Rural clusters 1	2	4	8	16	Mixed clusters 1	2	4	8	16	Urban clusters 1	2	4	8	16	average
$LLR_{ww}(DE, 50, \lambda)$																
λ = 66, 000	0.12	0.16	0.30	0.37	0.45	0.63	**0.66**	**0.76**	**0.81**	**0.87**	0.30	0.36	0.66	0.85	0.94	0.55
λ = 32, 000	0.19	0.26	0.44	0.52	0.57	**0.64**	**0.66**	**0.76**	0.80	0.86	0.41	0.49	0.77	0.92	0.97	0.62
λ = 23, 000	0.25	0.34	0.52	0.59	**0.60**	0.63	0.65	0.74	0.77	0.83	0.51	0.59	0.84	0.95	0.98	0.65
λ = 15, 000	0.36	0.47	0.55	**0.60**	0.57	0.58	0.58	0.65	0.68	0.73	0.66	0.73	0.91	0.98	**0.99**	0.67
λ = 7, 000	0.68	0.67	0.43	0.40	0.32	0.32	0.33	0.33	0.34	0.35	0.90	0.92	**0.97**	**0.99**	0.98	0.60
λ = 4, 000	0.85	0.68	0.41	0.38	0.35	0.29	0.29	0.28	0.27	0.27	**0.94**	**0.96**	**0.97**	0.98	0.93	0.59
λ = 500	**0.94**	**0.81**	**0.61**	0.57	0.56	0.45	0.44	0.41	0.39	0.38	0.45	0.42	0.35	0.32	0.29	0.49
$MLLR_{ww}(DE, 50)$	0.89	0.71	0.53	0.53	0.54	0.58	0.59	0.67	0.71	0.78	0.90	0.92	0.96	0.98	0.98	**0.75**
$LLR_{ww}(PE, 50, k)$																
k = 50% *population*	0.31	0.43	0.65	0.72	0.78	0.84	0.86	0.91	0.94	0.96	**0.27**	**0.33**	**0.61**	**0.82**	**0.91**	0.69
k = 40% *population*	0.37	0.49	0.71	0.79	0.83	0.85	0.87	0.92	0.94	0.96	0.22	0.26	0.52	0.73	0.85	0.69
k = 30% *population*	0.44	0.58	0.78	0.84	0.87	0.86	0.88	0.93	0.95	0.96	0.16	0.20	0.39	0.60	0.73	0.68
k = 20% *population*	0.57	0.70	0.86	0.90	0.92	0.90	0.92	0.95	0.96	**0.98**	0.11	0.13	0.22	0.34	0.48	0.66
k = 15% *population*	0.65	0.77	0.90	0.93	0.94	0.92	0.92	0.95	**0.97**	**0.98**	0.09	0.10	0.15	0.23	0.35	0.66
k = 10% *population*	0.78	0.87	0.95	0.97	0.97	**0.93**	**0.93**	**0.96**	**0.97**	**0.98**	0.08	0.09	0.12	0.18	0.27	0.67
k = 5% *population*	**0.90**	**0.95**	**0.98**	**0.99**	**0.99**	0.41	0.42	0.46	0.47	0.52	0.08	0.08	0.09	0.11	0.15	0.51
$MLLR_{ww}(PE, 50)$	0.86	0.93	0.97	0.98	0.98	0.88	0.90	0.93	0.95	0.96	0.19	0.24	0.49	0.72	0.85	**0.79**
$LLR_{ww}(NN, 50, s)$																
