Supporting Information

Supporting Information is provided with this submission.

Supporting Figure S1

For constant applied RF excitation field $B_1$ with no $R_1$ relaxation or off-resonance, $R_2$ determines the excitation dynamic for any pulse duration $p$ or nominal excitation angle $\alpha = \gamma B_1 p$. The effect varies over the range of relative $R_2$ (thick lines). To achieve any particular $\alpha < 2\pi$, the rotation is truncated at the appropriate $p$ (blue → yellow gradient). For magnetization across the range of $R_2$, the final orientations (dashed lines tracing the range over $R_2$) can differ greatly from the nominal excitation angle (dotted lines).

Supporting Figure S2

The trace of angles from 0° to 90° can be used to compute efficiency of a hard pulse excitation for any $T_2$ and nominal excitation angle $\alpha$. Excitation efficiency is $M_{xy}/\sin(\alpha)$ for $M_{xy}$ immediately following the excitation.

Supporting Figure S3

The trace of angles from 0° to 90° can be used to chart efficiency of a hard pulse saturation for any $T_2$ and nominal saturation angle $\alpha$. Saturation efficiency is $M_{xy}/(1 - \cos(\alpha))$ for $M_{xy}$ immediately following the saturation.
Supporting Figure S4

The trace of angles from 90° to 180° can be used to compute effectiveness of a hard pulse inversion for any $T_2$ and nominal inversion angle $\alpha$. Inversion effectiveness is $M_z/\cos(\alpha)$ for $M_z$ immediately following the inversion.
Supporting Information for Improved Cortical Bone Specificity in UTE MR Imaging

AUTHORS - Ethan M. Johnson, Urvi Vyas, Pejman Ghanouni, Kim Butts Pauly, John M. Pauly

Magnetic Resonance Systems Research Laboratory, Department of Electrical Engineering, Stanford University, Stanford, California.

Address correspondence to:
Ethan Johnson
Packard Electrical Engineering, Room 310
350 Serra Mall
Stanford, CA 94305-9510
TEL: (650) 725-7005
E-MAIL: ethanjohnson@stanford.edu

This work was supported by NIH P01 CA159992 and P41 EB015891 and by GE Medical Systems.

Approximate Word Count: 772

Submitted September 22, 2015, to Magnetic Resonance in Medicine as a Supporting Information for Improved Cortical Bone Specificity in UTE MR Imaging.
Concurrent excitation and relaxation dynamics

During RF excitation by constant applied $B_1$, if there is no $R_1$ relaxation or off-resonance, the ratio of $R_2$ to $R_2^{\text{crit}}$ or ‘relative $R_2$’ determines the excitation dynamic. Tracing the nominal $2\pi$ rotation induced by the field over a duration $(\gamma B_1)^{-1} = 2T_2^{\text{crit}}$ fully illustrates the magnetization orientations (Sup. Fig. S1) effected by any pulse duration $p$ or nominal excitation angle $\alpha = \gamma B_1 p$. Therefore this trace describes the effect of any hard pulse; the $90^\circ$ rotation discussed in *Improved Cortical Bone Specificity in UTE MR Imaging* is a special case of such. Without incorporation of other considerations, such as off-resonance or spatial selectivity, but given a maximum-$B_1$ constraint, a hard pulse is the maximally-efficient excitation, *i.e.* it is the best-possible approximation to an ideal excitation.

Excitations, saturations and inversions

The trace of excitation effects can be used to chart the efficacy of excitation, saturation and inversion hard pulses, which vary with $T_2$ and nominal excitation angle $\alpha$. An ideal excitation tips magnetization to angle $\alpha$ in the range of 0 to $\pi/2$, which creates transverse magnetization of $\sin(\alpha)$. The excitation efficiency of a real excitation is then $M_{xy}/\sin(\alpha)$ for $M_{xy}$ immediately following the excitation (Sup. Fig. S2). For example, a pair of excitations using 14 $\mu$T and 1.4 $\mu$T pulses for $22.5^\circ$ tip angles might be used to target magnetization from collagen-bound water in compact bone, which is attributed a $T_2$ of 0.4 ms. The pulses impose critical $T_2$ rates of 0.84 ms and 8.4 ms; the targeted magnetization then has relative $T_2$ of 0.48 and 0.05, and it is excited with efficiency of 87% and 35% under the excitations. If magnetization with $T_2$ of 10 ms is also present, it will be excited with efficiencies of 99% and 95%. Therefore a difference image will give the targeted $T_2$ an intensity weighting more than $10 \times$ that of the longer $T_2$.