s = 0.1	0.10	0.13	0.23	0.29	0.38	0.60	0.63	0.73	0.78	0.84	0.25	0.29	0.59	0.80	0.91	0.50
s = 0.25	0.12	0.16	0.28	0.35	0.43	0.62	0.66	0.75	0.80	0.86	0.26	0.31	0.60	0.82	0.92	0.53
s = 0.5	0.16	0.22	0.39	0.46	0.53	0.66	0.70	0.79	0.83	0.88	0.29	0.34	0.64	0.84	0.93	0.58
s = 1	0.43	0.51	0.66	0.72	0.74	0.76	0.78	0.85	0.88	0.91	0.38	0.44	0.71	0.87	**0.94**	0.71
s = 1.2	0.74	0.65	0.75	0.79	0.80	0.78	0.81	0.86	**0.89**	**0.92**	0.44	0.50	0.73	**0.88**	**0.94**	0.76
s = 1.4	0.83	0.77	0.81	0.83	0.83	**0.80**	**0.82**	**0.87**	**0.89**	0.91	0.50	0.56	**0.75**	**0.88**	0.93	0.80
s = 1.6	0.88	0.83	0.83	**0.85**	**0.84**	**0.80**	0.81	0.85	0.87	0.89	0.55	0.59	**0.75**	0.86	0.90	**0.81**
s = 1.8	0.91	0.87	**0.84**	**0.85**	0.83	0.77	0.79	0.82	0.83	0.86	0.57	**0.61**	0.72	0.82	0.85	0.80
s = 2	0.93	**0.88**	0.83	0.82	0.81	0.73	0.75	0.77	0.78	0.80	**0.58**	**0.61**	0.68	0.77	0.79	0.77
s = 2.4	0.93	**0.88**	0.78	0.76	0.73	0.63	0.65	0.66	0.65	0.66	0.56	0.57	0.57	0.63	0.61	0.68
s = 2.8	**0.94**	0.87	0.73	0.70	0.67	0.55	0.57	0.56	0.54	0.55	0.53	0.53	0.49	0.50	0.48	0.61
s = 4	**0.94**	0.84	0.64	0.61	0.59	0.47	0.47	0.44	0.42	0.42	0.48	0.47	0.39	0.37	0.34	0.53
s = 8	**0.94**	0.81	0.61	0.57	0.56	0.45	0.44	0.41	0.38	0.38	0.45	0.42	0.35	0.32	0.29	0.49
$MLLR_{ww}(NN, 50)$	0.91	0.83	0.77	0.78	0.77	0.73	0.75	0.81	0.83	0.87	0.51	0.54	0.68	0.82	0.90	0.77

Table 6

Estimated power of the parameter independent tests for global chain clustering.

		Fixed distance					Exponential distance					
	0.00	**0.5%**	**1%**	**2%**	**4%**	**8%**	**0.5%**	**1%**	**2%**	**4%**	**8%**	**average**
LR^h_{mm}(50%)	0.79	0.39	0.29	0.20	0.12	0.08	0.45	0.35	0.26	0.18	0.12	0.29
LR_{mm}(50%)	0.86	0.45	0.31	0.20	0.12	0.08	0.50	0.38	0.27	0.19	0.12	0.32
$LR_{wm}{}^h$(*ONE*, 50)	0.80	0.43	0.32	0.24	0.16	0.10	0.48	0.39	0.30	0.21	0.15	0.32
LR_{wm}(*ONE*, 50)	0.91	0.51	0.37	0.26	0.16	0.10	0.57	0.44	0.33	0.23	0.15	0.37
$LR_{wm}{}^h$(*AB*, 50)	0.79	0.41	0.32	0.23	0.16	0.10	0.48	0.38	0.29	0.21	0.15	0.32
LR_{wm} (*AB*, 50)	0.90	0.49	0.36	0.24	0.16	0.10	0.55	0.43	0.31	0.22	0.15	0.36
$LR_{wm}{}^h$ (*PB*, 50)	0.76	0.44	0.34	0.24	0.16	0.10	0.48	0.40	0.30	0.21	0.15	0.33
LR_{wm} (*PB*, 50)	0.85	0.50	0.36	0.25	0.16	0.10	0.55	0.43	0.32	0.22	0.15	0.35
LR_{ww} (*CB*, 50)	0.79	0.48	0.38	**0.28**	0.19	0.12	0.52	0.42	0.33	0.24	0.17	0.36
LR_{ww} (*DA*, 50)	0.85	0.47	0.35	0.25	0.16	0.10	0.51	0.41	0.31	0.22	0.15	0.34
LR_{ww} (*DP*, 50)	0.77	0.46	0.35	0.25	0.17	0.11	0.51	0.41	0.31	0.22	0.15	0.34
MLR_{ww} (*DE*, 50)	0.91	0.46	0.32	0.21	0.13	0.09	0.53	0.40	0.29	0.19	0.13	0.33
MLR_{ww}(*PE*, 50)	0.93	0.47	0.33	0.22	0.13	0.09	0.54	0.41	0.29	0.20	0.13	0.34
MLR_{ww}(*NN*, 50)	0.91	0.45	0.32	0.22	0.14	0.11	0.51	0.38	0.28	0.20	0.13	0.33
LLR^h_{wm} (*ONE*, 50)	0.82	0.42	0.32	0.24	0.17	0.12	0.50	0.40	0.30	0.23	0.16	0.33
LLR_{wm} (*ONE*, 50)	0.89	0.48	0.36	0.26	0.19	0.13	0.54	0.44	0.34	0.24	0.17	0.37
$LLR_{wm}{}^h$ (*AB*, 50)	0.75	0.37	0.29	0.22	0.16	0.11	0.44	0.36	0.28	0.21	0.15	0.30
LLR_{wm}(*AB*, 50)	0.87	0.45	0.34	0.25	0.18	0.12	0.51	0.41	0.31	0.23	0.17	0.35
$LLR_{wm}{}^h$(*PB*, 50)	0.58	0.34	0.26	0.19	0.14	0.10	0.37	0.31	0.24	0.18	0.14	0.26
LLR_{wm}(*PB*, 50)	0.74	0.45	0.34	0.26	0.18	0.12	0.50	0.41	0.32	0.24	0.17	0.34
LLR_{ww}(*CB*, 50)	0.44	0.35	0.31	0.26	**0.21**	**0.14**	0.36	0.33	0.28	0.22	0.17	0.28
LLR_{ww}(*DA*, 50)	0.59	0.36	0.31	0.25	0.19	0.13	0.40	0.34	0.28	0.22	0.16	0.29
LLR_{ww}(*DP*, 50)	0.43	0.34	0.29	0.25	0.20	**0.14**	0.35	0.32	0.27	0.22	0.17	0.27
$MLLR_{ww}$(*DE*, 50)	**1.00**	0.56	0.39	**0.28**	0.19	0.13	0.74	0.54	**0.39**	**0.26**	**0.18**	**0.42**
$MLLR_{ww}$(*PE*, 50)	0.99	0.49	0.36	0.24	0.16	0.10	0.60	0.45	0.33	0.23	0.15	0.37
$MLLR_{ww}$(*NN*, 50)	**1.00**	0.57	0.35	0.23	0.17	0.12	**0.82**	**0.58**	0.38	0.24	0.16	**0.42**
MEET(*DE*)	0.99	**0.62**	**0.41**	0.26	0.17	0.11	0.74	0.56	0.38	0.25	0.17	**0.42**

Table 7

Estimated power of the parameter independent tests for hot-spot clusters.