Analogously, a saturation pulse is intended to diminish the $M_z$ component of magne-
tization by tipping the magnetization to angle $\alpha$ in the range of 0 to $\pi/2$, which ideally leaves a longitudinal component of $1 - \cos(\alpha)$. Therefore the saturation efficiency of a real saturation pulse is $(1 - M_z)/(1 - \cos(\alpha))$ for $M_z$ immediately following the saturation (Sup. Fig. S3). This can be used, for example, to specify parameters for a simple rectangular long-$T_2$ suppression pulse, as well as to delineate its fundamental performance limits. To suppress longer-$T_2$ magnetization while retaining that with shorter-$T_2$, high and low efficiencies would be desired at the longer and shorter $T_2$ values respectively. For 10 ms and 0.4 ms $T_2$ magnetizations, the $B_1$ amplitude and tip angle can be chosen by centering the 1.4 decade difference in relative $T_2$ over the steepest change in efficiency. On this principle, $B_1$ should be set to place the longer $T_2$ just below the critical threshold and a tip angle near 90° should be used. With a 1.07 $\mu$T, 85° saturation pulse applied, the remaining normalised $M_z$ for the short-$T_2$ component will be 0.632 greater than that of the long-$T_2$ component. This can additionally be used to facilitate accurate $T_1$ mapping for short-$T_2$ components by saturation-recovery-type methods, as estimation of $T_1$ is premised upon knowledge of the $M_z$ remaining after the interrogation pulse.

Finally, an inversion ideally flips magnetization to an angle $\alpha$ in the range of $\pi/2$ to $\pi$ so that its polarity is negated. The effectiveness of an inversion is $M_z/\cos(\alpha)$ for $M_z$ immediately following the inversion (Sup. Fig. S4), which is 1 when maximally effective or is negative when ineffective. Analysis of contrast in pulse sequences that apply inversion pulses with short-$T_2$ magnetization present depends upon characterization of the influence that an inversion pulse has upon the short-$T_2$ components. In such analysis, the $\cos(\alpha)$ term that typically characterises inversion effects must be scaled by the efficacy to accurately represent $M_z$. For instance, a sequence that applies an inversion pulse and then performs one or more excitations and encodes will have some $T_2$-dependence in its $T_1$ contrast. Setting sequence parameters such as inversion time or flip angle to exploit the dependence necessitates incorporation of the inversion effectiveness. Following a 180° inversion pulse using $B_1$ of 20 $\mu$T, magnetization with $T_2$ of 0.4 ms, a relative $T_2$ of 0.7, is actually inverted with effectiveness of 0.46, which is much greater than 0, which might otherwise be assumed for a short $T_2$. 

2
As a consequence, if choosing parameters under an assumption of no inversion for short-$T_2$ components, an inversion time may be chosen that unintentionally nulls the component of interest!
List of Figures

S1 Excitation dynamics with significant $T_2$ and negligible $T_1$ . . . . . . . . . . 5
S2 Excitation efficiency with significant $T_2$ and negligible $T_1$ . . . . . . . . . . 6
S3 Saturation efficiency with significant $T_2$ and negligible $T_1$ . . . . . . . . . . 7
S4 Inversion effectiveness with significant $T_2$ and negligible $T_1$ . . . . . . . . . 8
Supporting Figure S1: For constant applied RF excitation field $B_1$ with no $R_1$ relaxation or off-resonance, $R_2$ determines the excitation dynamic for any pulse duration $p$ or nominal excitation angle $\alpha = \gamma B_1 p$. The effect varies over the range of relative $R_2$ (thick lines). To achieve any particular $\alpha < 2\pi$, the rotation is truncated at the appropriate $p$ (blue → yellow gradient). For magnetization across the range of $R_2$, the final orientations (dashed lines tracing the range over $R_2$) can differ greatly from the nominal excitation angle (dotted lines).
Supporting Figure S2: The trace of angles from 0° to 90° can be used to compute efficiency of a hard pulse excitation for any $T_2$ and nominal excitation angle $\alpha$. Excitation efficiency is $M_{xy}/\sin(\alpha)$ for $M_{xy}$ immediately following the excitation.
Supporting Figure S3: The trace of angles from 0° to 90° can be used to chart efficiency of a hard pulse saturation for any $T_2$ and nominal saturation angle $\alpha$. Saturation efficiency is $M_{xy}/(1 - \cos(\alpha))$ for $M_{xy}$ immediately following the saturation.
Supporting Figure S4: The trace of angles from 90° to 180° can be used to compute effectiveness of a hard pulse inversion for any $T_2$ and nominal inversion angle $\alpha$. Inversion effectiveness is $M_z / \cos(\alpha)$ for $M_z$ immediately following the inversion.