	Rural clusters					Mixed clusters					Urban clusters					
	1	**2**	**4**	**8**	**16**	**1**	**2**	**4**	**8**	**16**	**1**	**2**	**4**	**8**	**16**	**average**
LR_{mm}^{h} (50%)	**1.00**	**0.99**	0.97	0.97	0.97	0.94	0.94	0.94	0.94	0.95	0.92	0.90	0.89	0.91	0.93	0.94
LR_{mm} (50%)	0.99	**0.99**	0.95	0.95	0.94	0.89	0.89	0.89	0.90	0.91	0.86	0.84	0.84	0.88	0.89	0.91
LR_{wm}^{h} (*ONE*, 50)	0.99	**0.99**	0.97	0.97	0.97	0.94	0.94	0.95	0.96	0.96	0.89	0.90	0.90	0.94	0.96	0.95
LR_{wm}(*ONE*, 50)	0.99	**0.99**	0.95	0.95	0.95	0.90	0.91	0.92	0.93	0.94	0.85	0.85	0.85	0.91	0.93	0.92
LR_{wm}^{h} (*AB*, 50)	0.99	**0.99**	0.97	0.97	0.97	0.94	0.94	0.95	0.96	0.97	0.76	0.79	0.80	0.88	0.93	0.92
LR_{wm}(*AB*, 50)	0.99	0.98	0.95	0.95	0.95	0.90	0.91	0.92	0.93	0.94	0.69	0.72	0.74	0.84	0.89	0.89
LR_{wm}^{h} (*PB*, 50)	0.99	0.97	0.94	0.94	0.94	0.95	0.95	0.95	0.96	0.96	0.95	0.95	0.96	0.98	0.98	**0.96**
LR_{wm}(*PB*, 50)	0.98	0.97	0.91	0.92	0.90	0.92	0.92	0.93	0.93	0.94	0.91	0.91	0.92	0.95	0.96	0.93
LR_{ww}(*CB*, 50)	0.99	0.97	0.93	0.93	0.93	0.87	0.88	0.90	0.92	0.94	0.80	0.81	0.83	0.95	0.95	0.91
LR_{ww}(*DA*, 50)	0.99	0.98	0.94	0.95	0.95	0.90	0.90	0.92	0.93	0.95	0.67	0.68	0.70	0.84	0.91	0.88
LR_{ww}(*DP*, 50)	0.98	0.95	0.90	0.91	0.89	0.92	0.91	0.92	0.93	0.94	0.90	0.88	0.90	0.96	0.98	0.92
MLR_{ww}(*DE*, 50)	**1.00**	**0.99**	0.95	0.94	0.93	0.93	0.91	0.91	0.92	0.93	0.93	0.93	0.91	0.94	0.94	0.94
MLR_{ww}(*PE*, 50)	0.99	**0.99**	**0.98**	**0.98**	0.97	0.93	0.94	0.95	0.96	0.97	0.86	0.83	0.82	0.89	0.93	0.93
MLR_{ww}(*NN*, 50)	**1.00**	**0.99**	0.95	0.94	0.93	0.93	0.92	0.91	0.92	0.93	0.92	0.88	0.82	0.87	0.91	0.92
LR_{mm}^{h}(10%)	**1.00**	**0.99**	0.97	0.97	**0.98**	0.94	0.94	0.95	0.95	0.96	0.92	0.91	0.88	0.75	0.55	0.91
LR_{mm}(10%)	**1.00**	**0.99**	0.96	0.95	0.95	0.91	0.91	0.91	0.91	0.92	0.86	0.86	0.84	0.65	0.46	0.87
LR_{wm}^{h} (*ONE*, 10)	**1.00**	**0.99**	**0.98**	**0.98**	**0.98**	0.95	0.96	0.96	0.97	0.97	0.91	0.91	0.87	0.72	0.53	0.91
LR_{wm}(*ONE*, 10)	0.99	**0.99**	0.96	0.96	0.96	0.92	0.92	0.93	0.94	0.95	0.86	0.87	0.81	0.65	0.50	0.88
LR_{wm}^{h}(*AB*, 10)	0.99	**0.99**	**0.98**	**0.98**	**0.98**	0.95	0.95	0.96	0.97	0.97	0.75	0.77	0.66	0.45	0.31	0.84
LR_{wm}(*AB*, 10)	0.99	**0.99**	0.96	0.96	0.96	0.92	0.93	0.93	0.94	0.95	0.69	0.72	0.62	0.44	0.38	0.82
LR_{wm}^{h} (*PB*, 10)	0.99	0.98	0.96	0.96	0.96	**0.97**	**0.97**	**0.97**	**0.98**	**0.98**	**0.97**	**0.96**	0.95	0.91	0.78	0.95
LR_{wm}(*PB*, 10)	0.98	0.97	0.93	0.94	0.93	0.94	0.94	0.95	0.95	0.95	0.93	0.93	0.91	0.82	0.66	0.92
LR_{ww}(*CB*, 10)	0.99	0.97	0.95	0.96	0.96	0.91	0.92	0.94	0.95	0.96	0.91	0.90	0.87	0.77	0.64	0.91
LR_{ww}(*DA*, 10)	0.99	0.98	0.96	0.96	0.96	0.93	0.93	0.94	0.95	0.96	0.78	0.77	0.63	0.47	0.39	0.84
LR_{ww}(*DP*, 10)	0.98	0.96	0.91	0.92	0.91	0.94	0.93	0.94	0.95	0.96	**0.97**	0.95	0.92	0.84	0.68	0.92
MLR_{ww}(*DE*, 10)	**1.00**	**0.99**	0.96	0.96	0.95	0.93	0.92	0.93	0.94	0.95	0.93	0.93	0.89	0.75	0.55	0.91
MLR_{ww}(*NN*, 10)	**1.00**	**0.99**	0.96	0.96	0.96	0.95	0.93	0.93	0.94	0.96	0.92	0.88	0.78	0.58	0.42	0.88
LLR_{wm}^{h} (*ONE*, 50)	0.53	0.59	0.63	0.68	0.72	0.71	0.69	0.75	0.76	0.82	0.66	0.70	0.80	0.90	0.93	0.72
LLR_{wm}(*ONE*, 50)	0.31	0.37	0.44	0.49	0.52	0.66	0.66	0.73	0.76	0.82	0.52	0.58	0.77	0.90	0.94	0.63
$LLRwm^{h}$ (*AB*, 50)	0.63	0.74	0.84	0.87	0.89	0.70	0.68	0.74	0.76	0.82	0.39	0.43	0.47	0.65	0.69	0.69
LLR_{wm}(*AB*, 50)	0.37	0.48	0.62	0.66	0.68	0.64	0.65	0.72	0.75	0.81	0.37	0.42	0.63	0.81	0.88	0.63
LLR_{wm}^{h} (*PB*, 50)	0.03	0.02	0.02	0.02	0.02	0.37	0.35	0.39	0.38	0.42	0.91	0.91	**0.97**	**0.99**	**0.99**	0.45
LLR_{wm}(*PB*, 50)	0.07	0.07	0.10	0.11	0.12	0.51	0.52	0.59	0.61	0.67	0.78	0.81	0.93	0.97	0.98	0.52
LLR_{ww}(*CB*, 50)	0.09	0.11	0.19	0.24	0.31	0.49	0.53	0.64	0.70	0.77	0.31	0.37	0.67	0.87	0.95	0.48

NIH-PA Author Manuscript
NIH-PA Author Manuscript

	Rural clusters					Mixed clusters					Urban clusters					
	1	2	4	8	16	1	2	4	8	16	1	2	4	8	16	average
$LLR_{ww}(DA, 50)$	0.17	0.24	0.44	0.52	0.60	0.57	0.61	0.71	0.76	0.83	0.21	0.25	0.52	0.75	0.88	0.54
$LLR_{ww}(DP, 50)$	0.06	0.06	0.08	0.09	0.11	0.42	0.45	0.55	0.60	0.67	0.48	0.56	0.85	0.96	**0.99**	0.46
$MLLR_{ww}(DE, 50)$	0.89	0.71	0.53	0.53	0.54	0.58	0.59	0.67	0.71	0.78	0.90	0.92	0.96	0.98	0.98	0.75
$MLLR_{ww}(PE, 50)$	0.86	0.93	0.97	**0.98**	**0.98**	0.88	0.90	0.93	0.95	0.96	0.19	0.24	0.49	0.72	0.85	0.79
$MLLR_{ww}(NN, 50)$	0.91	0.83	0.77	0.78	0.77	0.73	0.75	0.81	0.83	0.87	0.51	0.54	0.68	0.82	0.90	0.77
$MEET(DE)$	0.20	0.22	0.23	0.21	0.23	0.93	0.90	0.84	0.82	0.83	0.94	0.92	0.96	0.98	**0.99**	0.68